Circle of fifths (Ger. Quintenzirkel)

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https://doi.org/10.1093/gmo/9781561592630.article.05806

Published in print: 20 January 2001

Published online: 2001

The arrangement of the tonics of the 12 major or minor keys by ascending or descending perfect 5ths, thus making a closed circle:

$$C-G-D-A-E-B-F\sharp = G\flat-D\flat-A\flat-E\flat-B\flat-F-C$$

Such an arrangement is dependent on an Enharmonic relationship somewhere in the circle; this is usually reckoned at F^{\sharp}/G^{\flat} for the major keys and at D^{\sharp}/E^{\flat} for the minor keys. Normally the system of Equal temperament is assumed for the circle of 5ths, with every note having an infinite number of enharmonic equivalents ($B^{\sharp} = C = D^{\flat} \flat$), though it is possible to use a tuning system in which certain 5ths are greater than the 12th part of the circle (e.g. pure 5ths) and others are commensurately smaller, so that the octave is still a closed circle. But in Pythagorean intonation, the system based entirely on pure 5ths, the 'circle' is open, since this system does not admit enharmonic equivalents. In the arrangement C - G - D - A - E - B - F # - C # - D # - A # - E # - B #, if all the 5ths are pure, then B^{\sharp} will be slightly higher than C. For this reason the arrangement of 5ths in the Pythagorean system can be represented by a spiral or coil, but not by a closed circle.

The circle of 5ths was first described by Heinichen in *Der General-bass* (1728), and has been used by theorists as a way of illustrating the relative harmonic 'remoteness' of one key from another, that is, the number of 5ths by which two notes are separated along the circle. This method of reckoning works well for the three primary harmonic functions – tonic, subdominant and dominant – but is at odds with the belief that supertonic relationships (e.g. between C and D, in C major), which are only two perfect 5ths 'wide', are in fact harmonically more remote than mediant relationships, which are three or four perfect 5ths wide.

Of the few compositions that circumnavigate the circle of 5ths, the two preludes 'through all the major keys' by Beethoven (op.39, ?1789) are the most famous.