A TRANSFORMATIONAL APPROACH TO JAZZ HARMONY

Michael McClimon

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Accepted by the Graduate Faculty, Indiana University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Doctoral Committee

Julian Hook, Ph.D.

Kyle Adams, Ph.D.

Blair Johnston, Ph.D.

Brent Wallarab, M.M.

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Michael McClimon

A TRANSFORMATIONAL APPROACH TO JAZZ HARMONY

Harmony is one of the most fundamental elements of jazz, and one that is often taken for granted in the scholarly literature. Because jazz is an improvised music, its harmony is more fluid and potentially more complex than that of other, notated traditions. Harmony in common-practice jazz (c. 1940–1965) is typically represented by chord symbols, which can be actualized by performers in any number of ways, and which might change over the course of a single performance.

This dissertation presents a transformational model of jazz harmony that helps to explain this inherent complexity. While other theories of jazz harmony require transcriptions into notation, the transformational approach enables analysis of chord symbols themselves. This approach, in which chord symbols are treated as first-class objects, is consistent with the way jazz harmony is usually taught, and with the way jazz musicians usually discuss harmony. Though transformational theory has been applied to later jazz, the aim of this study is rather different: the music under consideration here might be called "tonal jazz," in which functional harmonic progressions are still the rule.

After a general introduction, the first chapter introduces the transformational approach by developing a diatonic seventh-chord space. Chapter 2 expands this diatonic space to a fully chromatic space that focuses on the ii–V–I progression, laying the foundation for much of the work that follows. Chapter 3 extends the model to examine music in which root motion by thirds plays an important role, paying special attention to the way in which harmonic substitution interacts with more normative jazz harmony. Since the pioneering work of George Russell in the 1950s, many jazz musicians have drawn an equivalence between chords and scales; Chapter 4 develops a transformational approach to these chord-scales, enabling analyses of improvisations on tunes first analyzed in the preceding chapters. The final chapter centers on a single harmonic archetype, Rhythm changes, and brings together the theoretical framework in a series of analyses featuring solos by Johnny Griffin, Thelonious Monk, George Coleman, Sonny Rollins, and Sonny Stitt.

Julian Hook, Ph.D.

Kyle Adams, Ph.D.

Blair Johnston, Ph.D.

Brent Wallarab, M.M.

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5 Rhythm Changes

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Notes to the Reader

References to recordings in this dissertation are generally made only by giving the performer's name and album title; complete identifying information can be found in the discography.

When discussing a particular piece of jazz, the words "piece," "composition," and "work" all seem out of place. In general, I have adopted the word "tune" to mean roughly "the basic structure of a work, including (primarily) its melody and chord changes." This is in keeping with the way jazz musicians use the word: they may refer to a "16-bar tune," a "Rhythm tune," or "one of my favorite tunes" (all referring to the abstract structure of the tune and not simply the melody). When I am referencing a *particular* instantiation of a work (e.g. Bill Evans's recording of "Autumn Leaves" from *Portrait in Jazz*), I will make that clear.

In running text, dominant seventh chords are indicated with just a "7," major sevenths with "maj7," minor sevenths with "m7," and half-diminished sevenths with "m7♭5". In a minor key the tonic chord is often played with a major seventh, which is indicated "mM7." The progression Dm7–G7–Cmaj7 thus indicates a D minor seventh moving to a G dominant seventh moving to a C major seventh chord. In examples, chord symbols typically follow conventions used by *The Real Book*.

There are two ways mathematicians notate function composition: left-to-right and right-to-left. When combining two functions, f followed g, right-to-left orthography writes $g(f(x))$, while left-to-right orthography writes $(fg)(x)$. Right-to-left orthography is familiar to most readers, and we will use it when talking about transformations as functions: statements of the form $f(x)$ should be read from right to left. When discussing strings of transformations, however, right-to-left orthography can be confusing: $L(P(x))$ indicates the P operation applied to some object x, followed by the L operation (it is easy to see how the problem can proliferate when there are multiple operations involved). To avoid this problem, we will use left-to-right orthography denoted by the symbol $\bullet$; the operation P followed by the operation L is notated $P \bullet L$.[1] This combination of notation seems intuitive, but we will be explicit in situations where the orthography may be confusing.

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[1] This use follows that of Julian Hook; see *Exploring Musical Spaces* (New York: Oxford University Press, forthcoming), Section 2.3.

Chapter 1
Introduction

1.1 Problems of Jazz Analysis

When compared to a score by Beethoven (for example), the jazz lead sheet appears strikingly bare. The Beethoven score specifies nearly everything one might need to know in order to perform it. Though the minor details—dynamics, articulations, phrasing marks, and the like—will differ from piece to piece, we can usually depend on the presence of some basic information. It is rare for traditional scores not to include the instrumentation, for example, and a score that did not include the number of measures or which notes to play in combination with which which other notes would be very unusual indeed.

And yet, this is the usual state affairs for the jazz lead sheet, which is probably the most common form of a "jazz score."[1] Most lead sheets only include the basic outline of a melody, along with a set of "changes" that prescribe the harmonic structure of a piece. Beyond this most basic instruction, every other aspect is left up to the performers. Of these two elements (melody and harmony), harmony has a much larger role in determining the course of a particular jazz performance, so it seems appropriate to focus our analytical attention on it.

Jazz is essentially a harmonic music. In a typical jazz performance, the melody of the piece is heard only twice (at the beginning and the end), while the harmonic structure is heard throughout, determining the structure of the performance. Each soloist typically plays one or more "choruses," where each chorus is understood as a single iteration of the piece's harmonic structure. In marked contrast to a Beethoven score, jazz compositions usually remain unspecified when it comes to their contrapuntal structure: performers will typically improvise counterpoint that fits with the underlying harmonic framework. Harmony is the main restraining factor of a piece, and its primary method of coherence.

---

[1] This is not to say, of course, that there are not jazz compositions that do specify these minor details. These compositions are the exception, rather than the rule, in the music in which this study is interested.

The word "jazz"—which has been used at various times to describe McKinney's Cotton Pickers, Benny Goodman, Sun Ra, John Zorn, Tito Puente, and Brad Mehldau, among many others—is inescapably vague, so it will be useful at this point to delimit the terms of this study somewhat. Here I am interested in in what might be called "tonal jazz," which begins in the swing area and continues through hard bop, covering roughly the years 1940–1965. In this music, functional harmonic progressions are the norm; "tonal jazz" is meant in opposition to "modal jazz," where the rate of harmonic change is slower and the harmony is mostly non-functional.[2] This includes much of the music that most people think of when they hear the word "jazz," including big-band swing (Count Basie, much of Duke Ellington's music), bebop (Charlie Parker, Dizzy Gillespie, Thelonious Monk), and the mainstream jazz that followed bebop, known variously as "hard bop" or "post-bop" (John Coltrane, Sonny Rollins, Bill Evans, and many others). I intend the dates to be flexible, especially on the later end; given the strong influence of the bebop tradition on jazz and jazz pedagogy, the hard bop style continued to exist well beyond 1965, and many players today still play in the style.[3]

Now that we have delineated "jazz," we should explain exactly what we mean by "harmony." Harmony is of course one of the oldest topics in music theory, and as such has been hotly contested throughout its history. It is often found in opposition to counterpoint; in this view, counterpoint is concerned with individual melodic voices, while harmony is concerned with individual verticalities. In other traditions (most notably the Schenkerian tradition), harmony is understood to be an outgrowth of counterpoint: verticalities arise primarily through contrapuntal procedures. Furthermore, study of harmony is often broken down by genre: "tonal harmony" plays a different role than does "chromatic harmony" in both theoretical research and pedagogy.[4]

---

[2] Miles Davis's "So What" is probably the most well-known modal jazz piece; it is a 32-bar tune in which the first chord, Dm, lasts 16 bars, moves to E♭m for 8 bars, and back to Dm for the final 8. The term also describes other similar pieces, including the rest of Davis's album *Kind of Blue*, John Coltrane's recording of "My Favorite Things," and Herbie Hancock's "Maiden Voyage."

[3] As Scott DeVeaux puts it, bebop is "both the source of the present . . . and the prism through which we absorb the past. To understand jazz, one must understand bebop." *The Birth of Bebop: A Social and Musical History* (Berkeley: University of California Press, 1997), 3.

When jazz musicians refer to "harmony," they are typically referring to the changes themselves; that is, the chord symbols given on a lead sheet or arrangement. Even when they are not playing from sheet music, the chord symbol is the basic unit of harmonic understanding for jazz musicians. The reason for this is largely practical: a chord symbol is a concise way of referring to a particular sound, and improvising musicians must be able to understand this information quickly (when reading) and to recall it easily (when improvising).

Since the pioneering work of George Russell in the late 1950s, many jazz musicians conceive of an equivalence between a harmony (a chord symbol) and a scale.[5] The chord symbol Dm7 might imply a D dorian scale, for example, rather than simply the notes D–F–A–C. Because any of the notes of this scale will sound relatively consonant over a Dm7 chord, the chord symbol acts as a convenient shorthand for a particular "way of playing" for a jazz improvisor. This equivalence between chords and scales will be the focus of Chapter 4; for now it enough to note that understanding jazz harmony often involves more than understanding relationships between four-voice seventh chords.

When analyzing jazz harmony, it is often difficult to determine exactly what one should be analyzing. Lead sheets as circulated in fake books can be highly inaccurate, and often cannot be relied upon as a single source for any particular jazz performance, since it is rare that performers play directly from a lead sheet with no modifications.[6] In the case of jazz standards which may have originated elsewhere, we might wonder whether should we analyze the original sources. In many cases, however, the "jazz standard" version may be significantly different from the original version, reflecting a history of adaptation by generations of jazz musicians.[7] To make matters worse for the hopeful academic, this knowledge is often secret knowledge, not written down and learned only from more experienced musicians.

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[4] This is not meant to imply that "chromatic harmony" is not tonal; rather, studies that focus specifically on chromatic harmony often differentiate themselves from other tonal theoretical traditions.

[5] George Russell, *The Lydian Chromatic Concept of Tonal Organization*, 4th ed., vol. 1, *The Art and Science of Tonal Gravity* (Brookline, MA: Concept, 2001).

[6] Fake books are collections of lead sheets that traditionally were compiled anonymously and sold illegally, in order to avoid paying the copyright owners of the compositions they contained. The name "fake book" comes from the fact that with the melody and chord changes, jazz musicians can easily "fake" a tune they do not know. The most famous jazz fake book is ironically titled *The Real Book*, and was compiled in Boston in the early 1970s. In recent years, fake books have become mainstream, and most of them have now obtained proper copyright permissions. Hal Leonard now publishes the 6th edition of *The Real Book* (a nod to the five illegal editions); many of the notorious errors in the earlier editions have been corrected and it is now available for purchase legally. Further references to *The Real Book* in this document refer to this version unless otherwise noted. For a history of fake books, see Barry Kernfeld, *The Story of Fake Books: Bootlegging Songs to Musicians* (Lanham, MD: Scarecrow Press, 2006).

Many published jazz analyses rely on transcriptions of particular performances as a way to avoid some of these issues. In general, this solution works well, and I will certainly make use of transcriptions from time to time. This study, however, is interested in harmony more generally, and transcriptions can confuse matters somewhat. The kinds of questions I am interested in answering are of the type "What can we say about harmony in the piece 'Autumn Leaves?'" and less often of the type "What can we say about Bill Evans's use of harmony in the recording of 'Autumn Leaves' from *Portrait in Jazz*?" Furthermore, even transcriptions are not definitive when it comes to harmony: the pianist and guitarist might not be playing the same chord; the soloist might have a different harmony in mind than the rhythm section; or the bass player might play a bass line in such a way that affects our perception of the chordal root. Even in the course of a single performance, a group might alter a tune's harmonic progression, perhaps preferring some substitutions during solos and others during the head.[8]

This is a problem without one clear solution, and it may make more sense to use one method or another depending on the situation. Some compositions have canonical recordings—Coleman Hawkins's recording of "Body and Soul," for example—and in those circumstances determining the changes is usually unproblematic. Other compositions are more fluid, and different choruses might alter the basic structure within the course of a single performance (substituting a V7♭9 for a V13♯11 chord, for example). In these cases, I am interested in what Henry Martin has called the

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[7] By the time they become standards, many non-jazz compositions (Tin Pan Alley songs or traditional songs like "Back Home Again in Indiana") have been adapted, typically to additional harmonic motion to provide interest during solos. Some examples are more radical: the tune "Alice in Wonderland" (analyzed below), for example, is known to jazz musicians as a jazz waltz with one chord per bar. The main title music in the 1951 film from which it was taken, however, is in 4/4 with a relatively slower harmonic rhythm.

[8] The "head" is what jazz musicians call the statements of the melody in the course of a jazz performance. This melody is typically played once at the beginning of a performance and again at the end (often referred to as the "out head").

"ideal changes"; a hypothetical set of chords that we can use as a basis for understanding the many variations that might occur in actual performance.[9] These changes represent a sort of Platonic model of a composition; individual performances of "Autumn Leaves" can be seen as instances of some ideal AUTUMN LEAVES. Determining these ideal changes is often a process of mediating published lead sheets, recorded versions, and other sources; throughout this study I have tried to clarify exactly *what* harmony I am analyzing in any given example.[10]

In an attempt to answer some of these questions, this dissertation presents a transformational model of jazz harmony. While on the surface a transformational model may seem abstract and far-removed from the concerns of performing jazz musicians, harmony in jazz fits together nicely with David Lewin's famous "transformational attitude."[11] A jazz musician does not typically think of harmonies as a series of points in space, but rather as a series of "characteristic gestures" between them. Rather than focusing on an underlying tonality, a jazz musician often tries to "make the changes"—to fully engage with the sound of each individual harmony.

There is often quite a large gap between the way jazz is most commonly taught (in jazz studios and pedagogical books) and the way it has traditionally been understood by music theorists. Another goal of the present study is to use transformational methods in an attempt to narrow this gap, by bringing theoretical and mathematical rigor to materials that are often ignored by the theory community, and by applying established theoretical principles in a way that corresponds closely with the understanding of jazz musicians.

While we can never claim to know what jazz musicians think, we might get somewhat closer to an answer by examining jazz pedagogical materials. In the late 1960s, jazz began to be accepted into the academy, and many young jazz musicians began learning to play the music in schools, rather than exclusively from older musicians.[12] To supplement this teaching, a great deal of pedagogical material has appeared that aims to teach young musicians how to play jazz.

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[9] Henry Martin, *Charlie Parker and Thematic Improvisation* (Lanham, MD: Scarecrow Press, 1996), 5–6. See also David J. Heyer, "Vocabulary, Voice Leading, and Motivic Coherence in Chet Baker's Improvisations" (PhD diss., University of Oregon, 2011), 36–39.

[10] In places where I refer to a "tune" generically, I have provided at least two references to relatively straight-ahead recorded examples in the Appendix A.

[11] David Lewin, *Generalized Musical Intervals and Transformations* (New York: Oxford University Press, [1987] 2007), 159 (hereafter *GMIT).*

Unfortunately, there is little interaction between these pedagogical materials and music theoretical materials. Pedagogical materials, such as the recent *Berklee Book of Jazz Harmony*, often do not have bibliographies or mention recent work in the theoretical literature.[13] Likewise, most theoretical work does not refer to these harmonic handbooks which are staples of jazz education. Mark Levine's *Jazz Theory Book*, widely regarded in the jazz education world as *the* book on jazz theory, does not appear in any of the bibliographies in a special jazz issue of *Music Theory Online* (18.3), for example.[14]

In recent years the theory community has embraced pedagogical materials as a means of uncovering how historical musicians might have thought about their own music.[15] Though jazz pedagogues do not typically publish articles in music theory journals or otherwise consider themselves to be "music theorists," per se, the goal of their harmonic textbooks is quite similar to the goals of pedagogical books in music theory: to teach students how to think (or hear, or perform) in a particular style of music.

David Lewin points out in his introduction to transformational theory that when considering a particular musical passage, we often "conceptualize along with it, as one of its characteristic textural features, a family of directed measurements, distances, or motions of some sort."[16] I certainly hear these characteristic motions when I listen to jazz, and I think it is these motions that jazz pedagogues are emphasizing when they teach students to "make the changes." Despite its somewhat hostile mathematical appearance, transformational theory is an effective means of exploring these families of intuitions. Modeling sets of chord changes as transformations between harmonic objects, for example, allows the theoretical discourse to draw on the ways in which jazz musicians teach harmony, and can bring these two disparate areas somewhat closer together.

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[12] The Thelonious Monk Institute of Jazz maintains a web page on the history of jazz education in America at http://www.jazzinamerica.org/JazzResources/JazzEducation/Page.

[13] A noted exception here is Andy Jaffe's *Jazz Harmony* (Tübingen: Advance Music, 1996), which features an extended bibliography that includes many theoretical works.

[14] This issue of *Music Theory Online* is a Festschrift in memory of Steve Larson (September 2012), guest edited by Stephen Rodgers, Henry Martin, and Keith Waters.

[15] See, for example, Robert Gjerdingen, *Music in the Galant Style* (New York: Oxford University Press, 2007); and Giorgio Sanguinetti, *The Art of Partimento: History, Theory, and Practice* (New York: Oxford University Press, 2012).

[16] Lewin, *GMIT*, 16.

1.2 Theoretical Approaches to Jazz Harmony

Studies of jazz harmony in recent years have primarily taken the form of Schenkerian analyses that seek to uncover large-scale voice-leading structures in order to define tonality. Schenkerian analysis has proven to be an extremely useful tool for analyzing tonal music, and Steve Larson's pioneering work in applying its methods to jazz has undoubtedly expanded the field of jazz studies and brought jazz analysis into the theoretical mainstream. While theorists may disagree on exactly *how* we should apply these Schenkerian techniques, hardly anyone seems to doubt that they are the best way to examine tonal structures in jazz.

The touchstone of the Schenkerian jazz literature is Steve Larson's *Analyzing Jazz: A Schenkerian Approach*, which is the culmination of his work of the previous decades.[17] In this and all of his work, Larson advocates what might be called an "orthodox" Schenkerian approach. He treats the extended tones common in jazz (sevenths, ninths, elevenths, etc.) as standing in for tonic members at some deeper structural level.[18] Steven Strunk's important article on linear intervallic patterns in jazz also uses an orthodox approach, as does the work of Daniel Arthurs, David Heyer, and Mark McFarland.[19]

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17. Steve Larson, *Analyzing Jazz: A Schenkerian Approach* (Hillsdale, NY: Pendragon Press, 2009). This book makes use of much of his earlier work, including "Strict Use of Analytic Notation," *Journal of Music Theory Pedagogy* 10 (1996): 31–71; "Schenkerian Analysis of Modern Jazz: Questions about Method," *Music Theory Spectrum* 20, no. 2 (October 1998): 209–41; and "Composition versus Improvisation?," *Journal of Music Theory* 49, no. 2 (October 2005): 241–75.

18. Larson, *Analyzing Jazz*, 6.

19. Steven Strunk, "Linear Intervallic Patterns in Jazz Repertory," *Annual Review of Jazz Studies* 8 (1996): 63–115. See also, for example, Daniel Arthurs, "Reconstructing Tonal Principles in the Music of Brad Mehldau" (PhD diss., Indiana University, 2011); David J. Heyer, "Applying Schenkerian Theory to Mainstream Jazz: A Justification for an Orthodox Approach," *Music Theory Online* 18, no. 3 (September 2012); and Mark McFarland, "Schenker and the Tonal Jazz Repertory: A Response to Martin," *Music Theory Online* 18, no. 3 (September 2012). Arthurs's work on Mehldau is something of an outlier, for reasons explained further in note 38.

At levels close to the background, these orthodox analyses of jazz do not appear significantly different than analyses of classical music, and indeed that is part of their appeal.[20] Because most jazz is basically tonal music, these theorists are interested in showing its connection to the European classical tradition by using the same techniques to analyze both of them. This is especially important to note, since Schenkerian analysis has a well-known ethical component.[21] For Schenker, the compositions that were well-described by his theory were judged to be masterworks. By showing that jazz compositions can also be understood with these Schenkerian techniques, the implicit conclusion is that they too should be judged to be masterworks. Underlying these orthodox approaches, I think, is a desire to legitimize a place for jazz in academic music theory (I return to this point below).

An opposing group of theorists also supports the use of Schenkerian analysis, but argues that it should be adapted to account for tonal features specific to jazz. Principal among this group is Henry Martin, who advocates for the use of alternative *Ursätze* in jazz, including ascending or gapped *Urlinien*, non-triadic descents, and chromatic or neighbor-note *Urlinien*.[22] He argues that jazz pieces are "often influenced by a more African-American aesthetic that favors repetition and rhythmic interplay over voice-leading motion through descending linear progressions," and thus we should not feel obligated to adhere to traditional Schenkerian techniques.[23]

Martin draws in part on James McGowan's "dialects of consonance" in jazz, which describes different contextually stable notes of the tonic triad.[24] McGowan argues that consonance in jazz is stylistically defined, and that in certain styles we might hear extended tones as consonances, even though they would be dissonant in classical music. He describes three "principal dialects": the added sixth, common in early jazz and Tin Pan Alley standards; the minor seventh, particular to the blues; and the major seventh, common in later jazz performance.[25] For Schenker, the background is derived from the consonant tonic triad; Martin is able use these stylistically determined definitions of consonance as support for his stylistically informed background structures.

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[20] Throughout this document, the word "classical" used in this sense is used in its generic sense to stand for tonal music that participates in the Western art music tradition. This definition is imperfect, but is useful for distinguishing the music of Bach, Mozart, Beethoven, Schumann, and Wagner from that of Ellington, Basie, Parker, Monk, and Coltrane. I do not mean to imply that jazz cannot participate in the Western art music tradition; certainly at least some of it does.

[21] Nicholas Cook, "Schenker's Theory of Music as Ethics," *Journal of Musicology* 7, no. 4 (October 1989): 415–39.

[22] Henry Martin, "Schenker and the Tonal Jazz Repertory," *Tijdschrift voor Muziektheorie* 16, no. 1 (2011): 16–17. This work draws on earlier work of his own as well as that of others, including Allen Forte, *The American Popular Ballad of the Golden Era, 1924–1950* (Princeton University Press, 1995); and David Neumeyer, "Thematic Reading, Proto-backgrounds, and Registral Transformations," *Music Theory Spectrum* 31, no. 2 (October 2009): 284–324.

[23] Martin, "Schenker and the Tonal Jazz Repertory," 7.

[24] See James McGowan, "Dynamic Consonance in Selected Piano Performances of Tonal Jazz" (PhD diss., Eastman School of Music, 2005); and "Psychoacoustic Foundations of Contextual Harmonic Stability in Jazz Piano Voicings," *Journal of Jazz Studies* 7, no. 2 (October 2011): 156–91.

McGowan's work is among the minority in recent years that does not feature a Schenkerian bent; in addition to his dialects of consonance, he is interested in applying (paleo-) Riemannian functional analysis to jazz.[26] Despite this Riemannian focus, however, he is not interested in transformational analysis: David Lewin's name appears nowhere in his bibliography. Closer to my own interests is John Bishop's 2012 dissertation, which also turns to transformations and mathematical group theory to close the gap between jazz pedagogy and jazz theory.[27] Bishop focuses primarily on the interplay of pure triads in jazz, and connects harmony to chord-scales via these triads.

Other important non-Schenkerian models of jazz harmony are found in earlier works of Martin and Strunk. In his dissertation, Martin advocates a syntactic approach based on the circle of fifths, in which chains of descending fifths point towards a tonic pitch.[28] Steven Strunk's early theory of jazz harmony is a layered approached that draws on the Schenkerian concept of analytical levels to uncover a single tonal center for a jazz tune.[29] These older models tend to fall more in line with how jazz musicians themselves discuss harmony, and we will return to them below.[30]

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[25] McGowan, "Dynamic Consonance," 76–79 and throughout. The dialects are particularly clear in final tonic chords, especially in the case of the characteristic tonic major-minor seventh of the blues.

[26] Ibid. On the distinction between "paleo-" and "neo-" Riemannian analysis, see Steven Rings, "Riemannian Analytical Values, Paleo- and Neo-," in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Rehding (New York: Oxford University Press, 2011), 487–511.

[27] John Bishop, "A Permutational Triadic Approach to Jazz Harmony and the Chord/Scale Relationship" (PhD diss., Louisiana State University, 2012).

[28] Henry Martin, "Jazz Harmony" (PhD diss., Princeton University, 1980); see also his "Jazz Harmony: A Syntactic Background," *Annual Review of Jazz Studies* 4 (1988): 9–30.

[29] Steven Strunk, "The Harmony of Early Bop: A Layered Approach," *Journal of Jazz Studies* 6 (1979): 4–53.

[30] I have a suspicion that these earlier models of harmony tend reflect jazz musician's intuitive understanding because they are not interested in legitimizing jazz analysis for the academy, as I suggest above. Both Strunk and Martin's articles appear in the *Journal of Jazz Studies/Annual Review of Jazz Studies* (the journal was renamed in 1981), while more recent articles on jazz harmony have appeared in music theory journals: *Music Theory Spectrum*, *Journal of Music Theory*, and the *Dutch Journal of Music Theory*, for example. As jazz research has moved from the fringes into the theoretical mainstream, I think it has grown more removed from jazz practice itself. As I mention above, one of the

Given the prevalence of Schenkerian techniques in jazz analysis and its proven explanatory power in other tonal repertories, it is reasonable to ask why a different approach like transformational theory is useful or necessary. In order to answer this question, I would first like to problematize the Schenkerian focus of recent jazz analysis. Steve Larson's "Schenkerian Analysis of Modern Jazz: Questions about Method" will serve as a useful foil; it is one of the fundamental articles in the field, and its titular questions will help to guide the discussion here.[31]

In the article, Larson asks three main questions we must answer if we are to take seriously the suggestion that such Schenkerian analysis is appropriate for jazz:

1. Is it appropriate to apply to improvised music a method of analysis developed for the study of composed music?
2. Can features of jazz harmony (ninths, elevenths, and thirteenths) not appearing in the music Schenker analyzed be accounted for by Schenkerian analysis?
3. Do improvising musicians really intend to create the complex structures shown in Schenkerian analysis?[32]

Larson's answer to the first question is yes. Many of Schenker's own methods were of course developed for improvisatory music, and even if they had not been, they have proven to be explanatory for such music.[33] I agree with Larson on this point, and don't have anything in particular to add. Certainly we should not expect that our theories can prove useful only for the music for which they are designed; in fact as theorists we generally hope that the opposite is true, and that our theories have broader applications than originally intended!

The second question has generated much more discussion in the literature regarding exactly how we might apply Schenkerian methods to jazz, already discussed above. While this discussion is

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goals of the present study is to show how we might narrow this gap while still approaching the music with the necessary theoretical rigor.

[31] This article appears with only slight changes as the second chapter of Larson's book, *Analyzing Jazz*. Garrett Michaelsen critiques Schenkerian analysis in similar ways in "Analyzing Musical Interaction in Jazz Improvisations of the 1960s" (PhD diss., Indiana University, 2013), 7–11.

[32] Larson, "Schenkerian Analysis of Modern Jazz," 210.

[33] In the opening of *Free Composition*, Schenker refers to improvisation as "the ability in which all creativity begins"; *Free Composition [Der freie Satz]*, ed. and trans. Ernst Oster (New York: Longman, [1935] 1979). Schenker's first *Erlauterungsausgabe* (explanatory edition) was indeed of an improvisatory work, Bach's *Chromatic Fantasy and Fugue*.

mostly one of the mechanics of analysis, it raises another point in which I am interested: is a jazz musician's conception of musical space (or musical structure) the same as a classical musician's? The orthodox Schenkerians argue that it is not, and that jazz is a fundamentally triadic music (since at some deep structural level all of the extended tones are reduced away). Those that favor a modified approach disagree with this characterization, but still agree that Schenkerian analysis is the best way to approach jazz harmony. My own reactions to this question overlap with my answer to Larson's third question, so I will return to it shortly.

Larson's third question regards compositional or improvisational intent. The argument is perhaps obvious: because Schenkerian analysis depends on uncovering long-range voice-leading plans, how could improvising musicians possibly hold such plans in their memory while playing? At some level, we might not even be interested in the answer to this question. Schenkerian analysis has proven explanatory, after all, for music that was doubtlessly composed without Schenkerian methods in mind (namely, music written before Schenker's birth). Nevertheless, Larson spends a great deal of time addressing this particular point, so we should see to it here as well.

After dismissing the possible intentional fallacy of this question, Larson turns to pianist Bill Evans as an example of how a jazz musician could produce complicated long-range voice-leading patterns while improvising. To do so, he relies heavily on an interview that Bill Evans gave on Marian McPartland's radio program, *Piano Jazz*.[34] Evans discusses how he always has a basic structure in mind while playing:

> McPartland: Well, when you say structure, you mean like, one chorus in a certain style, another . . .
>
> Evans: No, I'm talking about the abstract, architectural thing, like the theoretical thing.[35]

Evans goes on to demonstrate how he has certain structural features in mind (harmonic and melodic arrivals). Larson then shows how Evans's accompanying commentary can be understood as explaining voice-leading events like prolongation and linear progressions, and provides voice-leading analyses of his playing.[36]

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[34] The interview was recorded on November 6, 1978. The program is available online at http://www.npr.org/2010/10/08/92185496/bill-evans-on-piano-jazz, and was released on CD under Evans's name as well: *Marian McPartland's "Piano Jazz" Radio Broadcast* (The Jazz Alliance TJA-12038-2, 1993).

[35] Larson, "Schenkerian Analysis of Modern Jazz," 219.

I think that Larson's use of Schenker's methods to analyze the music of Bill Evans is well justified, but I am less sure of the extent to which they are applicable to jazz more generally. Larson anticipates this objection, noting that it might be offered on two grounds: "first, that Evans was unusually talented as an improviser; and second, that his way of thinking was radically different from that of other jazz musicians."[37] Certainly Evans was not a typical jazz musician: he was white, and studied at the Mannes College of Music, at that time a cradle of Schenkerian activity in this country.[38] Larson suggests that Evans was such an influential pianist that his Schenkerian improvisational tendencies might have influenced other musicians with whom he played. While this may be true, Evans was probably not seen as influential until after 1960, and this study is concerned primarily with music before that time (or at least, with musicians whose style was well established by that time).[39]

Larson allows that the first objection is justified: "That Evans was an unusually talented improviser—and that Schenkerian analysis can show this—is a principal argument of this article."[40] This statement is representative of the legitimizing enterprise of the application of Schenker's methods to jazz mentioned above. It also contains a dangerous implication, made explicit in Larson's closing paragraphs:

> Much jazz improvisation lacks the relationships that reward long-range hearing, and consists, as [André] Hodeir observes, of "disconnected bits of nonsense." . . . But the fact that jazz musicians often say that "a jazz improvisation should tell a story" suggests that many jazz musicians are concerned with creating and experiencing global relationships. That they do not always achieve this goal in performance is not surprising—the task is difficult. But there are exceptions.

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[36] See especially the table in Larson, "Schenkerian Analysis of Modern Jazz," 229.

[37] Ibid., 239.

[38] Brad Mehldau, who is the focus of Daniel Arthurs's work, is quite similar to Evans, in that he is a classically-trained white pianist with a strong acknowledged influence of the European classical tradition.

[39] Bill Evans made his first recordings as a leader in 1958, but he was not widely known until his appearance on Miles Davis's *Kind of Blue* in 1959. Shortly after he left Davis, he won wide acclaim with his trio with Scott LaFaro and Paul Motian, whose groundbreaking first album was *Portrait in Jazz*, released in 1960.

[40] Larson, "Schenkerian Analysis of Modern Jazz," 239.

> Is Schenkerian analysis applicable only to jazz performances that are exceptions? No, Schenkerian analysis may be applied to any jazz performance—and it may show the shortcomings of that performance.[41]

Far from "retaining Schenker's methods but not his epistemology, his specific insights into music but not the system of beliefs that supported them" (as Nicholas Cook suggests that modern Schenkerians often do), Larson seems to be using Schenkerian analysis to judge certain performances as "masterworks" and others as inferior, much in the way done by Schenker himself.[42] Performances by musicians who do not share Evans's interest in the "abstract, architectural thing" may well be excellent performances when judged by other value systems.[43] Furthermore, Schenkerian analyses of jazz often focus on what turns out to be the least interesting part of a jazz piece. Jazz is essentially tonal music, so it is not at all surprising (to me, at least) that it is often possible to reveal an *Ursatz* from a particular performance.

Whether or not jazz musicians are thinking in a Schenkerian manner is not really the point, however. As Steven Rings puts it in the introduction to his book, "any analytical act will . . . leave a surplus—a vast, unruly realm of musical experience that eludes the grasp of [a] single analytical model. Corners of that vast realm may nevertheless be illuminated via other analytical approaches, but those approaches will leave their own surpluses. And so on."[44] Schenkerian analysis typically focuses on long-range voice leading in order to reveal an underlying diatonic framework, while deemphasizing (some would say reducing) surface details, including much harmonic chromaticism.

This focus on harmony as a first-class object is something that is at the heart of much jazz pedagogical material, and might help constitute a different set of analytical values by which we can understand jazz. Rather than having to consign musicians who we cannot understand with Schenkerian analysis to a second tier of appreciation, we can instead try to understand them on something of their own terms. A Schenkerian analysis of Coltrane's "Giant Steps" solo might reveal that it is "lacking an artistically convincing relationship among structural levels," but it would likely be difficult to find a jazz musician who did not hold the composition up as an example of Coltrane's supreme mastery of the music.[45]

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[41] Larson, "Schenkerian Analysis of Modern Jazz," 240–41.

[42] Cook, "Schenker's Theory of Music as Ethics," 439.

[43] In particular, value systems that do not stem from the European classical tradition. Michaelsen, drawing on George Lewis's distinction between "Afrological" and "Eurological" modes of improvising, suggests that Schenkerian analysis is a particularly Eurological method of analysis; his own theory is designed to address interaction in improvisation, which is a more Afrological value. See "Analyzing Musical Interaction," 4–5, 11–12. Lewis first introduces the terms in "Improvised Music after 1950: Afrological and Eurological Perspectives," *Black Music Research Journal* 16, no. 1 (Spring 1996): 91–122.

[44] Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 5.

1.3 Transformational Theory

Given the current dominance of Schenkerian theory in the study of jazz harmony, we might ask what transformational thinking brings to the table. Transformational theory in recent years has focused on neo-Riemannian analysis, with particular emphasis on efficient voice leading and non-functional, chromatic progressions. Steven Rings has written that this focus on neo-Riemannian theory has "led to a view that some works are divvied up into some music that is tonal . . . and some that is transformational." Continuing, he argues that to do so "is to misconstrue the word transformational, treating it as a predicate for a certain kind of music, rather than as a predicate for a certain style of analytical and theoretical thought."[46] As he is right to point out, there is nothing about transformational theory that necessitates its restriction to this locally chromatic music; his book uses the theory to explain, as he says, "specifically *tonal* aspects of tonal music." It is this use of "transformational" that I wish to bring to bear on jazz, which is essentially tonal music.

Fundamental to the Schenkerian approach is the relatively equal balance of harmony and voice leading; for jazz musicians, though, this balance is heavily weighted toward the harmonic. Schenkerian analysis tends to deemphasize certain harmonies with the aim of exposing an underlying diatonic framework. This goal is in contrast with the typical goal of a performing jazz musician, for whom individual chords have first-class status.

Transformational theory too, often treats harmonies as first-class objects, and thus makes it especially appropriate for analyzing jazz. "Often," only because transformational theory can be used for more than simply examining harmony: Lewin's Generalized Interval Systems (explained in detail below) only require a set of elements, a group of intervals, and a function mapping pairs of elements of the set into the group of intervals. Most commonly the elements of the set are harmonies, but they do not have to be.[47]

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[45] Larson, "Schenkerian Analysis of Modern Jazz," 241. Though there are many published analyses of "Giant Steps," I am unaware of any that use Schenkerian methods.

[46] Rings, *Tonality and Transformation*, 9.

Mathematical music theories have become especially widespread in recent years.[48] Many of these models focus only on triads; while these models are valuable, nearly all chords in jazz are (at least) seventh chords. Because the neo-Riemannian literature is relatively well-known, it will be useful here to limit our focus to those theories that deal in some way with non-triadic music. This work falls basically into two categories: work that deals exclusively with a single type of chord, and work that deals with musical objects of different types.

Most of the studies dealing with a single type of chord are concerned with members of set class (0258), the half-diminished and dominant seventh chords. In a 1998 article, Adrian Childs develops a model for these chord types that is closely related to standard neo-Riemannian transformations on triads.[49] Edward Gollin's article in the same issue of the *Journal of Music Theory* explores three-dimensional Tonnetze in general, with special focus on the dominant and half-diminished seventh chords.[50]

In general, neo-Riemannian-type operations on the (0258) tetrachords turn out to be somewhat less useful than their triadic counterpoints, owing to the symmetry of set class (0258).[51] Any one tetrachordal Tonnetz can only show a subset of all of the (0258) tetrachords, while the familiar triadic Tonnetz of course shows all 24 major and minor triads. Recognizing this limitation, Jack Douthett and Peter Steinbach present a model that also includes minor sevenths and fully diminished seventh chords, using a digram they refer to as the “Power Towers.”[52] While Douthett and Steinbach’s description accounts for two of the three main types of seventh chords commonly used in jazz (it is missing the crucial major seventh), all of these neo-Riemannian models focus on parsimonious voice leading. While this focus is valuable, it will not prove to be terribly useful for the functional harmony in which this study is interested.

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[47] One of the main subjects of Ring's book is a GIS that describes "heard scale degrees," and does not include harmony at all; see *Tonality and Transformation*, 41–99, and throughout. Lewin provides several examples of non-harmonic GISes in *GMIT*, 16–24.

[48] As evidenced in part by the number of books published in the last several years; in addition to Rings, there is Richard Cohn, *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature* (New York: Oxford University Press, 2012); and Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011).

[49] Adrian P. Childs, "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords," *Journal of Music Theory* 42, no. 2 (October 1998): 181–93.

[50] Edward Gollin, "Some Aspects of Three-Dimensional *Tonnetze*," *Journal of Music Theory* 42, no. 2 (1998): 195–206. Because the terms are in common use in the transformational literature, "Tonnetz" and the plural "Tonnetze" are rendered without italics throughout this study.

[51] Gollin refers to the differences between the two-dimensional triadic Tonnetz and his three-dimensional tetrachordal version as "degeneracies" (Ibid., 200). Child's cubic representation only shows 8 of the possible 24 (0258) tetrachords: those related by parsimonious voice leading to a single diminished seventh chord ("Beyond Neo-Riemannian Triads," 188).

The other group of transformational models consists of what Julian Hook has termed “cross-type transformations”: he extends Lewin’s definition of a transformation network to allow for transformations between objects of different types.[53] This category of transformations contains the inclusion transformations (discussed by both Hook and Guy Capuzzo) which map a triad into the unique dominant or half-diminished seventh chord that contains it and vice versa.[54] Also included in this category are more general approaches for relating set classes of different cardinalities, including Joseph Straus’s formulation of atonal voice leading and Clifton Callender’s split and fuse operations.[55] Finally, Dmitri Tymoczko’s continuous tetrachordal space can accommodate *all* four-note chords, but, as Hook notes, Tymoczko downplays (and sometimes ignores) the transformational aspects of his geometric models.[56]

Though I have mentioned these cross-type transformational works only in passing here, we will return to some of them in some detail below, where they will be more relevant. Because Hook does not strictly define what constitutes a "type" in a cross-type transformation, his formulation will allow us to apply transformational procedures rigorously in situations where we might wish to consider objects to be members of different types, even though they may be identical in some other typological system.[57]

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[52] Jack Douthett and Peter Steinbach, “Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition,” *Journal of Music Theory* 42, no. 2 (1998): 255–56.

[53] “Cross-Type Transformations and the Path Consistency Condition,” *Music Theory Spectrum* 29, no. 1 (April 2007): 1–40. Lewin’s definition of a transformational network is in *GMIT*, 193–97.

[54] Julian Hook, “Uniform Triadic Transformations,” *Journal of Music Theory* 46, nos. 1/2 (Spring–Autumn 2002): 57–126; Guy Capuzzo, “Neo-Riemannian Theory and the Analysis of Pop-Rock Music,” *Music Theory Spectrum* 26, no. 2 (October 2004): 177–99.

[55] Joseph Straus, “Uniformity, Balance, and Smoothness in Atonal Voice Leading,” *Music Theory Spectrum* 25, no. 2 (October 2003): 305–52; Clifton Callender, “Voice-Leading Parsimony in the Music of Alexander Scriabin,” *Journal of Music Theory* 42, no. 2 (1998): 219–33.

[56] An explanation of this four-dimensional space is found in Tymoczko, *A Geometry of Music*, 93–112; much of this work is based on Dmitri Tymoczko, “The Geometry of Musical Chords,” *Science* 313, no. 5783 (July 2006): 72–74; and Clifton Callender, Ian Quinn, and Dmitri Tymoczko, “Generalized Voice-Leading Spaces,” *Science* 320, no. 5874 (April 2008): 346–48. Hook points out Tymoczko’s relationship to transformational theory in his review of Tymoczko’s book, ¶13–14.

1.4 Aside: Lead Sheet Notation

As mentioned in Section 1.1, jazz musicians often begin learning a particular tune with a lead sheet, and the changes found there form the harmonic foundation of a particular performance. Because this dissertation is interested in jazz harmony generally, lead sheets serve as a useful abstraction of the countless possible instantiations of any one tune, and considering them them briefly here will prove fruitful for the rest of this study.[58]

A lead sheet typically gives only a melody and a set of chord changes: Figure 1.1 gives the *Real Book* lead sheet for John Klenner and Sam Lewis's "Just Friends."[59] It is a very typical example, and nearly everything about the page is designed to make it easy for a jazz musician to "fake" a performance of the tune on the bandstand: the anonymous compilers of the book provide the composer and lyricist's names and a sample recording; the music is split into four-measure chunks to make the phrases and the form of the tune clear; and there is almost no extraneous information—even the key signature is omitted on all lines but the first. The melody is given for the head, and the changes are provided for the rhythm section (usually piano, bass, and drums, but sometimes other instruments like guitar) and for solos. All other aspects of the tune need to be negotiated prior to performance (how the tune will begin and end, for example).

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[57] In particular, it will be useful to consider the IIm7♭5 chord (a half-diminished seventh) as a different type than the V7 chord (a dominant seventh) given their different functional roles in jazz harmony, despite the fact that they are of the same set class.

[58] It is worth mentioning that not all jazz musicians read music; jazz is largely an aural tradition, and many early jazz musicians did not read, instead learning the music by ear. For students learning jazz today, however, learning to read chord changes is an essential part of their training. For an ethnomusicological survey of jazz musicians' relationships with lead sheets, see Paul Berliner, *Thinking in Jazz: The Infinite Art of Improvisation* (Chicago: University of Chicago Press, 1994), 71–76.

[59] This lead sheet is taken from the older, illegal *Real Book* (249). While the newer Hal Leonard edition maintains most of the original selections, in some cases they do not: "Just Friends" does not appear until vol. 4 of the Hal Leonard *Real Book*.

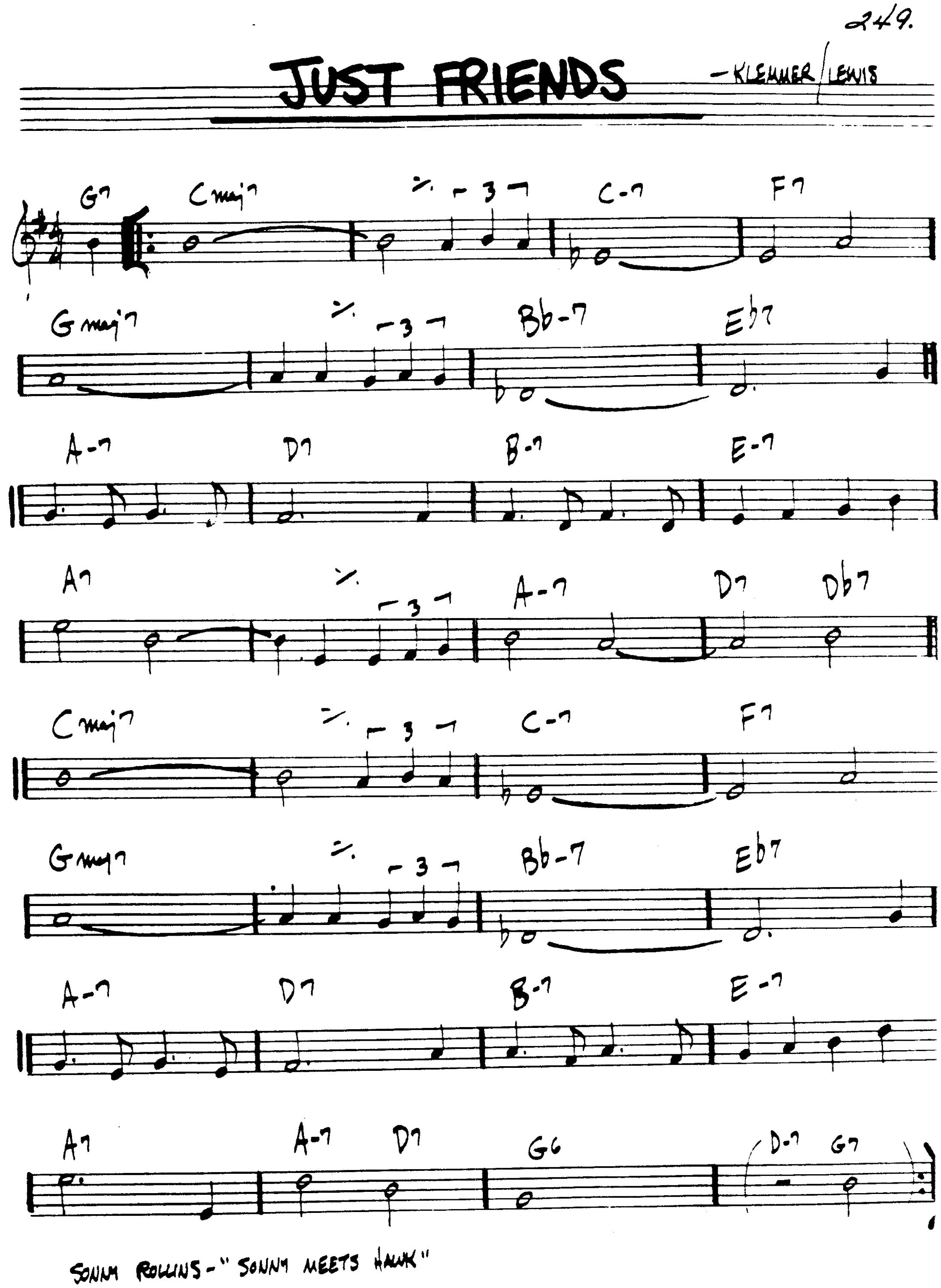
249.
JUST FRIENDS
– KLENNER / LEWIS
G7
Cmaj7
%
3
C-7
F7
Gmaj7
%
3
Bb-7
Eb7
A-7
D7
B-7
E-7
A7
%
3
A-7
D7
Db7
Cmaj7
%
3
C-7
F7
Gmaj7
%
3
Bb-7
Eb7
A-7
D7
B-7
E-7
A7
A-7
D7
G6
D-7
G7
SONNY ROLLINS – "SONNY MEETS HAWK"

Figure 1.1. A sample lead sheet of "Just Friends" (John Klenner/Sam Lewis).

The Real Book uses standard conventions for labeling chords. A chord symbol consists of the chord root (referred to by a letter name), and a symbol indicating the quality: the most common of these are are the dominant seventh (simply "7"), minor seventh ("–7") and major seventh ("maj7").[60] Thus, the opening of "Just Friends" (G7–Cmaj7) consists of a G dominant seventh chord moving to a C major seventh chord. This abstraction is extremely useful for a performing musician, but leaves something to be desired if pressed into use as an analytical system. The chord symbols do not explicitly tell us, for example, that G7–Cmaj7 is a typical V^7–I^7 progression in C major; that kind of knowledge is implicit for experienced musicians and analysts.

Complicating the problem somewhat is that chord symbols are imprecise by design. In most situations, jazz musicians do not want to be told exactly what notes they should play (if they did, they probably would not have become jazz musicians); instead, they treat chord symbols only as guidelines. A G7 chord would certainly include the root, third, and seventh (G, B, and F), but might also include the ♯11 (C♯), ♭9 (A♭), or ♯5 (D♯), depending on the situation: the melody might suggest certain alterations, a performer might prefer some alterations over others, or an improvisor may work themselves into a dissonant portion of a solo where a bare dominant seventh in the piano would sound especially out of place.

To illustrate the flexibility of chord symbols, Figure 1.2 gives two realizations of the first eight measures of "Just Friends." The first is Mark Levine's, from early in his book on jazz piano; it uses what he calls "three-note voicings" (the root, third, and seventh).[61] The second realization is my own, and features many alterations to the basic outlines given by the chord symbols. Both of these realizations are valid interpretations of the given chord symbols, and are meant to reinforce the point that chord symbols, while only a guideline, indeed represent something important about a given harmonic progression. For all of their imprecision, chord symbols represent a reality for performing jazz musicians, and as such will be foundational for our work on harmony here.

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[60] Jamey Aebersold publishes a free book on his website that serves as an introduction to the very basics of jazz; the section titled "Nomenclature" is especially useful for deciphering chord symbols. *Jazz Handbook* (New Albany, IN: Jamey Aebersold Jazz, 2010), 15, http://www.jazzbooks.com/mm5/download/FQBK-handbook.pdf.

[61] Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 21. Realizing chord symbols is perhaps most important for pianists and guitarists (who are most often charged with realizing them in performance), and any introductory text on these instruments will be filled with voicings to be used in different situations.

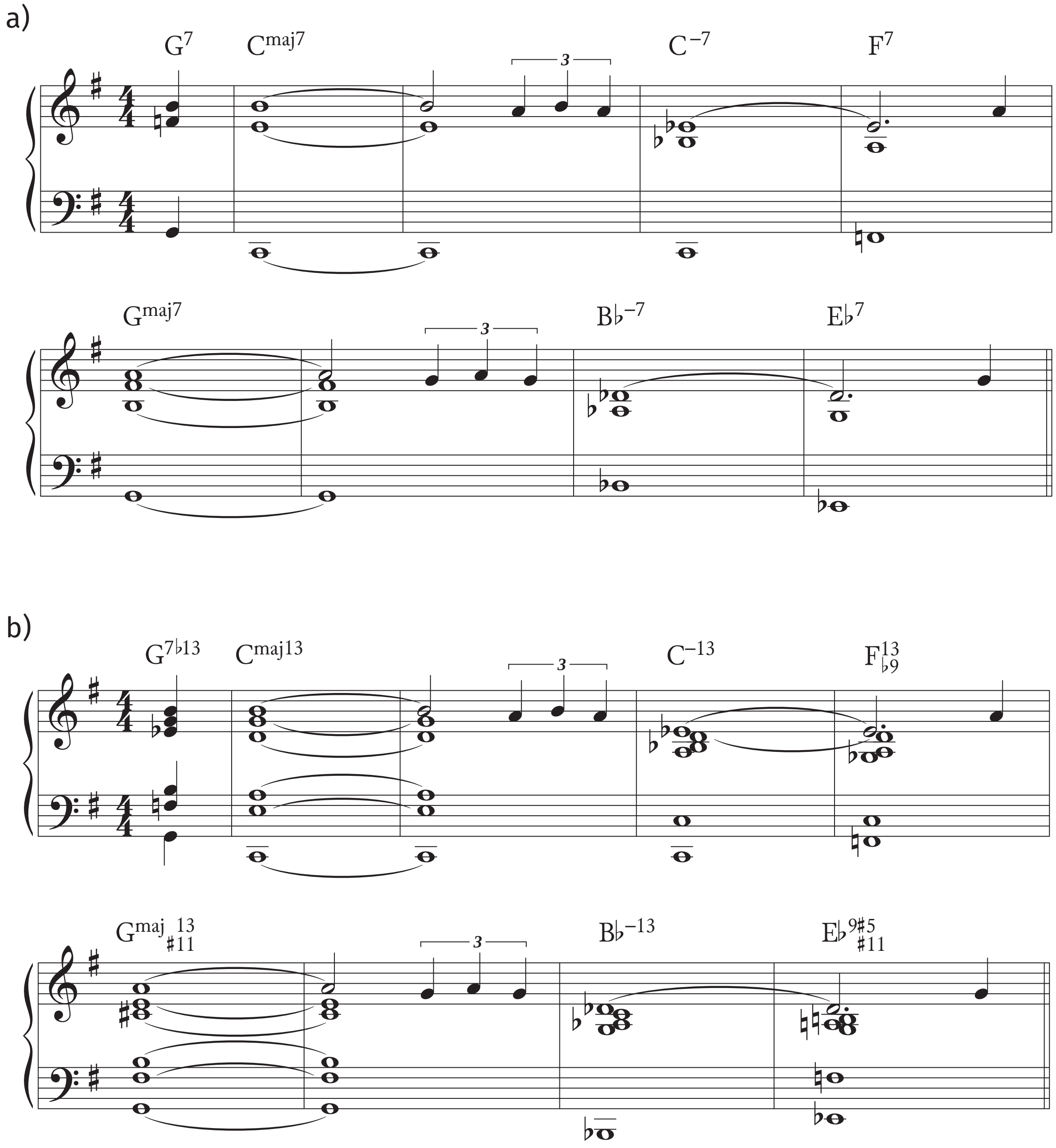
a)
G7
Cmaj7
3
C−7
F7
Gmaj7
3
B♭−7
E♭7
b)
G7♭13
Cmaj13
3
C−13
F13 ♭9
Gmaj 13 ♯11
3
B♭−13
E♭9♯5 ♯11

Figure 1.2. Two piano realizations of "Just Friends," mm. 1–8.
a) Using three-note voicings (from Mark Levine, *The Jazz Piano Book*).
b) Using more alterations/extensions (chord symbols reflect voicings used).

1.5 Diatonic Chord Spaces

It will be easiest to introduce the transformational approach to jazz harmony developed in this study by way of an example. Much of this dissertation will be interested in the development of various musical spaces and motions that are possible within them. This kind of approach was first developed by David Lewin in *Generalized Musical Intervals and Transformations*, and a review of his approach will be useful before moving on to more involved examples.

1.5.1 INTERVALS AND TRANSFORMATIONS

Figure 1.3 shows the chord changes to the A section on the jazz standard "Autumn Leaves."[62] Jazz musicians sometimes refer to this progression as a "diatonic cycle": it uses only seventh-chords found in the G-minor diatonic collection.[63] As in classical music, the leading tone is raised in the dominant chord so that the resulting chord is D7, not Dm7. We can easily arrange this progression around the familiar circle of fifths, placing the tonic G minor at the top of the circle (see Figure 1.4).

While this arrangement around the diatonic circle of fifths makes intuitive sense, it can also represent what Lewin has called a Generalized Interval System (GIS). Generalized Interval Systems are Lewin's way of formalizing the "directed measurements, distances, or motions" that we often understand as "characteristic textural features" of a given musical space.[64] Though we will unpack

C-^{7} | F^{7} | $\text{B}\flat^{\text{maj7}}$ | $\text{E}\flat^{\text{maj7}}$ |

$\text{A-}^{7\flat5}$ | D^{7} | G- | ||

Figure 1.3. The changes to "Autumn Leaves" (Joseph Kosma), A section.

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[62] *The Real Book* gives these changes in E minor, but most recorded performances are in G minor. I have transposed the given changes to reflect the most common performance key. Steven Strunk analyzes both "Autumn Leaves" and "How My Heart Sings" (analyzed below) as examples of 10–7 linear intervallic patterns in "Linear Intervallic Patterns in Jazz Repertory," 96–97.

[63] Emile De Cosmo's etude book titled simply *The Diatonic Cycle* (North Bergen, N.J.: EDC Publications, 1970) gives many different solo patterns possible over the diatonic cycle in all twelve major and minor keys.

[64] Lewin, *GMIT*, 16.

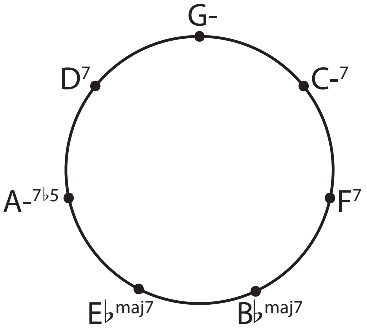

Figure 1.4. The changes to "Autumn Leaves," arranged around the diatonic circle of fifths.

this definition using the harmonies from the A section of "Autumn Leaves" as an example, Lewin's formal definition is as follows:

> A *Generalized Interval System* (GIS) is an ordered triple (S, IVLS, int), where S, the *space* of the GIS, is a family of elements, IVLS, the *group of intervals* for the GIS, is a mathematical group, and int is a function mapping S × S into IVLS, all subject to the two conditions (A) and (B) following.
> (A): For all r, s, and t in S, int(r,s)int(s,t) = int(r,t)
> (B): For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s,t) = i.[65]

The first element in a GIS is a family of elements, S, which Lewin also calls a musical space. Preceding this formal definition, he gives a number of examples of musical spaces, including the familiar diatonic and pitch and pitch-class spaces, along with less familiar musical spaces like frequency space, time point space, and various durational spaces.[66] In our "Autumn Leaves" example, we are interested in the (unordered) set of harmonies in the G-minor diatonic collection: $S = \{\text{Gm, Am7}\flat\text{5, B}\flat\text{maj7, Cm7, D7, E}\flat\text{maj7, F7}\}$.[67]

With the first element of a GIS satisfied, we must then define a group of intervals (IVLS). Though we could perhaps imagine a number of different ways to define intervals among elements of the set S (a point to which we will return later), the most obvious is to measure distances in diatonic steps between chord roots. Because we are interested in abstract chord roots and not the actual pitches played by some bass player or pianist's left hand, we will use diatonic pitch classes. This has the effect of modularizing the set of possible intervals, changing IVLS from $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (as it would be in diatonic *pitch* space) to $\{0, 1, \ldots, 6\}$ (diatonic pitch *class* space).[68] Arithmetic in this group is mod-7, exactly in the way that arithmetic using the more familiar chromatic pitch class space, $\{0, 1, 2, \ldots, 11\}$, is mod-12.

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[65] Lewin, *GMIT*, Definition 2.3.1 (26). *GMIT* contains many terms that Lewin renders in all capitals; I have rendered them here in small capitals (except when quoting directly) in order to reduce their typographical impact.

[66] Ibid., 16–25.

[67] The tonic harmony is given as it is in *The Real Book*, simply as Gm. In performance, a musician might choose to play this chord with a major seventh (GmM7), a sixth (Gm6), or even a minor seventh (Gm7; this is less likely since minor seventh chords are most commonly ii chords, not tonics).

Lewin specifies that IVLS must be a mathematical group, and we will take care here to show that IVLS = $\{0, 1, \ldots, 6\}$ is indeed such a group. A group is a set of elements, G, and a binary operation, $\otimes$, that satisfies the four group axioms:

- Closure: for $a, b \in G$, then $a \otimes b$ must be an element of G.
- Associativity: for $a, b, c \in G$, then the equation $(a \otimes b) \otimes c = a \otimes (b \otimes c)$ must be true.
- Identity element: There exists an element $e \in G$ such that for any element $a \in G$, $a \otimes e = e \otimes a = a$ is true.
- Inverses: For any element $a \in G$, there exists a unique element $a^{-1} \in G$ such that $a \otimes a^{-1} = a^{-1} \otimes a = e$ is true.[69]

To show that our IVLS is a group, it is sufficient to show that the set $\{0, 1, \ldots, 6\}$ under some binary operation satisfies the group axioms. The binary operation is simply addition mod-7 (which we will notate using the usual $+$ sign instead of the abstract $\otimes$ used above). We can then show that the set IVLS is closed: for any two elements $a, b \in \text{IVLS}$, $a + b$ is also an element of IVLS ($1 + 3 = 4; 4 + 4 = 1; 1 + 0 = 1$; and so on). Modular addition, like its non-modular counterpart, is associative: $(3 + 4) + 5 = 3 + (4 + 5)$, and likewise for any chosen elements of IVLS. The identity element for addition is 0, which combined with any element $a \in \text{IVLS}$ gives a itself. Inverses in the group are simply complements mod-7 (the number that when added to the given number gives 0, mod-7): $2^{-1} = 5; 1^{-1} = 6$; and so on. The integers modulo *n* are labeled $\mathbb{Z}_n$, so we may also refer to IVLS in our "Autumn Leaves" example as the group $\mathbb{Z}_7$.

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[68] In chromatic pitch space, we might say that the interval between C4 and A3 is -3, while the interval between C4 and A5 would be $+21$. In chromatic pitch class space, however, the interval is calculated mod-12 (because octaves are equivalent), and both of these intervals are equal to 9.

[69] Lewin's definition of a mathematical group is spread over several pages in the first chapter of *GMIT* (4–6). For other definitions in works of music theory, see Rings, *Tonality and Transformation*, 12–13, and Julian Hook, *Exploring Musical Spaces* (New York: Oxford University Press, forthcoming), Ch. 5. Any introductory mathematical text on group theory will contain the group axioms; see, for example, Israel Grossman and Wilhelm Magnus, *Groups and Their Graphs* (Washington, D.C.: Mathematical Association of America, 1964), 10–14 and Nathan Carter, *Visual Group Theory* (Washington, D.C.: Mathematical Association of America, 2009), 51.

The last element of a GIS is an interval function that maps $S \times S$ into IVLS. In other words, the interval from one element of S to another must be a member of the group $\mathbb{Z}_7$. In our "Autumn Leaves" example, the interval from one element of S to another is simply the number of steps in the G-minor diatonic collection (always counting upward) between the two elements. Thus, int(D7, Gm) = 3, since G, the root of the second chord, lies 3 diatonic steps above D, the root of the first. Likewise, int(Am7♭5, B♭maj7) = 1; int(F7, E♭maj7) = 6; and so on.

We now have all of the elements of a GIS, but we must still prove that Lewin's conditions A and B are satisfied, which we will do by example. Condition A states that for all r, s, and t in S, int(r,s)int(s,t) = int(r,t). In our "Autumn Leaves" example, int(Cm7, D7) and int(D7, E♭maj7) (both interval 1) must combine to equal int(Cm7, E♭maj7) (interval 2). Second, for every chord *s* in S and every interval *i* in $\mathbb{Z}_7$, there must be a unique chord *t* which satisfies the equation $\text{int}(s, t) = i$. For example, there must be exactly one chord that lies 2 units above Cm7: namely, E♭maj7. It is easy to confirm that the two conditions are also true for any choice of $r, s, t \in$ S.

Before shifting our focus from generalized intervals to transformations, I want to return to the structure of IVLS, the group $\mathbb{Z}_7$. Above, we measured intervals by the number of G-minor diatonic steps between chord roots: a single step corresponded to the interval 1. We can also say that the group $\mathbb{Z}_7$ is a cyclic group, *generated* by the interval 1. (Cyclic groups are notated C_n, where *n* is the size of the group; we could therefore label the group in question C_7.) Counting diatonic steps is not the only way we might consider measuring intervals in the space, however. Given the ubiquitous descending fifths of "Autumn Leaves," we might instead like to measure distance by the number of descending fifths between chord roots. Though this generation has no noticeable effect on the abstract group structure of IVLS—the group is C_7 in either case—it does affect the last element of a GIS, the interval function.[70] As shown in Figure 1.5, int(Am7♭5, D7) = 3 when

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[70] The group C_7 can be generated by any of its members, since 7 is prime. In general, a cyclic group can be generated by a member only if the two are relatively prime. The group C_{12}, for example, can be generated only by 1, 5, 7, or 11; put another way, only the chromatic scale and circle of fifths (or fourths) cycle through all 12 pitches in the chromatic octave before returning to the starting point.

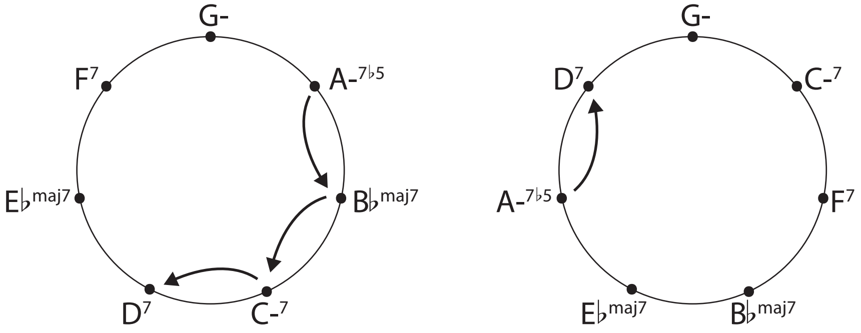

Figure 1.5. The "Autumn Leaves" GIS, generated by diatonic step (left) and descending fifth (right).

measured by diatonic steps (the left figure), but int(Am7♭5, D7) = 1 when measured by descending fifths (the right figure).[71]

Generalized Interval Systems are but one part of Lewin's project; we will now turn our attention to the "transformations" of the book's title. Much has been made of the "transformational attitude" that accompanies the shift from generalized intervals to transformations that occurs in the later chapters of *GMIT*. As Lewin has it, GIS thinking represents a Cartesian, observer-oriented position, examining musical objects as points in abstract space. This is in contrast to the transformational attitude, which Lewin describes as "much less Cartesian" in what is perhaps the most-cited portion of the book:

> Given locations s and t in our space, this attitude does not ask for some observed measure of extension between reified "points"; rather it asks: "If I am *at* s and wish to get to t, what characteristic gesture (e.g. member of STRANS) should I perform in order to arrive there?[72]

The GIS perspective outlined above is an intervallic perspective: we developed a system that allowed us to say that "the distance from Am7♭5 to D7 is 3." By replacing, as Lewin does, "the concept of interval-in-a-GIS" with "the concept of transposition-operation-on-a-space,"[73] we can convert this GIS statement into a transformational one: "transposing the root of Am7♭5 by three diatonic steps gives D7." The transformational statement is more active, replacing distance metrics with verbs like "transpose."[74]

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[71] To avoid confusion, we will use only the GIS that measures distance in diatonic steps from this point on.

[72] Lewin, *GMIT*, 159. Henry Klumpenhouwer argues that the dichotomy between Cartesian GIS thinking and what he calls "anti-Cartesian" transformational thinking is the central theme of *GMIT*; see "In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's *Generalized Musical Intervals and Transformations*," *Music Theory Spectrum* 28, no. 2 (Fall 2006): 277–89. For more on the "transformational attitude," see Rings, *Tonality and Transformation*, 24–29; Ramon Satyendra, "An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory," *Journal of Music Theory* 48, no. 1 (Spring 2004): 102–3 and 116–17; and Julian Hook, "David Lewin and the Complexity of the Beautiful," *Intégral* 21 (2007): 172–77.

Though there is certainly a difference in the language used in GIS statements and that used in transformational statements, Lewin takes care to note that the two attitudes are not diametrically opposed:

> we do not have to choose *either* interval-language *or* transposition-language; the generalizing power of transformational theory enables us to consider them as two aspects of one phenomenon, manifest in two different aspects of this musical composition.[75]

While we might prefer interval-language in some contexts and transformation-language in others, the two attitudes are quite closely related; any GIS statement can be converted into a transformational one by using the mechanism Lewin describes just before the famous passage in *GMIT*.

By combining the space S of a GIS with an operation-group on S, we can derive a transformational system. This operation-group must be simply transitive on S (hence the reference to STRANS in the quote above): "given any elements s and t of S, then there exists a unique member OP of STRANS such that OP(s) = t."[76] Lewin then states that in any GIS, "there is a unique transposition-operation T satisfying T(s) = T, namely $T = T_{\text{int(s,t)}}$."[77] In familiar chromatic pitch-class space, the unique transposition T_k is that operation where k is the interval in semitones between the two pitches: the interval between C and E♭ is 3, and the operator T_3 maps C onto E♭.

Converting our "Autumn Leaves" GIS into a transformational system, then, is only slightly more complicated than this chromatic pitch-class space example, since ordinary transposition will not work intuitively. To avoid confusion with the traditional transposition operator, we will instead use the lowercase, t_k.[78] This diatonic transposition operator will be used in almost the same way, however: the operation t_k is that transposition which transposes the root of a chord k steps inside the G-minor diatonic collection. With this understanding, the conversion works as expected: the interval between Am7♭5 and D7 is 3, so the operator t_3 maps Am7♭5 onto D7 in the space $S = \{\text{Gm, Am7}\flat\text{5, B}\flat\text{maj7, Cm7, D7, E}\flat\text{maj7, F7}\}$.[79] The same statement may also be written as Am7♭5 $\xrightarrow{t_3}$ D7, if we want to emphasize the idea of a transformation as a mapping operation.

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[73] Lewin, *GMIT*, 157.

[74] Ramon Satyendra describes the two attitudes as being noun-oriented (GISes) vs. verb-oriented (transformations); "An Informal Introduction," 102–3.

[75] Lewin, *GMIT*, 160, emphasis original. Julian Hook clarifies this point in "David Lewin and the Complexity of the Beautiful," 172–77.

[76] Lewin, *GMIT*, 157.

[77] Ibid.

Now that we have explored the underlying mathematics, we are in a position to reexamine the A section of "Autumn Leaves," (as given in Figure 1.3), as well as the circle-of-fifths arrangement in Figure 1.4. The progression is, quite simply, a series of t_3 operations within the G-minor diatonic space; each chord root descends by diatonic fifth. In a mod-12 chromatic space (like the ones we will begin developing in the next chapter) it can be difficult to make sense of this harmonic progression. The chord qualities can be confusing—two major sevenths in a row followed immediately by a half-diminished seventh—and it is sometimes hard to remember where the tritone in the bass falls in the progression, given the prevalence of perfect-fifth bass motion in chromatic space. Understood in the context of this diatonic space, however, the harmonic motion becomes much clearer.

It is reasonable to pause at this point and ask what advantages this transformational approach brings. After all, we began our discussion of "Autumn Leaves" by noting that jazz musicians sometimes refer to the progression of the A section as a "diatonic cycle," and it may seem as though we have gone through a great deal of mathematical rigmarole simply to arrive back at our starting point. There would seem to be very little difference between describing a progression as a "diatonic cycle" and describing it as "a series of t_3 transformations in the G-minor diatonic set of seventh chords." And yet, this is almost exactly the point. Transformational theory allows us a means to formalize what is often intuitive knowledge for jazz musicians, thereby narrowing the gap between the way jazz musicians discuss harmony and the way music theorists often do.

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[78] The diatonic transposition operator t_k is described in Julian Hook, "Signature Transformations," in *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha M. Hyde, and Charles J. Smith (Rochester: University of Rochester Press, 2008), 139–40.

[79] The intervals chosen for measurement in the GIS do have an impact on the associated transformational system: if we had used the descending fifths generation as described above, the the operator t_1 (not t_3) would map Am7♭5 onto D7, since the interval in the underlying GIS would be different.

1.5.2 Analytical Applications

Before concluding this chapter, I want to examine "Autumn Leaves" in a bit more detail, then move on to a few other analytical examples. The full chord changes to "Autumn Leaves" are given in Figure 1.6. The opening A section, discussed at length above, can be understood as a series of t_3 operations in the G-minor diatonic set. The bridge modulates to the relative major, but we can still understand this passage using the same transformational system. After repeating the G-minor ii–V–I progression that concludes the A section in the first four measures of the bridge, the entire progression is transposed up a third (a larger-scale t_2) to repeat the ii–V–I in the key of B♭. Despite this modulation, the connections from chord to chord are all t_3 operations, continuing the chain that has been present since the beginning (see Figure 1.7).[80]

There are three passages in this piece that are not simply t_3 operations: the connection from Gm to Am7♭5 that begins the bridge; the connection from B♭maj7 at the end of the bridge to Am7♭5 that follows; and the third and fourth bars of the final section, Gm7–C7–Fm7–B♭7. The first of these, Gm–Am7♭5, is a t_1 transformation. The bridge begins by retracing the same harmonic ground as the last four bars of the A section (a ii–V–I progression in G minor); the connection to the bridge, then, can be understood as reversing the two descending fifths that ended the A section. This observation can be represented algebraically ($t_3^{-1} \bullet t_3^{-1} = t_4 \bullet t_4 = t_1$) or graphically (by taking two steps counterclockwise in the circle of fifths in Figure 1.4).[81]

The next transformation that breaks the series of t_3 operations is the t_6 from B♭maj7 to Am7♭5 at the end of the bridge. This t_6 is easily understood as a combination of two t_3 operations, by imagining that there is a "missing" E♭maj7 chord in the last bar of the bridge.[82] Interpolating

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[80] Lewin's definition of a transformation network appears is at *GMIT*, 196. We will delay an in-depth explanation until the next chapter, at which point it will be more relevant.

[81] Recall the discussion in the preface (page xiv): the symbol • here represents left-to-right function composition.

[82] This observation has not gone unrecognized by jazz musicians: at least one recording (Vince Guaraldi, on the album *A Flower is a Lovesome Thing*) includes the VImaj7 chord in this measure, though most do not.

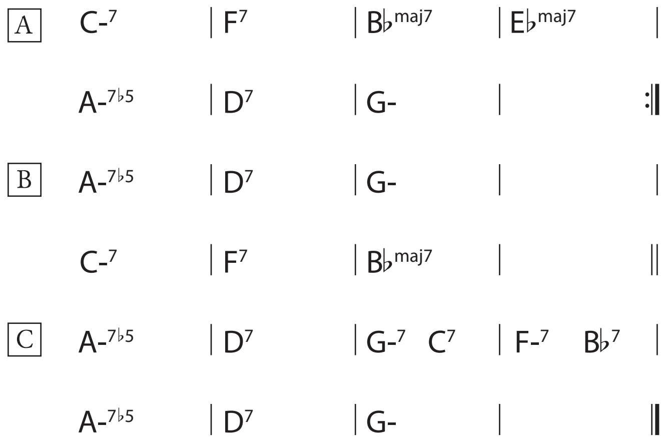

Figure 1.6. The complete changes to “Autumn Leaves.”

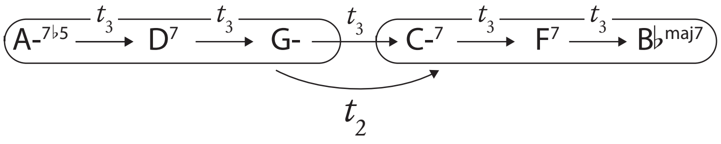

Figure 1.7. A transformation network for “Autumn Leaves,” bridge.

this chord allows us to hear the progression (beginning on Cm7) as identical to that of the first eight bars, now displaced to span a formal boundary, as shown in Figure 1.8.[83]

The only four chords in “Autumn Leaves” that cannot be understood in the GIS developed above are those in the progression Gm7–C7–Fm7–B♭7. While the chord roots belong to the G-minor collection, the qualities are incorrect. This progression consists of two ii^7–V^7 progressions, the first in F and the second in E♭ (a tonic that, like the one at the end of the bridge, does not actually appear in the music). These ii–V progressions are best situated in chromatic, rather than diatonic, space; for now, we will pass over this progression until we develop such a space in the next chapter.

It is relatively rare for tunes to be as systematically diatonic as “Autumn Leaves” (though Bart Howard’s “Fly Me to the Moon” comes close); instead, pieces often make use of diatonic cycles in

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[83] Hearing this parallelism also requires hearing the Gm7 in the third bar of the final A section as equivalent to the cadential Gm in the first. The chord symbol Gm is ambiguous by nature; if a performer wanted to bring out this parallelism, she could perhaps play the tonic chords in the first A section as minor seventh chords.

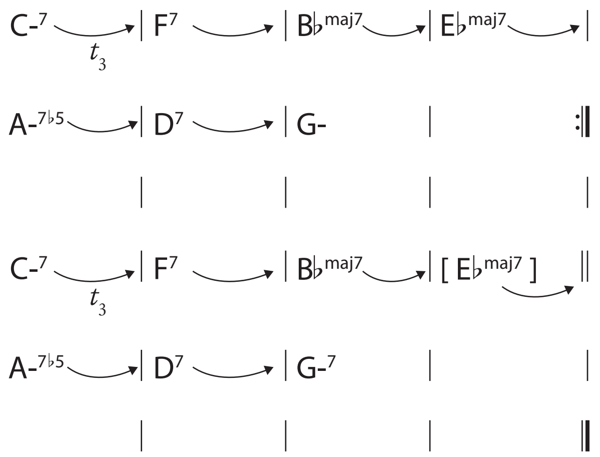

Figure 1.8. The diatonic cycle of "Autumn Leaves," with a hypermetrically displaced copy spanning the formal boundary at the end of the bridge.

only a portion of a piece before moving on to contrasting music. A relatively straightforward example of this is Sammy Fain and Bob Hilliard's "Alice in Wonderland," the changes to which are shown in Figure 1.9.[84] Like "Autumn Leaves," "Alice in Wonderland" begins with a minor diatonic cycle in mm. 1–7 (also beginning on a iv chord), shown here in A minor. We can modify our GIS from above simply by changing the set (S is now the set {Am, Bm7♭5, Cmaj7, Dm7, E7, Fmaj7, G7}); the group IVLS and the interval function are identical.[85] The harmony in m. 7 is an Am7 chord, but because the quality of the tonic seventh chord is ambiguous in minor keys, this chord is easily understood as tonic.

The second eight measures are all diatonic in the key of C major; the linking E♭7 chord signals a shift between diatonic collections.[86] This move to the relative major is a common one, and the only difference between the two diatonic sets is the shift of E7 (in A minor) to Em7 (in C major). The progression here is not as systematic as the first eight (see the annotations in Figure

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[84] The most well-known recording of "Alice in Wonderland" is probably on Bill Evans's *Portrait in Jazz*. These changes are as given in *The Real Book*, and as played by Evans. This figure gives the changes to the second ending of the first sixteen measures (the first ending contains a ii–V in D minor to return to the opening).

[85] The interval functions are identical, with the caveat that diatonic steps are to be counted in the A-minor collection, not the G-minor.

[86] The E♭7 chord is a tritone substitute for A7, which is of course the dominant of the following Dm chord. This harmonic motion is not easily understood in either the A-minor or C-major diatonic spaces; we will return to tritone substitutes in the next chapter.

D-^{7} | G^{7} | C^{maj7} | F^{maj7} |

$\text{B-}^{7\flat5}$ | E^{7} | A- | $\text{E}\flat^{7}$ |

D-^{7} | G^{7} | E-^{7} | A-^{7} |

D-^{7} | G^{7} | C^{maj7} | ‖

Figure 1.9. Changes to “Alice in Wonderland” (Sammy Fain/Bob Hilliard), mm. 1–16.

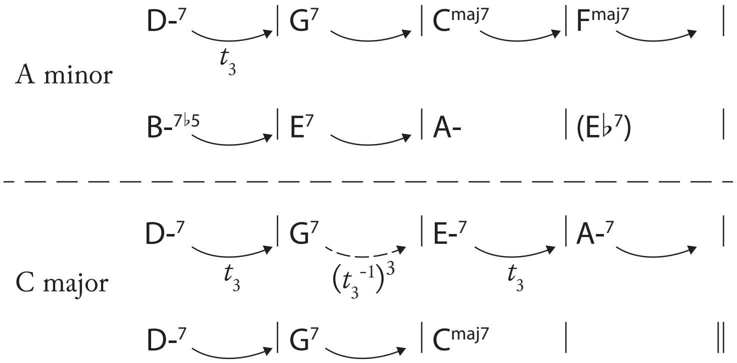

Figure 1.10. The changes to “Alice in Wonderland,” with transformational labels between harmonies.

1.10). After a C-major ii–V in mm. 9–10, the harmony moves up a step to Em7–Am7.[87] The progression G7–Em7 seems to move backwards in the cycle, almost as if realizing the phrase is in danger of arriving at C major too soon. We can capture this intuition by choosing to understand the progression not as a forward-directed t_5, but as an algebraically-equivalent combination of three ascending fifths: $t_3{}^{-1} \bullet t_3{}^{-1} \bullet t_3{}^{-1}$ (which we could also write as $t_3{}^{-1}$ raised to the third power: $(t_3{}^{-1})^3$).

Throughout *GMIT*, Lewin is clear that transformational theory is a means of expressing our “intuitions” about a musical passage in a mathematically rigorous way.[88] As he puts it: “If I want to change Gestalt 1 into Gestalt 2 . . . , what sorts of admissible transformations in my space (members of STRANS or otherwise) will do the best job?”[89] Our explication of diatonic

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[87] It is this Am7 that confirms we are in a C-major diatonic space; it would be more typical to transpose the ii–V progression exactly to Em7–A7.

[88] Steven Rings refers to Lewin’s “intuitions” as “apperceptions”; see *Tonality and Transformation*, 17–21.

[89] Lewin, *GMIT*, 159.

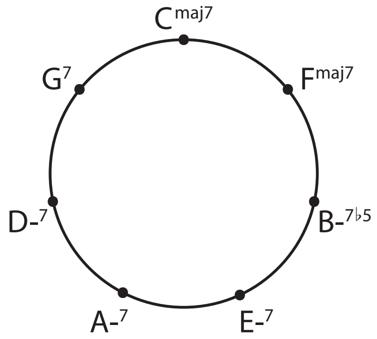

Figure 1.11. A diatonic cycle in C major.

seventh-chord spaces may appear to stem from the desire to label everything a t_3 and move on, confident that we could justify our labels mathematically if called upon to do so. This, though, could not be further from the truth; developing the space allows us a powerful means to capture intuitions (or apperceptions) like the one above. Though the operation t_5 does map G7 to Em7 in the C-major diatonic space, hearing this connection as $t_3^{-1} \bullet t_3^{-1} \bullet t_3^{-1}$ instead represents the idea of stepping backwards through the circle of descending fifths (an observation that may not be easy to show using other methods of analysis).

Both examples of outright diatonic cycles we have seen thus far have been in minor keys. In general, minor-key cycles are easier to use than those in major keys (a C-major diatonic cycle is given for reference in Figure 1.11). In minor, the raised leading tone in the V chord means that the cycle consists of a ii–V–I in the relative major and a ii–V–I in the tonic joined by the VImaj7 chord (as we saw in the second cycle in Autumn Leaves, Figure 1.8). In a major-key cycle, the lone half-diminished seventh chord is followed immediately by three minor seventh chords (in C major, Bm7♭5–Em7–Am7–Dm7), which by comparison is a relatively unusable tonal progression.

Nevertheless, Earl Zindars's standard "How My Heart Sings" (the changes to which are given in Figure 1.12) does indeed contain a major-key cycle.[90] Unlike the previous two examples, this tune begins on the iii chord, at which point it begins the t_3 cycle. Once again, it is the Am7 chord

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[90] The canonical recording of this tune is again by Bill Evans, on the album *How My Heart Sings!* Evans seems to have a propensity towards tunes with diatonic cycles: *Portrait in Jazz* also contains a very well-known recording of "Autumn Leaves."

E-^{7}	A-^{7}	D-^{7}	G^{7}
C^{maj7}	F^{maj7}	B-$^{7\flat5}$	E$^{7\flat9}$
A-^{7}	A$\flat^{\circ7}$	A-^{7}/G	F♯-$^{7\flat5}$

Figure 1.12. Changes to “How My Heart Sings” (Earl Zindars), mm. 1–12.

that alerts us that this progression takes place in diatonic, rather than chromatic, space (A7 would seem to make more harmonic sense, as the dominant of the following D minor). After reaching the tonic in m. 5, the t_3s continue making their way to A minor, four measures later.

Notably, the chord in m. 8 is an E7♭9—this chord belongs not to C major, but rather to A minor. At some point, then, the progression shifts from taking place in a C-major cycle (the E chord is a minor seventh in the opening bar) to an A-minor cycle. Unlike the abrupt shift to the relative major in “Alice in Wonderland” (signaled by E♭7), this modulation is gradual, using a traditional pivot chord.[91] In a transformational reading, the progression maintains the t_3 sequence throughout the first nine measures, but the underlying diatonic set changes almost imperceptibly from C major to A minor. Zindars uses this modulation in order to negotiate the unusual succession of chord qualities in the major diatonic cycle: by beginning the progression on iii in a major key and modulating to the relative minor before returning to it, he is able to have his cake and eat it too—the chord root E appears first as a minor seventh and again later as a dominant of the relative minor.

One more example will suffice to conclude our discussion of diatonic cycles: Jerome Kern’s “All the Things You Are” (the changes are given in Figure 1.13).[92] Though this tune does not contain a cycle as explicit as the examples we have seen so far, viewing this piece through the lens of diatonic seventh chord space reveals relationships that may otherwise go unnoticed.

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[91] Exactly which chord functions as the pivot is of little importance, so long as it happens before the E dominant seventh. I am inclined to hear the Fmaj7 as a pivot (functioning simultaneously as IV and VI), in order to keep both ii–V–I progressions (in C major and A minor) intact.

[92] These changes are again taken from *The Real Book*. The Am7♭5 chord in the sixth bar of the A′ section is not included in some charts of this tune, so I have put it in parentheses here. (The older, illegal *Real Book* as well as another illegal fake book called simply *The Book* both omit this chord.)

A	F-^{7}	B♭-^{7}	E♭^{7}	A♭^{maj7}
	D♭^{maj7}	G^{7}	C^{maj7}	
A′	C-^{7}	F-^{7}	B♭^{7}	E♭^{maj7}
	A♭^{maj7}	(A-$^{7♭5}$) D^{7}	G^{maj7}	E$^{7♯9}$
B	A-^{7}	D^{7}	G^{maj7}	
	F♯-$^{7♭5}$	B^{7}	E^{maj7}	C$^{7♯5}$
A″	F-^{7}	B♭-^{7}	E♭^{7}	A♭^{maj7}
	D♭^{maj7}	G♭^{7}	C-^{7}	B^{o7}
	B♭-^{7}	E♭^{7}	A♭^{maj7}	

Figure 1.13. Changes to “All the Things You Are” (Jerome Kern).

“All the Things You Are” begins in the key of F minor, and progresses through a diatonic cycle (a chain of t_3 operations in an F-minor diatonic seventh-chord GIS) until arriving on D♭maj7 in m. 5.[93] At this point, the chord roots continue to descend by diatonic fifth in the key of F minor, but the qualities of the chords rooted on G and C have been altered, from Gm7♭5 and C7 (as they would be in the F-minor GIS) to G7 and Cmaj7. This arrival on a C major chord, rather than a C dominant seventh, has the effect of a half cadence in the prevailing key of F minor. The half-cadential C-major chord also serves as a linking chord to the next phrase, which contains a diatonic cycle in the key of C minor. Like the first A section, this phrase also veers away from the cycle to end in a half cadence on G major.

The bridge of this tune is usually described as being made up of two ii–V–I progressions, the first in G major and the second in E major. While this is true, we might also understand this progression as an alteration of a diatonic cycle in E minor. The bridge begins on a iv chord, which initiates a ii–V–I in the relative major. After two bars of Gmaj7, a t_6 takes us to F♯m7♭5. Just as in the bridge to "Autumn Leaves," we can understand this t_6 as a combination of two t_3 operations, interpolating a missing Cmaj7 chord.[94] Though the cycle of the bridge (and its associated GIS) is in the key of E minor, the final chord is an E *major* seventh, a kind of Picardy third; in this way the bridge, like the A sections, can end on a major chord.

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[93] I tend to hear the opening of “All the Things You Are” in F minor, though it is certainly possible to hear it in A♭ major instead. In either case, the diatonic GIS is nearly identical; the only difference is the quality of the C chord (Cm7 in A♭, and C7 in F minor). Readers who prefer to hear the opening in A♭ can easily make the necessary alterations to the commentary here.

The final A section of "All the Things You Are" is an expanded version of the first, now ending in the overall tonic of A♭. It begins, after a linking C7♯5 chord, by outlining a series of t_3 operations that lead to D♭maj7.[95] At this point we might expect the t_3s to continue, leading to Gm7♭5–C7–Fm (the tonic of the cycle), but instead we see G♭7–Cm7–B°7. This progression is clearly not diatonic, so we cannot say much about it at this point. It is interesting to note, however, that the harmony four bars from the end is a B♭m7, which is exactly where the chain of t_3s would have arrived, had it continued (see Figure 1.14). After this phrase expansion, the piece closes with a ii–V–I in the key of A♭.

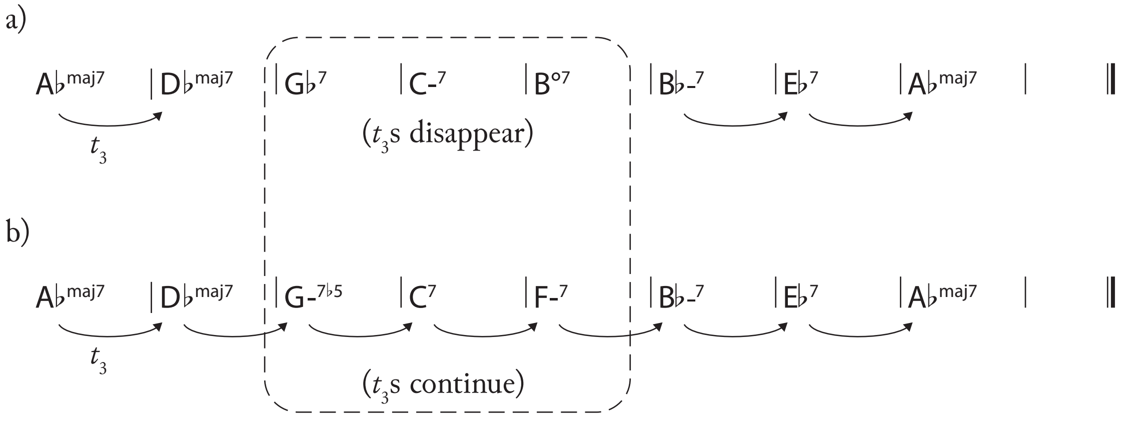

Figure 1.14. The final nine bars of "All the Things You Are."
a) The changes as written.
b) A hypothetical version that continues the t_3 cycle in F minor/A♭ major.

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[94] Again, this implicit diatonicism has not gone unnoticed; Ahmad Jamal arrives emphatically on Cmaj7 in the fourth bar of the bridge on *Jamal at the Pershing, Vol. 2*.

[95] If the linking dominant did not appear at the end of the bridge, we would see the succession Emaj7–Fm7. This succession is remarkably similar to Lewin's SLIDE operation: retaining the third of the chord while moving the root and fifth by half-step; see *GMIT*, 178. Here, the seventh is also retained as a common tone; we will return to this operation (which we will call SLIDE_7) in the next chapter.

Again, we might pause to ask what is gained by hearing "All the Things You Are" in diatonic, rather than chromatic, space. After all, there is never a clear statement of a diatonic cycle in the manner of "Autumn Leaves" or "Alice in Wonderland," and music in chromatic space (as we will begin to see in the next chapter) still tends to descend by fifth. Without the guiding influence of F minor, though, the succession of chord qualities at the beginning is difficult to make sense of: two minor chords, followed by a dominant seventh, then two major chords, all seemingly unrelated to the phrase-concluding tonic C major.[96] Understanding the first 8 bars as diatonic, ending with a tonicized half cadence, makes sense of the chord qualities, and eliminates the difficult-to-explain third relation A♭M–CM that results from a desire to hear both V–I progressions in the A section as tonic-defining.[97]

Hearing "All the Things You Are" diatonically allows us to listen to sections of music at once: the first eight bars are in F minor, the next eight in C minor, the bridge in E minor, and the last twelve return to F minor before shifting to the relative major, A♭, for the final cadence. When we hear the tune as a chain of t_3 operations in shifting diatonic spaces, our attention is drawn to the connections between the spaces—key areas—themselves, rather than the (comparatively boring) series of descending diatonic fifths that occur within them. While transformational analyses are often accused of privileging chord-to-chord connections to the detriment of long-range hearing, in this case the GIS framework developed above allows us to hear over longer distances where chord-to-chord connections may fail to do so (while, crucially, still recognizing the importance of chord-to-chord connections for a jazz musician aiming to "make the changes").

Though they do appear occasionally, most jazz tunes do not contain diatonic cycles, and thus we will need to expand the transformational approach introduced here to account for a larger portion of jazz practice. Chapter 2 will outline an approach to chromatic space that will help to understand the ii–V–I progressions that we passed over in our discussion of diatonic space.

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[96] Henry Martin analyzes the A section simply as a series of descending fifths (taking note of the aberrant root tritone D♭–G) ending on C major. His analysis assumes a chromatic space, in which the harmonic progressions are more difficult to make sense of: he notes that "a certain tonal ambiguity pervades this piece" ("Jazz Harmony," 15–19.).

[97] Hearing the third-relations does have the nice side effect, absent from our analysis here, that all of the phrase-ending major-sevenths in the first sixteen bars (A♭, C, E♭, G) spell out the tonic major-seventh chord; ibid., 19 makes this observation.

Transformational approaches, and neo-Riemannian theories in particular, have flourished partly because of their ability to explain non-functional progressions that contain primarily root motion by thirds. Chapter 3 will draw upon this literature to approach jazz, especially common after bebop, that is more dependent on thirds than fifths for structure. Though harmony is crucially important to performing jazz musicians, much of the jazz pedagogical literature equates chords with scales: a Dm7 chord is functionally equivalent to a D dorian scale, for example. Chapter 4 will develop a transformational approach for these "chord-scales," treating scales as first-class harmonic objects. Finally, Chapter 5 will bring the theoretical work of the early chapters to a close, by taking a close analytic look at tunes based on George Gershwin's "I Got Rhythm."

CHAPTER 2
ii–V Space

Because most jazz is not purely diatonic, we need to expand our transformational system to account for more chromatic examples. This chapter will begin that work, taking the very common ii–V–I progression as its basis. Most of the spaces we will develop in this chapter will fall under the broad category of "fifths spaces," but at the end of the chapter we will have occasion to return to the diatonic space of Chapter 1 to see how they might be enhanced with the chromatic spaces introduced here.

2.1 A Descending Fifths Arrangement

2.1.1 FORMALISM

The most common harmonic progression in jazz is undoubtedly the ii^7–V^7–I^7 progression (hereafter, simply ii–V–I, or often just ii–V). It is the first progression taught in most jazz method books, and the only small-scale harmonic progression to have an entire Aebersold play-along volume dedicated to it.[1] The progression is so prevalent that many jazz musicians describe tunes in terms of their constituent ii–Vs; a musician might describe the bridge of "All the Things You Are" (shown in Figure 2.1) as being "ii–V to G, ii–V to E, then V–I in F."[2] Given the importance of this progression for improvising jazz musicians, it seems natural to use it as the basis for developing a more general transformational model of jazz harmony.

Earlier versions of this chapter were presented at the annual meetings of the Music Theory Society of the Mid-Atlantic (Philadelphia, PA, March 2013) and the Society for Music Theory (Milwaukee, WI, November 2014). I am grateful for the many helpful comments and questions I received at these conferences (especially those from Stefan Love, Brian Moseley, and Keith Waters).

[1] Jamey Aebersold, *The II–V⁷–I Progression*, Jamey Aebersold Play-A-Long Series, vol. 3 (New Albany, IN: Jamey Aebersold Jazz, 1974). The Aebersold play-along series is a staple of jazz pedagogues; most contain a selection of tunes, along with a CD of a rhythm section so that students can practice with a recording. The ii–V volume is number three of well over 100, and includes the phrase "the most important musical sequence in jazz!" on the cover.

[2] Our diatonic analysis of "All the Things You Are" in the previous chapter notwithstanding, the ubiquity of ii–V–I progressions means that many jazz musicians are apt to hear the progression as successions of ii–Vs, even in cases where a diatonic pattern may be present.

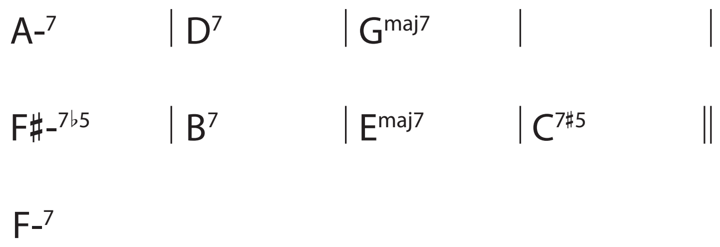

Figure 2.1. The bridge of “All the Things You Are” (Jerome Kern).

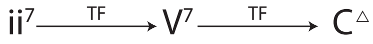

Figure 2.2. A transformation network for a ii–V–I in C major: Dm7–G7–Cmaj7.

Figure 2.2 shows a transformation network for a single ii–V–I progression; we will begin by developing the formal apparatus for this progression, after which we can begin to combine ii–V–I progressions to form a larger musical space.[3] This figure, with its combination of general Roman numerals and specific key centers, is designed to reflect how jazz musicians tend to talk about harmony; we might read this network as “a ii–V–I in C.” The combination of Roman numerals and key areas bears some similarity to Fred Lerdahl’s chordal-regional space, but Figure 2.2 is a transformation network, while chordal-regional space is strictly a spatial metaphor.[4]

We have encountered one transformation network already (Figure 1.7 in the previous chapter), but we have yet to define the concept formally. Transformation networks are a major part of David Lewin’s project in *GMIT*, and have been thoroughly covered in the literature, so we will need to consider the formalism only briefly here.[5] A transformation network consists of objects of some kind (here, they are chords) represented as vertices in a graph, along with some relations (transformations) between them, represented as arrows. In Lewin’s definition, all of the objects in a transformation network must be members of a single set *S*, and the transformations must be

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[3] The triangle on the C chord in this figure indicates a major seventh. The triangle (instead of “maj7”) is intended to save space and reduce clutter in the graphical representations.

[4] Fred Lerdahl, *Tonal Pitch Space* (New York: Oxford University Press, 2004), 96–97.

[5] Lewin’s definition of a transformation network is in *Generalized Musical Intervals and Transformations* (New York: Oxford University Press, [1987] 2007), 196. For a relatively concise summary, see Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 110–16.

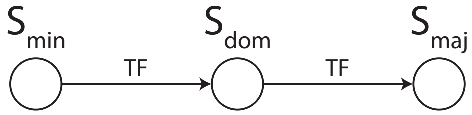

Figure 2.3. The underlying transformation graph for a single ii–V–I progression.

functions from *S* into *S* itself.[6] The transformations in Figure 1.7 are indeed Lewinian transformations (mappings in the G-minor diatonic set), but the ii–V–I transformation network is more complex.

The transformation TF in Figure 2.2 is in fact a *cross-type* transformation, as defined by Julian Hook.[7] Hook expands Lewin's definition of a transformation network to include objects of different types, necessary to define transformations in the ii–V–I progression. The progression contains musical objects of three types of diatonic seventh chords: minor, dominant, and major sevenths (in the key of C major, the progression is Dm7–G7–Cmaj7). Using Hook's relaxed definition, we are free to define transformations from any set of objects to any other; to understand the figure above, we need to define the transformation TF such that it maps ii^7 chords to V^7 chords, and V^7 chords to I^7 chords.

Before defining the transformations, however, we first need to define the sets themselves. To help with this, Figure 2.3 shows the underlying transformation graph of the transformation network in Figure 2.2. Throughout *GMIT*, Lewin is careful to distinguish transformation graphs from transformation networks: a graph is an abstract structure, showing only relations (transformations) between unspecified set members, while a network realizes a graph, specifying the actual musical objects under consideration.[8] Because cross-type transformation graphs contain objects of different types, a node in a cross-type transformation graph must be labeled with the set from which the node contents may be drawn (even in the abstract transformation graph).[9] In

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[6] *GMIT*, Definitions 9.3.1 (196) and 1.3.1 (3).

[7] Julian Hook, "Cross-Type Transformations and the Path Consistency Condition," *Music Theory Spectrum* 29, no. 1 (April 2007): 1–40.

[8] *GMIT*, 195–96 and throughout. See also Hook, "Cross-Type Transformations," 6–8.

[9] Ibid., 7.

Figure 2.3, the nodes are labeled simply S_{min}, S_{dom}, and S_{maj}, which we can understand as the sets of minor, dominant, and major seventh chords, respectively.

While at its core the ii–V–I progression contains three types of seventh chords, in reality a jazz musician might add any number of extensions or alterations to this basic structure. Given this practice, defining the archetypal progression as being composed of four-note set classes (seventh chords) seems unnecessarily restrictive. In order to allow for some freedom in the chord qualities, we will consider only chordal roots, thirds, and sevenths; these pitches are sufficient to distinguish the three chord qualities in a ii–V–I.[10]

In this chapter, we will represent a chord with an ordered triple $X = (x_r, x_t, x_s)$, where x_r is the root of the chord, x_t the third, and x_s the seventh. The definitions of the three sets are as follows:[11]

$$S_{\text{min}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 3; x_s - x_r = 10\} \qquad \text{ii}^7 \text{ chords}$$

$$S_{\text{dom}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 10\} \qquad \text{V}^7 \text{ chords}$$

$$S_{\text{maj}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 11\} \qquad \text{I}^7 \text{ chords}$$

The definitions are intuitive, and have clear musical relevance: ii^7 chords have a minor third (interval 3) and minor seventh (interval 10), V^7 chords have a major third and minor seventh (intervals 4 and 10), and I^7 chords have a major third and major seventh (intervals 4 and 11). Defining the chords this way rather than as four-note set classes offers the great advantage of flexibility. Using the ordered-triple representation, the progressions Dm7–G7–Cmaj7 and Dm9(♭5)–G7♭13♯9♭9–Cmaj7♯11 are understood as equivalent, since the roots, thirds, and sevenths are the same: both progressions are represented $(2, 5, 0)$–$(7, 11, 5)$–$(0, 4, 11)$. Because the sets are defined in pitch-class space, the three sets all have cardinality 12: each pitch class is the root of exactly one ii^7, V^7, and I^7 chord.

---

[10] In fact, many jazz piano texts begin with "three-note" or "shell" voicings, consisting only of chordal roots, thirds, and sevenths; see, for example Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 17–22; and Joe Mulholland and Tom Hojnacki, *The Berklee Book of Jazz Harmony* (Boston: Berklee Press, 2013), 211–12.

[11] Here and throughout this chapter, pitch classes are represented as mod-12 integers, with $C = 0$; all calculations are performed mod-12.

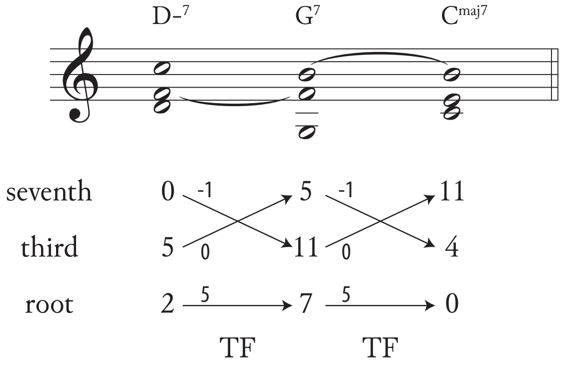

Figure 2.4. Voice leading in the ii–V–I progression.

With the space of the nodes defined, we can now formulate the transformation representing a ii–V–I, which we will call simply "TF":

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{min}}, \quad \text{then} \quad \text{TF}(X) = Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 1, x_t) \in S_{\text{dom}}$$

$$\text{If } Y = (y_r, y_t, y_s) \in S_{\text{dom}}, \quad \text{then} \quad \text{TF}(Y) = Z = (z_r, z_t, z_s) = (y_r + 5, y_s - 1, y_t) \in S_{\text{maj}}$$

Again, these definitions are designed to be musically relevant; the voice-leading diagram in Figure 2.4 illustrates this more clearly.[12] The root of the second chord is a fifth below the root of the first ($y_r = x_r + 5$), the third of the second chord is a semitone below the seventh of the first ($y_t = x_s - 1$), and the seventh of the second chord is a common tone with the third of the first ($y_s = x_t$). In Lewin's transformational language, if a jazz musician is "at a ii^7 chord" and wishes to "get to a V^7 chord," the transformation that will do the best job is TF: "move the root down a fifth and the seventh down a semitone to become the new third." (Recall that we may also write ii$^7 \xrightarrow{\text{TF}}$ V^7, rather than TF(ii^7) = V^7.) Note that the transformation TF is also valid between V^7 and I^7 (the second equation above, involving sets S_{dom} and S_{maj}). TF is both one-to-one and onto for sets of ordered triples; it maps each ii^7 to a unique V^7, and each V^7 to a unique I^7. As such, its inverse (TF^{-1}) is well defined, and allows motion backwards along the arrows shown in the transformation graph in Figure 2.3.

---

[12] This figure represents what Joseph Straus calls "transformational voice leadings" in his study of atonal voice leading; "Uniformity, Balance, and Smoothness in Atonal Voice Leading," *Music Theory Spectrum* 25, no. 2 (October 2003): 305–52.

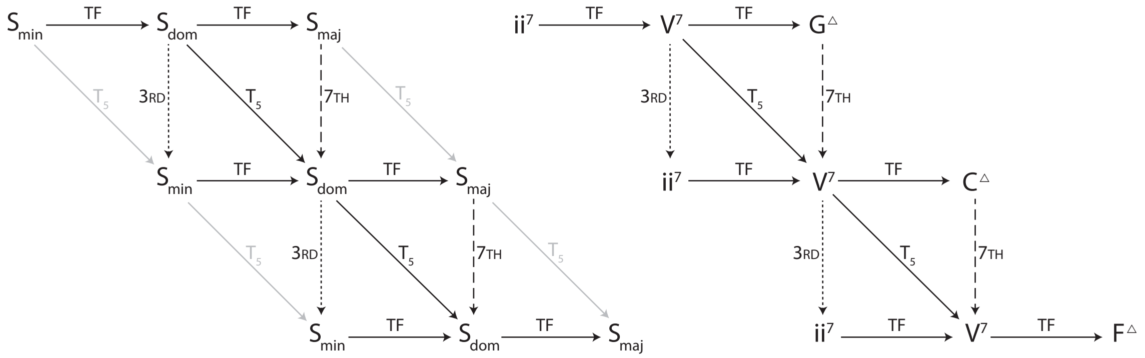

Figure 2.5. A transformation graph (left) and transformation network (right) for a small portion of ii–V space.

It is worth mentioning here that TF and TF_{T} (which we will define in the next section) are well-defined operations for any ordered triple of members of the integers mod-12 (i.e., a member of the set $\mathbb{Z}_{12} \times \mathbb{Z}_{12} \times \mathbb{Z}_{12}$). There is nothing mathematically incorrect about the statement $(0,1,2) \xrightarrow{\mathrm{TF}} (5,1,1) \xrightarrow{\mathrm{TF}} (10,0,1)$, for example, but this succession has little musical relevance for the applications under consideration here. Because Hook does not formally define what he means by a "type," the formulation allows for situations like this one, in which the three types are all members of a single larger set.[13] The advantage for defining TF as a cross-type transformation is that the content of a single node in the transformation graph is restricted to members of a 12-element set of specific ordered-triple configurations.

With this understanding of the transformations involved in a single ii–V–I progression, we can continue to see how we might connect multiple progressions in order to form a larger ii–V space. Because root motion by descending fifth is extremely common in jazz, we might consider connecting ii–V–I progressions by descending fifth; Figure 2.5 illustrates this arrangement both as a transformation graph and a transformation network. This descending fifths arrangement means that all of the chords sharing a root are aligned vertically (directly below Gmaj7 is G7, which is itself above Gm7). This arrangement allows us to define two more transformations, which we will call simply 7TH and 3RD:

---

[13] Hook himself makes this clear, noting that for any two sets S and T it is possible to define a single-type transformation in the union set S ∪ T, though it is not always clear how a function defined on one set should be extended to cover both. He also notes that even when a single-type transformation is possible, "the cross-type approach is often simpler and more natural," which certainly seems to be the case here. "Cross-Type Transformations," 5n8.

$$\begin{aligned} &\text{If } L = (l_r, l_t, l_s) \in S_{\text{maj}}, && \text{then} \quad 7\textsc{th}(L) = M = (m_r, m_t, m_s) = (l_r, l_t, l_s - 1) \in S_{\text{dom}} \\ &\text{If } M = (m_r, m_t, m_s) \in S_{\text{dom}}, && \text{then} \quad 3\textsc{rd}(M) = N = (n_r, n_t, n_s) = (m_r, m_t - 1, m_s) \in S_{\text{min}} \end{aligned}$$

Like the TF transformation, the 7TH and 3RD transformations have clear musical relevance: each lowers the given note by a semitone. Although adjacent progressions are connected by descending fifth, the T_5 labels connecting adjacent ii^7 chords and I^7 chords are shown in gray in the graph (and omitted in the network, and in later examples), since these chords are not often directly connected in jazz.

By extending the network of Figure 2.5, we arrive at the entirety of ii–V space, as shown in Figure 2.6. Because ii–V space includes cross-type transformations, it does not easily form a Lewinian GIS.[14] Considered more generally, though, it is easy to see that by considering a single ii–V–I progression as a unit, ii–V space maps cleanly onto ordinary pitch-class space. As Figure 2.6 makes clear, we can consider the ii–V–I in C as being three perfect fifths above the ii–V–I in E♭ (or put transformationally, the T_3 operation transforms a ii–V–I in C to one in E♭). This formulation does not allow us a means to say, for example, that "the ii chord in C is *x* units away from the V chord in E♭," but because ii–V–Is are rarely split up, falling back on normal pitch-class distance is sufficient in most situations.[15]

---

[14] It would be possible to form a GIS by considering all ordered triples as the group, as suggested above. While this is possible, defining an interval function in this group is much more difficult: such a function would need to account for the 36 ordered triples in ii–V space (ii^7, V^7, and I^7 chords) as well as the many more (1692) that are not included in the space. Such a function is conceivable, but would not in any case reflect the musical realities ii–V space is interested in portraying.

[15] ii–V space is a directed graph, so in circumstances where the pitch-class distance metric is somehow not sufficient, we can instead rely on the standard way of measuring distance in a directed graph: by counting the number of edges in the shortest path between two chords. The distance from ii^7 of C to V^7 of E♭ is then 4: ii to V in C (1 edge), then 3 T_5s to V of E♭.

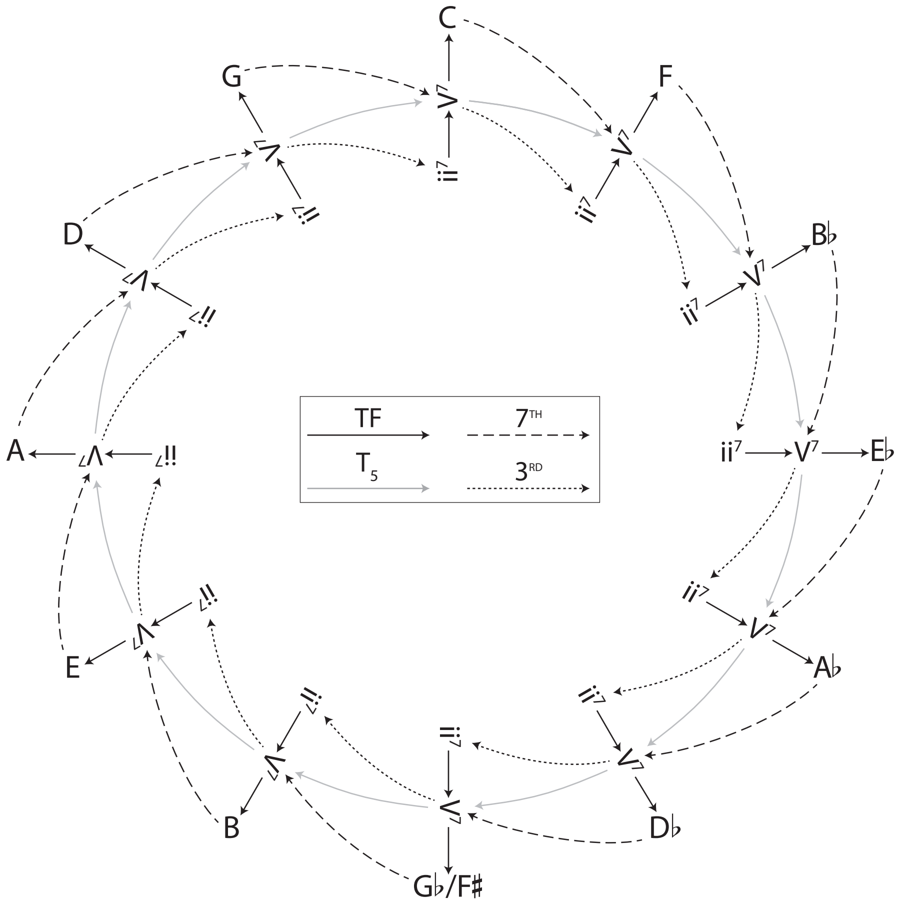

Figure 2.6. The complete ii–V space, arranged around the circle of fifths.

2.1.2 Analytical Interlude: Lee Morgan, “Ceora”

Though we will return to the formalism a bit later, we have defined enough of ii–V space at this point to see how it might be useful in analysis. To do so, we will examine Lee Morgan’s composition “Ceora,” first recorded on the 1965 album *Cornbread*. The changes for the A section are given in Figure 2.7, and the accompanying moves in ii–V space are shown in Figure 2.8.[16] “Ceora” is in the key of A♭ major, and begins with the progression I–ii–V–I in the first three bars, staying within a single horizontal slice of ii–V space. This is followed immediately by a ii–V–I progression in D♭, a fifth lower (mm. 4–5).

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[16] These changes are taken from *The Real Book*, and reflect what is played on the *Cornbread* recording. In this figure, the circle indicates the tonic, while the numbers on the labels indicate the order of transformations. Because “Ceora” uses only a part of the space, the circle of Figure 2.6 has been squared off here so that the labels are easier to read.

At this point, we might expect the ii–Vs to continue in descending fifths, but the potential ii^7 chord in G♭ is substituted with Dm7, its tritone substitute, which then resolves as a ii–V in C.[17] Instead of resolving to C major in m. 7, this ii–V resolves instead to C *minor*: both the seventh and third of the expected Cmaj7 are lowered to become Cm7. (This progression is extremely common, and is one of the principal means of maintaining harmonic motion in the course of a jazz tune.)

A similar progression in B♭ follows, leading to a B♭m7 chord in m. 9. We then hear a ii–V progression in the tonic in mm. 9–10, but the expected A♭maj7 does not materialize; the E♭7 chord moves instead to Cm7 as ii of B♭ (a northwesterly move in the space). This progression repeats in mm. 11–13, leading once again to the Dm7 chord first heard in m. 5. The repeated upward motions in the space have the effect of ramping up the tonal tension in the passage; not only do the dominants fail to resolve as expected, but their stepwise rising motion takes the music far away from the tonic A♭. To release this harmonic tension, the ii–V in C resolves at m. 15 to C minor, at which point the harmonic rhythm doubles and the progression follows the normal descending fifths pattern to reach the tonic that begins the B section in m. 17.

The B section of "Ceora" (shown in Figure 2.9) follows much of the same trajectory as the A section until the last four bars; the only differences are the addition of the ♭5 in the Cm7 and the ♯9 in the F7 in mm. 11–12 of the section. Because we have defined chords and transformations only in terms of chordal roots, thirds, and sevenths, neither of these changes affect our transformational reading of the passage. Instead of ramping up to ii^7 of C as in the A section, the ii–V in B♭ resolves to B♭m7 in m. 12. This B♭m7 becomes the ii chord of a ii–V–I in tonic, resolving in m. 15 of the section. A final ii–V in the last measure provides additional harmonic interest, and functions as a turnaround to lead smoothly back to A♭maj7 to begin the next chorus.

At this point, we have successfully mapped all of the chords in "Ceora" to their associated locations in ii–V space; it is reasonable to ask, though, whether this mapping of chords to space

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[17] We will return to the concept of tritone substitutes in the next section.

A♭^{maj7}	B♭–^{7} E♭^{7}	A♭^{maj7}	E♭–^{7} A♭^{7}
D♭^{maj7}	D–^{7} G^{7}	C–^{7}	F$^{7\sharp 9}$
B♭–^{7}	E♭^{7}	C–^{7}	F^{7}
D–^{7}	G^{7}	C–^{7} F^{7}	B♭–^{7} E♭^{7}

Figure 2.7. Changes for the A section of “Ceora” (Lee Morgan).

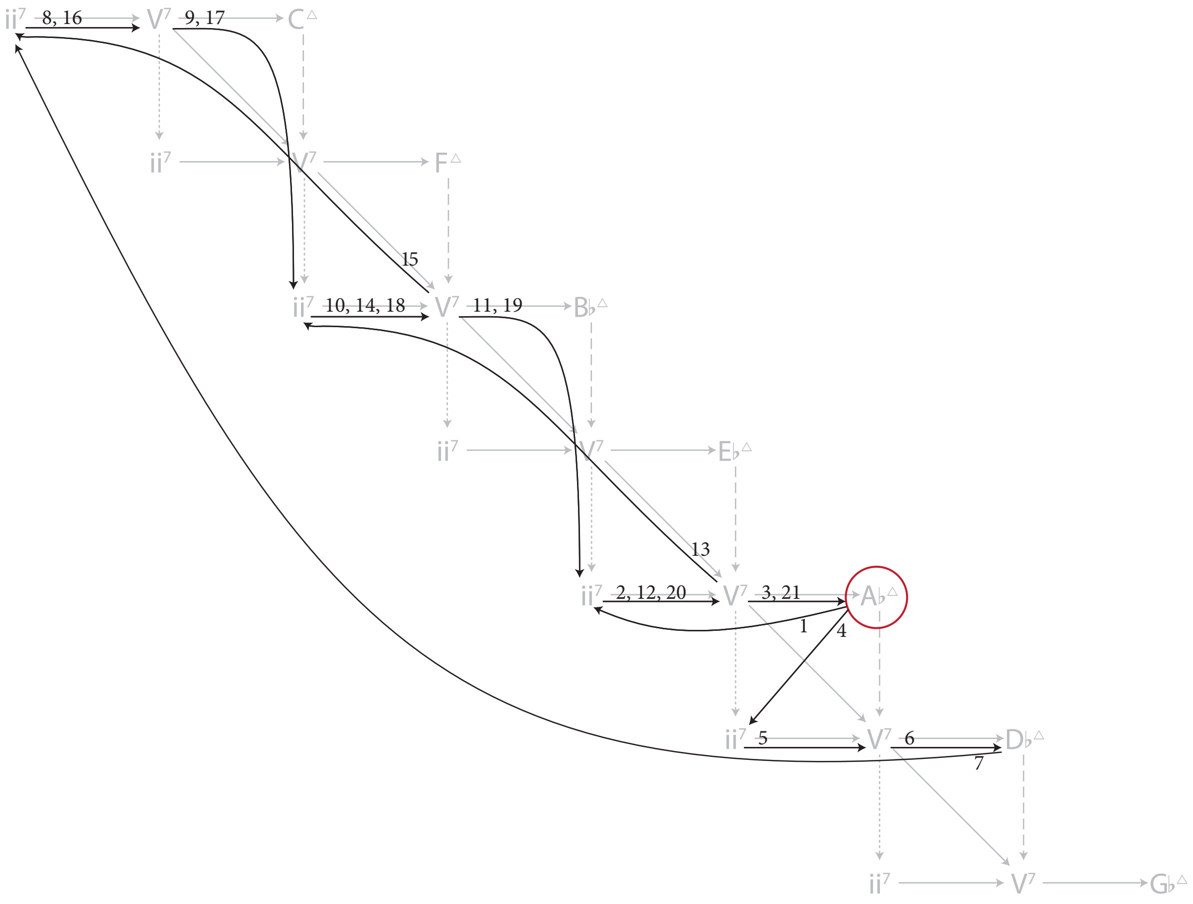
ii7
8, 16
V7
9, 17
C△
ii7
V7
F△
15
ii7
10, 14, 18
V7
11, 19
B♭△
ii7
V7
E♭△
13
ii7
2, 12, 20
V7
3, 21
A♭△
1
4
ii7
5
V7
6
D♭△
7
ii7
V7
G♭△

Figure 2.8. The A section of “Ceora” in ii–V space.

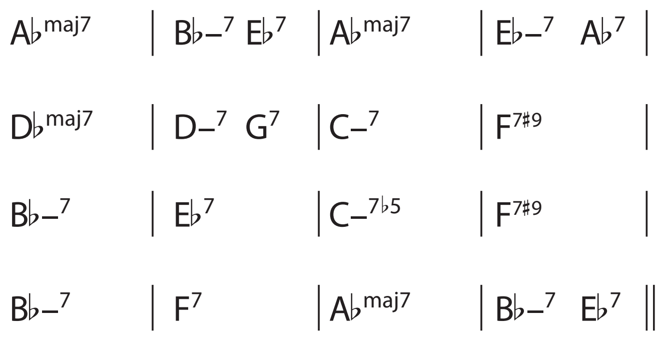
A♭maj7 | B♭–7 E♭7 | A♭maj7 | E♭–7 A♭7 |
D♭maj7 | D–7 G7 | C–7 | F7♯9 |
B♭–7 | E♭7 | C–7♭5 | F7♯9 |
B♭–7 | F7 | A♭maj7 | B♭–7 E♭7 ||

Figure 2.9. Changes for the B section of "Ceora" (Lee Morgan).

locations should even count as "analysis." After all, ii–V space contains each minor, dominant, and major seventh chord exactly once, so we did not even need to make any decisions as to where in the space a particular chord should go. Have we, in fact, learned anything about "Ceora" from our exploration of ii–V space?

The answer, I think, is yes. Although we could criticize ii–V space for being simply a particular arrangement of common harmonic progressions in jazz, similar arrangements have proven themselves useful in many areas of music theory: the circle of fifths, the neo-Riemannian Tonnetz, the pitch-class "clock face," and countless others.[18] One of the benefits of ii–V space is that it allows us to easily visualize common harmonic motions in jazz. The succession of chord symbols that make up the changes to "Ceora" may make immediate sense to an experienced jazz musician, but ii–V space allows others to to make sense of these relationships more clearly. The fact that our analysis in ii–V space may seem obvious is in fact a feature, not a bug; such a criticism reveals that ii–V space, with all its mathematical formalism, can clarify information that may otherwise remain hidden in the raw data of the chord symbols.[19]

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[18] For a study of many different musical spaces (and a defense of their use), see Julian Hook, *Exploring Musical Spaces* (New York: Oxford University Press, forthcoming), Chapter 1 and throughout.

[19] Music analysis is in many ways similar to the field of data visualization: both often involve revealing the underlying structure of what might otherwise seem like an undifferentiated stream of data. As Edward Tufte tells us, visualizations are often "more precise and revealing" than other, mathematical means of analyzing data. *The Visual Display of Quantitative Information*, 2nd ed. (Cheshire, CT: Graphics Press, 2001), 13–14.

2.2 Tritone Substitutions

2.2.1 FORMALISM

There is an important aspect of jazz harmony that has not yet been considered in our discussion of ii–V space. Crucial to harmony beginning in the bebop era is the tritone substitution: substituting a dominant seventh chord for the dominant seventh whose root is a tritone away.[20] Because tritone-substituted dominants are functionally equivalent, both the progressions Dm7–G7–Cmaj7 and Dm7–D♭7–Cmaj7 may be analyzed as ii–V–I progressions in the key of C.

This functional equivalence means that a tritone-substituted dominant can act as a shortcut to an otherwise distant portion of ii–V space. In the circle-of-fifths arrangement of Figure 2.6, keys related by tritone are maximally far apart (diametrically opposed on the circle), but in jazz practice, G7 and D♭7 are functionally identical (both dominant-function chords in C major). To account for this progression in our space, we need to somehow bring these chords closer together; one solution is to connect two segments of the space by T_6 in a sort of "third dimension," as shown in Figure 2.10. The topology of this space is more complicated than the ordinary circle of fifths, however. Once a progression reaches the bottom of the "front" side of the figure, it reappears at the top of the "back" side (G♭ at the bottom is listed again as F♯ at the top); likewise, progressions disappearing off the bottom of the back side reappear at the top of the front side (C major is given in both locations).

This arrangement of key centers is topologically equivalent to a Möbius strip, which is somewhat easier to see by focusing only on the dominant seventh chords, as shown in Figure 2.11.[21] By wrapping this figure into a circle and gluing the left and right edges together with a half-twist (so that the two G7 chords and the D♭7/C♯7 match up), we arrive at the desired Möbius

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[20] The tritone substitution has been discussed extensively in the literature, so we will not discuss it at any length here. See, for example, Nicole Biamonte, "Augmented-Sixth Chords vs. Tritone Substitutes," *Music Theory Online* 14, no. 2 (June 2008); Henry Martin, "Jazz Harmony: A Syntactic Background," *Annual Review of Jazz Studies* 4 (1988): 11; and Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 360–65.

[21] A similar diagram can be found in Werner Pöhlert, *Basic Harmony*, trans. Jürgen Krohn and Norman Bowie (Werner Pöhlert Publications, 1989), 5, and can be seen implicitly in Figure 1-1 of Martin, "Jazz Harmony."

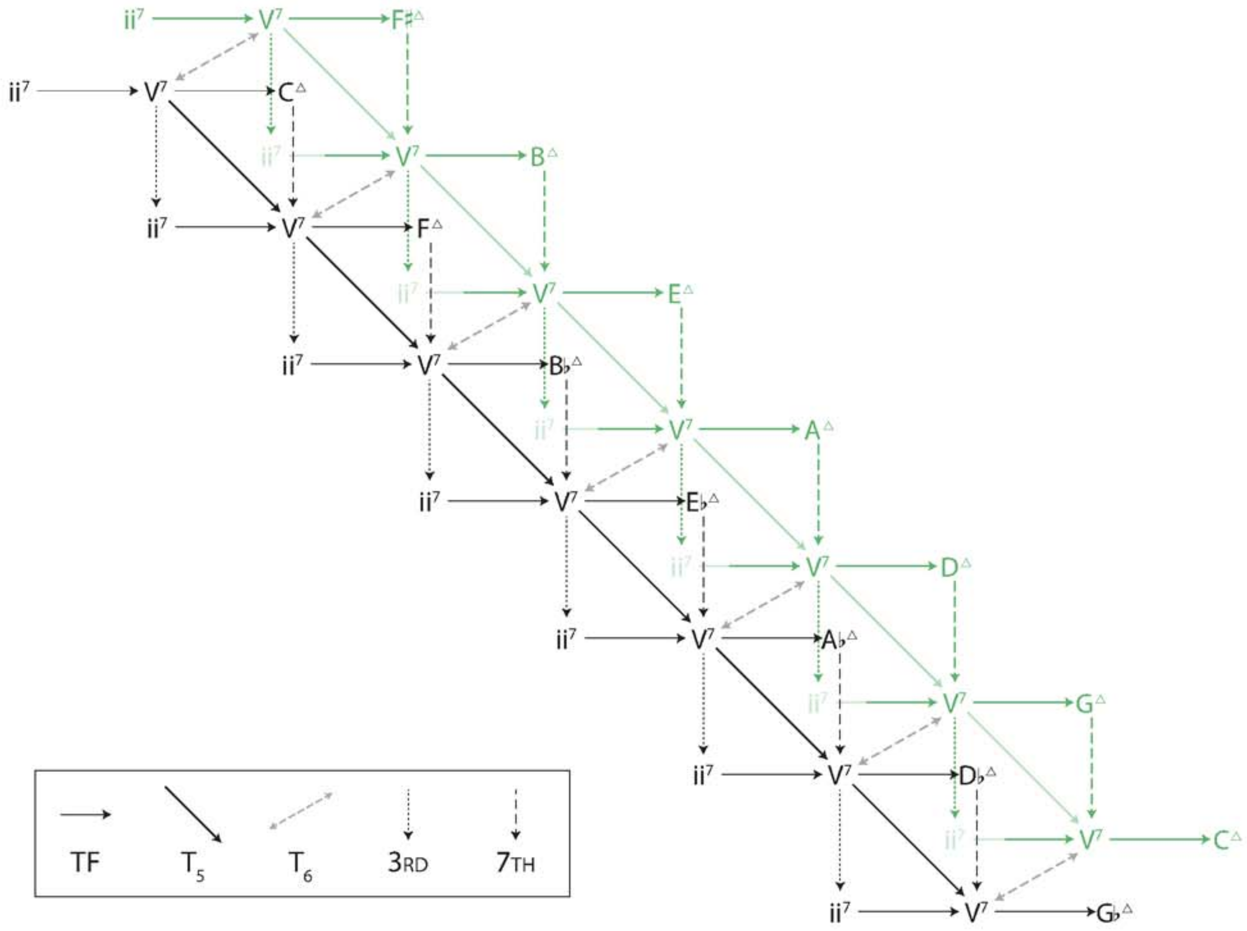
ii7
V7
F♯Δ
ii7
V7
CΔ
ii7
V7
BΔ
ii7
V7
FΔ
ii7
V7
EΔ
ii7
V7
B♭Δ
ii7
V7
AΔ
ii7
V7
E♭Δ
ii7
V7
DΔ
ii7
V7
A♭Δ
ii7
V7
GΔ
ii7
V7
D♭Δ
ii7
V7
CΔ
ii7
V7
G♭Δ
TF
T5
T6
3RD
7TH

Figure 2.10. The complete ii–V space, showing tritone substitutions.

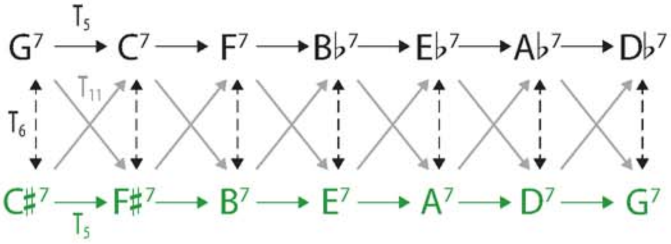
T5
G7
C7
F7
B♭7
E♭7
A♭7
D♭7
T11
T6
C♯7
F♯7
B7
E7
A7
D7
G7
T5

Figure 2.11. The Möbius strip at the center of ii–V space.

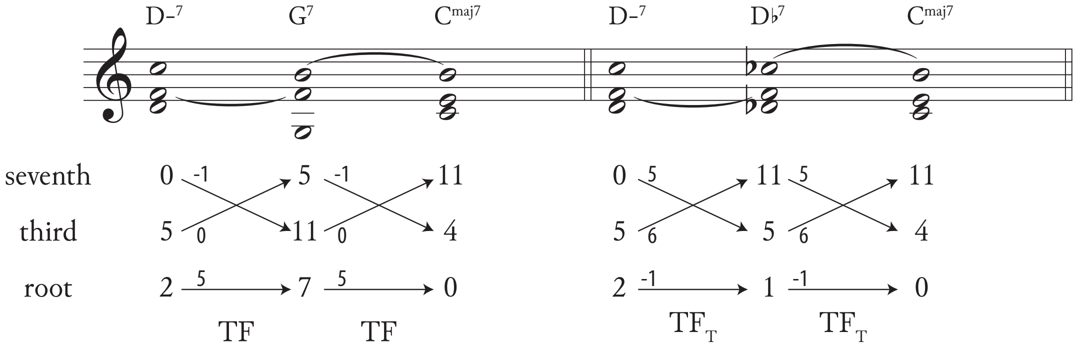

Figure 2.12. Voice leading in the TF and $\mathrm{TF_T}$ transformations, compared.

strip. Though the underlying topology is easier to visualize this way, it is difficult to include all of the other progressions (the ii–Vs themselves) in this diagram, so we will continue to use the "three-dimensional" version of Figure 2.10, with the understanding that this topology remains in effect. In any case, the arrangement of keys into the front and back sides is arbitrary, and may be repositioned as necessary; it is often convenient to have the tonic key (when there is one) centrally located at the front of the space.

While we could navigate this space using only the transformations TF and T_6, it is convenient to define another transformation to help with a common progression like Dm7–D♭7–Cmaj7. We will call this transformation $\mathrm{TF_T}$, to highlight its relationship to the more normative TF:

$$\text{If } X = (x_r, x_t, x_s) \in S_{\min}, \text{ then } \mathrm{TF_T}(X) = Y = (y_r, y_t, y_s) = (x_r - 1, x_s + 5, x_t + 6) \in S_{\mathrm{dom}}$$
$$\text{If } Y = (y_r, y_t, y_s) \in S_{\mathrm{dom}}, \text{ then } \mathrm{TF_T}(Y) = Z = (z_r, z_t, z_s) = (y_r - 1, y_s + 5, y_t + 6) \in S_{\mathrm{maj}}$$

The $\mathrm{TF_T}$ transformation represents a tritone substitution, but it transforms bass motion by fifth into bass motion by semitone (*not* bass motion by tritone); the voice-leading diagram in Figure 2.12 clarifies the relationship with the ordinary TF. Because TF and T_6 commute, $\mathrm{TF_T}$ can be considered as either TF followed by T_6, or vice versa. With this new transformation, we can understand the progression A♭m7–D♭7–Cmaj7 as a substituted ii–V–I in C: A♭m7 $\xrightarrow{\mathrm{TF}}$ D♭7 $\xrightarrow{\mathrm{TF_T}}$ Cmaj7.

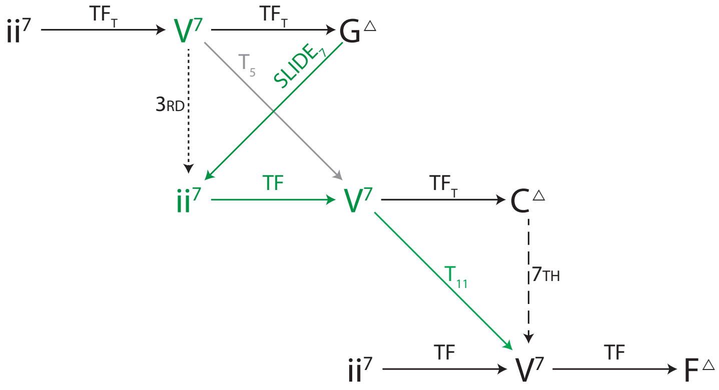

Figure 2.13. A transformation network for a small portion of ii–V space, with tritone substitutions.

The introduction of tritone substitutes complicates the space somewhat; Figure 2.13 shows a transformation network of the same portion of the space as in Figure 2.5, but with some chords replaced with their tritone substitutes (shown in green).[22] The relationship between a substituted dominant in G major (A♭7) and the unsubstituted ii^7 chord in C (A♭m7) is still, of course, a 3RD transformation. The substituted V^7 in C moving to the diatonic V^7 in F changes the transposition from a descending fifth to a descending half-step, as indicated by the T_{11} arrow.

Perhaps most interesting in this tritone-substituted space is the new relationship between a major seventh chord and the substituted ii^7 in the progression a fifth below (in this figure, between Gmaj7 and A♭m7). Normally there is no voice-leading connection between these two chords, but with the substituted ii^7, the third and seventh are both held as common tones, and the root and fifth of the chord both ascend by semitone (from G–B–D–F♯ to A♭–C♭–E♭–G♭).[23] Because of its similarity to the standard SLIDE transformation, with the addition of the common tone seventh, we will call this transformation SLIDE$_7$:[24]

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[22] Determining the structure of the underlying transformation graph of this network is straightforward, so I have not included a figure of it here.

[23] Though we are defining chords as ordered triples in this chapter, I have included the fifth in this description to highlight the relationship to the triadic SLIDE, which maintains the root and fifth of a triad while changing the quality of the third.

[24] The SLIDE transformation was introduced by David Lewin (*GMIT*, 178), but has since become a part of of the standard set of Neo-Riemannian transformations. SLIDE$_7$ is defined here only as a transformation from I^7 chords to ii^7 chords, but of course the triadic SLIDE is an involution (two successive applications of SLIDE to any triad will result in the same triad).

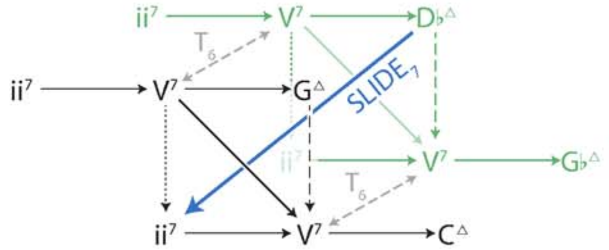


Figure 2.14. The SLIDE$_7$ transformation from D♭maj7 to Dm7 in mm. 5–6 of "Ceora."

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{maj}}, \quad \text{then} \quad \text{SLIDE}_7(X) = Y = (y_r, y_t, y_s) = (x_r + 1, x_t, x_s) \in S_{\text{min}}$$

In jazz this progression occurs frequently when moving between key centers related by half step, though it is uncommon in classical music.[25] We have encountered this transformation once already: the motion from D♭maj7 to Dm7 in mm. 5–6 of "Ceora" is indeed a typical SLIDE$_7$ transformation (see Figure 2.14).

2.2.2 Analytical Interlude: Charlie Parker, "Blues for Alice"

Equipped with these new tritone-substitution transformations, we can now analyze somewhat more complicated music; Charlie Parker's "Blues for Alice" will serve as a useful first example (the changes are given in Figure 2.15).[26] The essential structure of the blues is present: the tune arrives on a subdominant in m. 5, and on a home-key ii–V in m. 9 of the twelve-bar form.

Parker elaborates this basic structure with a series of stepwise descending ii–V progressions (see Figure 2.16). The first of these is a diatonic descent: m. 2 jumps from the tonic F major to a

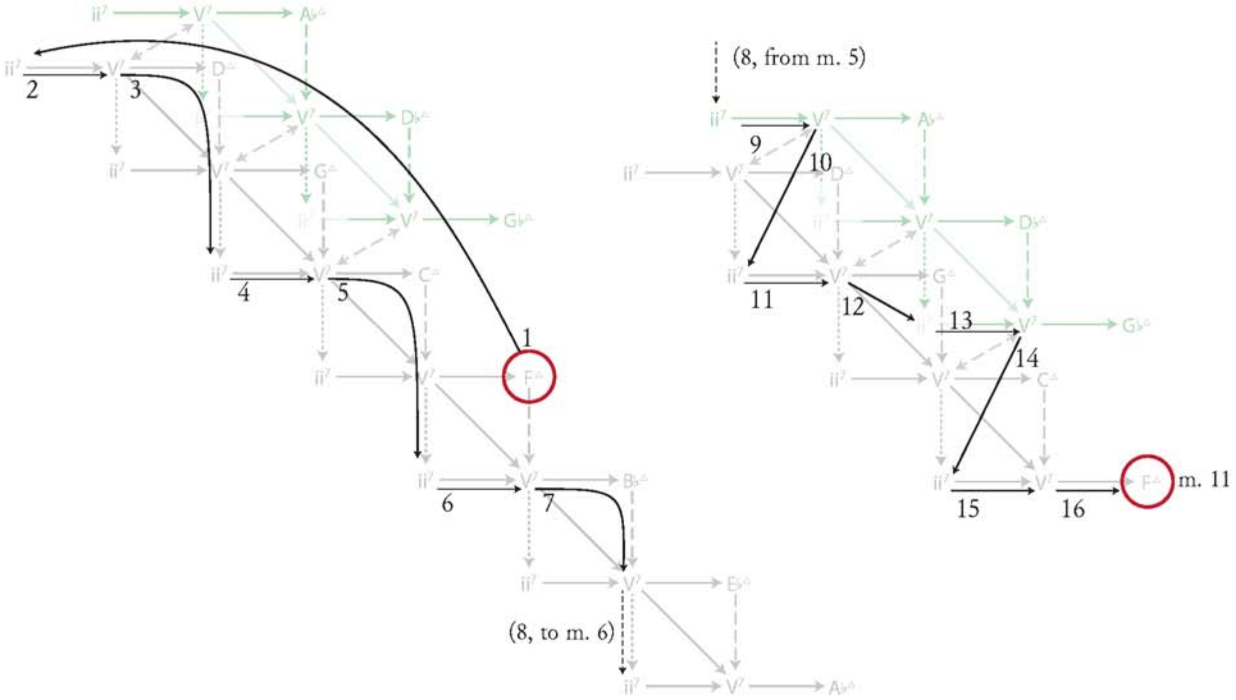
---

[25] The SLIDE$_7$ transformation can be found in a chromatic sequence in the second movement of the Fauré string quartet, mm. 36–39. Julian Hook analyzes this passage from a number of different mathematical perspectives in "Contemporary Methods in Mathematical Music Theory: A Comparative Case Study," *Journal of Mathematics and Music* 7, no. 2 (2013): 89–102.

[26] This progression is often known as the "Bird Blues," though Mark Levine calls it the "descending blues" in *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 228. Like many sets of Parker changes, several different versions exist; the changes here represent a mediation of these sources. *The Real Book* gives Am7 (a vi chord) instead of Fmaj7 in m. 11; Levine's *Jazz Theory Book* gives D♭7 (a tritone substitute) instead of G7 in the second half of m. 3; and the *Charlie Parker Omnibook* omits both D7 chords (the first in the second half of m. 7, the second in m. 11) and the tonic in m. 11 is an F7. Most of these differences are minor, and over the course of a recorded performance the changes might vary among all of these versions. Other compositions that contain this progression include Parker's own "Confirmation," Sonny Stitt's "Jack Spratt," and Toots Thielemans's "Bluesette."

F^{maj7}	$E-^{7\flat5}$ A^7	$D-^7$ G^7	$C-^7$ F^7
$B\flat^7$	$B\flat-^7$ $E\flat^7$	$A-^7$ D^7	$A\flat-^7$ $D\flat^7$
$G-^7$	C^7	F^{maj7} D^7	$G-^7$ C^7

Figure 2.15. Changes to “Blues for Alice” (Charlie Parker).



ii⁷
V⁷
A♭^Δ
D^Δ
D♭^Δ
G^Δ
G♭^Δ
C^Δ
F^Δ
B♭^Δ
E♭^Δ
1
2
3
4
5
6
7
(8, to m. 6)
(8, from m. 5)
9
10
11
12
13
14
15
16
m. 11

Figure 2.16. An analysis of “Blues for Alice” in ii–V space: mm. 1–5 (left), mm. 6–11 (right).

ii–V in D, which resolves (via the 7TH and 3RD transformations) to a Dm7 chord as the ii^7 of C major. We first saw this progression in "Ceora" (mm. 6–7), where we noted that it was a very common way of maintaining harmonic motion; instead of a ii–V resolving to its tonic, it resolves to the minor seventh chord with the same root. Because this progression is so common, it is useful to define it as its own transformation, which we will call EC (for "evaded cadence"). Unlike TF, EC is useful only as a transformation from V^7 chords to ii^7 chords:

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{dom}}, \text{ then } \text{EC}(X) = Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 2, x_t - 1) \in S_{\text{min}}$$

EC is of course equivalent to TF • 7TH • 3RD, but only when the starting chord is a V^7 chord (a member of S_{dom}). In ii–V space, EC can be represented by starting on a dominant, then following one arrow to the right and two arrows downward. The structure of the space immediately shows that EC is impossible beginning on a ii^7 chord; we can follow a single arrow to the right, but there is only one downward arrow from a V^7 chord.[27]

This pattern of stepwise descending ii–Vs continues until arriving at the subdominant B♭ in m. 5, which includes the standard blues alteration of the lowered seventh.[28] The intuition that this B♭7 is in fact a stable harmony, rather than a descending-fifth transposition of F7, can be captured somewhat in our transformational labels. Instead of labeling this progression $\text{F7} \xrightarrow{T_5} \text{B}\flat 7$, we might instead label it as $\text{F7} \xrightarrow{\text{TF} \bullet \text{7TH}} \text{B}\flat 7$; this designation expresses the notion that the B♭7 chord is heard as a resolution to a stable chord (the TF transformation) that has merely been inflected with the lowered seventh (the 7TH transformation). Combining this with the rest of mm. 2–5, it is easy to construct a transformation network:

$$\text{Em7}\flat 5 \xrightarrow{\text{TF}} \text{A7} \xrightarrow{\text{EC}} \text{Dm7} \xrightarrow{\text{TF}} \text{G7} \xrightarrow{\text{EC}} \text{Cm7} \xrightarrow{\text{TF}} \text{F7} \xrightarrow{\text{TF} \bullet \text{7TH}} \text{B}\flat 7$$

---

[27] At least, impossible if we want to stay within the three sets we are studying in this chapter. Like the other transformations we have defined, EC is an admissible transformation on the set of all mod-12 ordered triples: if we begin with a Dm7 chord, $(2, 5, 0)$, EC gives us the triple $(7, 10, 4)$, which of course is not a major, minor, or dominant seventh chord.

[28] The major-minor seventh chord as a stable chord is characteristic of the blues; see, for example, James McGowan, "Psychoacoustic Foundations of Contextual Harmonic Stability in Jazz Piano Voicings," *Journal of Jazz Studies* 7, no. 2 (October 2011): 158–59 and throughout. This fact is somewhat obscured in ii–V space, since major-minor sevenths appear in the space only as V^7 chords; we will return to this limitation later in Section 2.3.2.

After this B♭7 chord, Parker uses a *chromatic* stepwise pattern of ii–Vs (mm. 5–10), which we can understand as a tritone-substituted version of the earlier descending fifths pattern:

$$\text{B}\flat\text{m7} \xrightarrow{\text{TF}} \text{E}\flat 7 \xrightarrow{\text{EC}_\text{C}} \text{Am7} \xrightarrow{\text{TF}} \text{D7} \xrightarrow{\text{EC}_\text{C}} \text{A}\flat\text{m7} \xrightarrow{\text{TF}} \text{D}\flat 7 \xrightarrow{\text{EC}_\text{C}} \text{Gm7}$$

(Here, EC_C is the chromatic variant of EC, equivalent to $\text{TF} \bullet \textsc{slide}_7$, applied to a dominant seventh chord.) Once this sequence arrives on Gm7 as the ii chord of the tonic F, there is a ii–V–I progression in the home key. After the resolution in m. 11, the progression moves backwards through fifths space to begin a VI–ii–V turnaround to F major to begin the next chorus.[29]

So far, we have not said very much about the first two chords in "Blues for Alice": Fmaj7–Em7. In ii–V space, these chords are relatively far apart (4 edges): $\text{Fmaj7} \xrightarrow{\text{7TH}} \text{F7} \xrightarrow{T_6} \text{B7} \xrightarrow{T_5} \text{E7} \xrightarrow{\text{3RD}} \text{Em7}$. Because ii–V space prioritizes functional relationships, chord progressions that are close in terms of voice leading often appear quite distant in the space. In reality, a musician would probably *not* think of this move as being distant, since the two chords are so close to one another in pitch space: to rephrase again in Lewinian terms, the "characteristic motion" that does the best job in taking us from Fmaj7 to Em7 is something like "move the root down a half step and both the third and seventh down a whole step." This transformation is easy enough to define, but would not be part of ii–V space proper; inevitably, the space cannot tell us everything we want to know about jazz harmony. The space is designed to show typical harmonic motions, so progressions that do not seem to lie well in ii–V space demand other explanations (and indeed, often they are voice-leading explanations).

2.3 A Few Extensions

2.3.1 MINOR TONIC CHORDS

As it has been developed thus far, ii–V space has a glaring omission: it requires that all tonic chords be major sevenths. Certainly there are jazz tunes in minor keys, and thus there is a need to account for the tonic minor. We have already seen ii–V progressions that resolve to minor chords—we called that transformation EC in the previous section—but the only minor chords in the space are ii^7 chords, not tonics. One of the advantages of ii–V space is that it is easily extended to account for harmonic features specific to particular situations; in this section, we will do just that to allow for stable minor tonic chords.

---

[29] A "turnaround" is what jazz musicians call a short progression that leads from a chord (often a tonic chord) back to itself. They appear most commonly at the ends of forms, and provide harmonic interest during solos, when a player might play several choruses in a row. The ii–V appears frequently in this formal location, as do many of its variants: vi–ii–V, iii–VI–ii–V, iii–♭III7–ii–♭II7, etc.

The minor ii–V–i progression is usually played as ii7♭5–V7♯9–i mM7.[30] Because we are working with ordered triples of only roots, thirds, and sevenths in this chapter, the alteration of the fifth and ninth have no effect on our definitions of S_{min} and S_{dom} (defined in Section 2.1 above). We do, however, need to formally define the set of minor-major seventh chords (chords with a minor third and major seventh):

$$S_{\text{mM7}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 3; x_s - x_r = 11\}$$

With this definition in place, we can explore how this set interacts with the three sets we have seen already in this chapter. The 3RD transformation works intuitively, and transforms a major seventh chord to a minor-major seventh with the same root:

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{maj}}, \quad \text{then} \quad \text{3RD}(X) = Y = (y_r, y_t, y_s) = (x_r, x_t - 1, x_s) \in S_{\text{mM7}}$$

Likewise, the 7TH transformation transforms a minor-major seventh into a minor-minor seventh with the same root:

$$\text{If } Y = (y_r, y_t, y_s) \in S_{\text{mM7}}, \quad \text{then} \quad \text{7TH}(Y) = Z = (z_r, z_t, z_s) = (y_r, y_t, y_s - 1) \in S_{\text{min}}$$

It will also be useful to define versions of the TF and TF_T transformations that transform a dominant seventh into a *minor* tonic, equivalent to TF • 3RD or TF_T • 3RD. We will call them

---

[30] The extensions used for the dominant chord in a minor ii–V is quite flexible: Aebersold's *II–V⁷–I Progression* gives the quality as 7♯9, but Mark Levine usually gives the chord symbol simply as "alt." Levine includes the minor ii–V in the category of "melodic minor scale harmony," and "alt." is short for the "altered scale" (the seventh mode of melodic minor). The G altered scale is G–A♭–B♭–B♮–C♯–E♭–F♮–G, and is sometimes called the "diminished whole-tone" scale, since it begins as an octatonic scale and ends as a whole-tone scale, or the "super-locrian" scale, the locrian mode with a flatted fourth. This sound could be expressed with a number of different chord symbols—G7(♭9♯9♯11♭13) or G7(♭5♯5♭9♯9), for example—so jazz musicians typically write "alt." See *The Jazz Theory Book*, 70–77. We will return to this equivalence between chords and scales in Chapter 4.

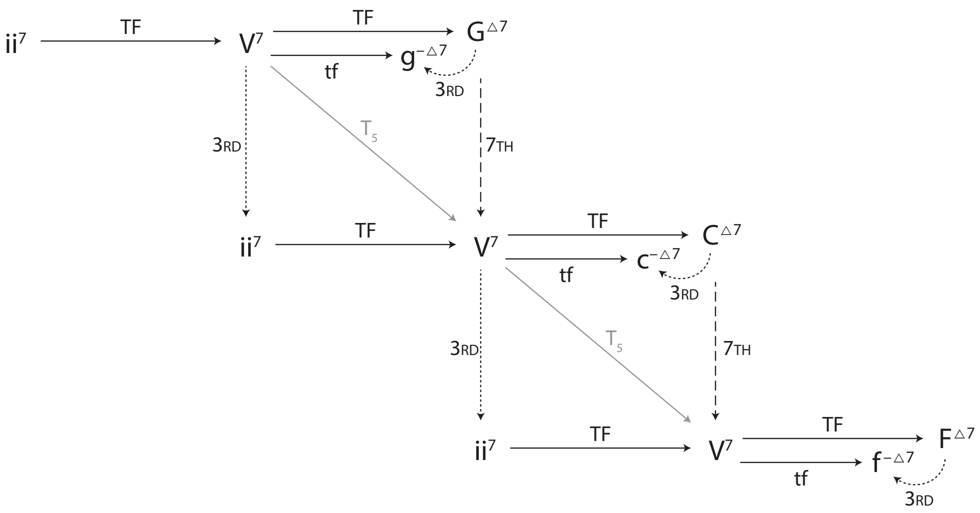


Figure 2.17. A small portion of ii–V space, including minor tonic chords.

simply "tf" and "tf_T" (the lowercase here is meant to parallel the use of lowercase letters to indicate minor triads):

$$\text{If } X = (x_r, x_t, x_s) \in S_\text{dom}, \quad \text{then} \quad \text{tf}(X) = Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 2, x_t) \in S_\text{mM7}$$

$$\text{If } X = (x_r, x_t, x_s) \in S_\text{dom}, \quad \text{then} \quad \text{tf}_\text{T}(X) = Y = (y_r, y_t, y_s) = (x_r - 1, x_s + 4, x_t + 6) \in S_\text{dom}$$

Note that unlike the standard TF and TF_T transformations, tf and tf_T only transform V^7 chords to I^7 chords; the same transformations do *not* hold for ii^7 to V^7.

Figure 2.17 shows a small portion of ii–V space that includes minor tonic chords. Because most jazz tunes do not contain exclusively minor chords, this figure gives both major and minor tonic chords in every key. The transformations defined in the previous paragraph are readily apparent in the space, with the exception of the 7TH transformation from a minor-major seventh to a minor seventh—GmM7 moving to Gm7 as ii^7 of F major, for example. Though we will not do so here, determining how to fill in the figure with tritone substitutions, or to conform it around the circle of fifths in the manner of Figure 2.6, is easy enough to imagine (if not to draw, given the added complexity of the minor-major sevenths).

C–		$\mathrm{G}-^{7}$	C^{7}
$\mathrm{F}^{\mathrm{maj7}}$		$\mathrm{F}-^{7}$	$\mathrm{B}\flat^{7}$
$\mathrm{E}\flat^{\mathrm{maj7}}$	$\mathrm{E}\flat-^{7}$ $\mathrm{A}\flat^{7}$	$\mathrm{D}\flat^{\mathrm{maj7}}$	$\mathrm{D}-^{7\flat5}$ $\mathrm{G}^{7\flat9}$ ‖
$\mathrm{C}-^{[\mathrm{maj7}]}$			

Figure 2.18. Changes to “Solar” (Miles Davis).

By way of a brief example, Figure 2.18 gives the changes for Miles Davis’s “Solar.” This tune is in C minor, though that is not immediately apparent from the changes themselves; in the canonical recording of this piece (from Davis’s own *Walkin’*), the C minor chords are played as minor-major sevenths, and the piece ends on a CmM7 chord. The fact that the only tonic chord appears in the opening bar of the form gives performances of this tune even more of cyclical quality than is usual in jazz. By not arriving on a tonic at the end of the short 12-bar form, Davis achieves a formal overlap at every chorus: the opening tonic serves simultaneously as the harmonic resolution of the previous chorus and the formal beginning of the next.

The analysis in ii–V space is mostly unremarkable, but it is given in Figure 2.19. Note that this figure has replaced Cmaj7 at the top of the space with a minor tonic, CmM7, and as such there is no arrow given between the C-minor tonic and V^{7}/F. This analysis, though, is not possible in the ii–V space of the previous section, since the C-minor harmony of the first bar is decidedly *not* a ii^{7} chord (it would be ii of B♭, and there is no B♭ major harmony in the piece at all).

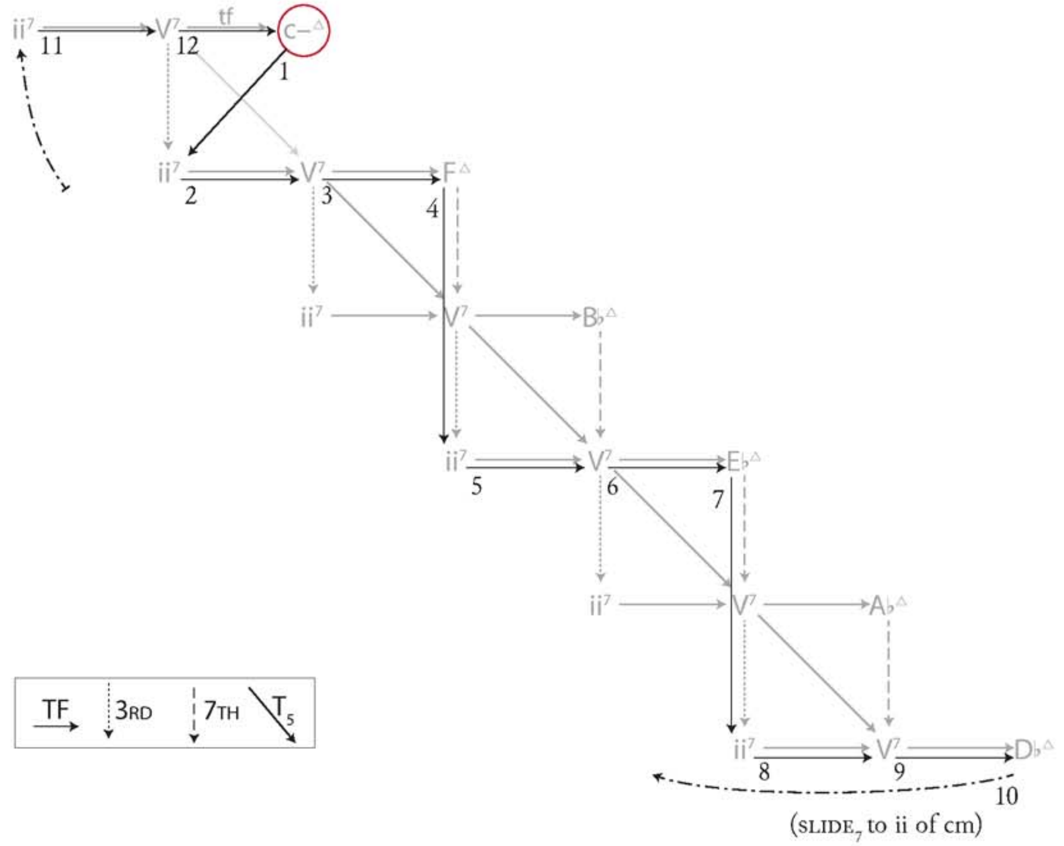


Figure 2.19. An analysis of “Solar” in ii–V space, with C minor tonic.

2.3.2 OTHER KINDS OF TONIC CHORDS

We have now solved the problem of tonic chords that happen to be minor-major seventh chords, but in fact the problem is more general: it would be nice to have some way of allowing for any kind of tonic chord we might find in real music. As mentioned in Section 1.2, James McGowan has argued for what he calls three “dialects of consonance” in jazz (extended tones we might consider consonant): the added sixth, the minor seventh, and the major seventh.[31] Both of the approaches in this chapter so far have focused only on the major-seventh dialect, when it appears atop both major and minor triads. Many Tin Pan Alley tunes end with tonic add-6 chords (which appear

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[31] James McGowan, “Dynamic Consonance in Selected Piano Performances of Tonal Jazz” (PhD diss., Eastman School of Music, 2005), 76–79.

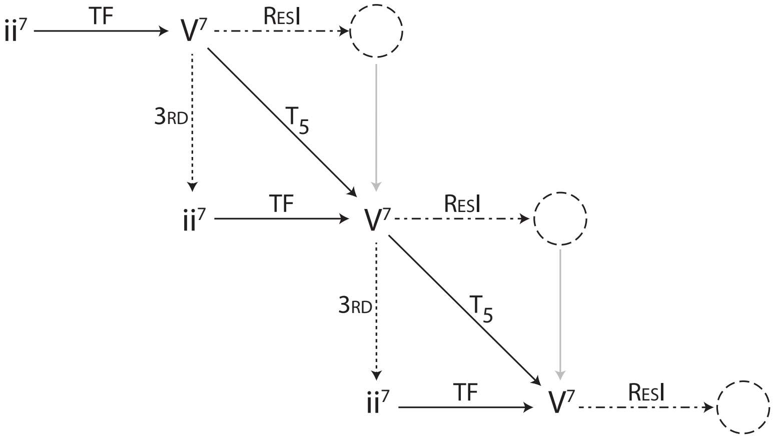

Figure 2.20. A generic version of ii–V space, with unspecified tonic chords.

nowhere in ii–V space), and as we noted in our discussion of "Blues for Alice", it is very common for a blues tonic to be a major-minor seventh chord (which appear only as V^7 chords in the space).

The solution to this shortcoming of the space is to introduce some general transformation (which we might call "ResI") that could be redefined as needed for each style.[32] The generic space would then appear as it does in Figure 2.20. This space is still arranged in perfect fifths, and the basic shape of the ii–V–I progressions is still present, but the quality of the tonic chords is unspecified. Before using this space in analysis, of course, we must actually define what we mean by a "tonic chord" in a given situation. Because ii–V space contains cross-type transformations, this means we need to define both the *set* of tonic chords and the transformation ResI, from S_{dom} to the set of tonics.[33] (By defining ResI to be equivalent to TF and defining tonic chords to be members of S_{maj}, for example, the generic space here becomes the specific layout of ii–V space first presented in Figure 2.10.)

Again, it will be easiest to demonstrate exactly how this generic space can be actualized by means of an example. In our analysis of "Blues for Alice" in Section 2.2.2 above, we noted that the

---

[32] The name of the transformation ResI is inspired by Steven Rings's use of the transformation "ResC" in the first chapter of *Tonality and Transformation*, 25-27.

[33] In practice, ResI will almost always be a transformation that moves the root of a V^7 down a perfect fifth. In theory, however, there is no limitation on the definition of ResI. It is possible, for example, to construct a space where tritone substitutes are normative by defining ResI to be equal to TF$_{\text{T}}$; in this case, the gray arrows in Figure 2.20 would represent the transformation 7th • T_6.

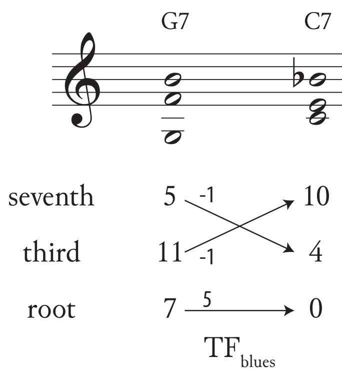

Figure 2.21. Voice leading in the $\mathrm{TF}_{\mathrm{blues}}$ transformation.

B♭7 chord in m. 5 served as the resolution of the ii–V in the preceding bar, but contained the lowered seventh, which is typical for the blues. There, we tried to capture the intuition that the B♭7 was stable by labeling the transformation as TF • 7th: a resolution merely inflected with the lowered seventh. This transformation, though, still results in the B♭7 as a dominant seventh chord (it appears in the space only as V^7 of F).

The generic ResI transformation offers a better solution, in that we can define a “blues TF,” which resolves a V^7 to a tonic major-minor seventh:

$$
\begin{aligned}
&S_{\mathrm{IMm7}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 10\} \\
&\text{If } X = (x_r, x_t, x_s) \in S_{\mathrm{dom}}, \quad \text{then} \quad \mathrm{TF}_{\mathrm{blues}}(X) = Y = (y_r, y_t, y_s) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = (x_r + 5, x_s - 1, x_t - 1) \in S_{\mathrm{IMm7}}
\end{aligned}
$$

Note that $\mathrm{TF}_{\mathrm{blues}}$ is equivalent to T_5, but is defined in a way to demonstrate its similarity to TF (see the voice leading in Figure 2.21; $\mathrm{TF}_{\mathrm{blues}}$ is undefined on ii^7 chords). We have also defined the set S_{IMm7}, the set of tonic major-minor seventh chords; this seems intuitive, but is somewhat complicated. S_{IMm7} is exactly equivalent to S_{dom}—in the language of set theory, they are the same set. The difference between them is not structural, but interpretive: S_{IMm7} is the set of *tonic* major-minor seventh chords, while S_{dom} is the set of *dominant* major-minor sevenths. This distinction allows us to capture the difference between B♭7 as a stable resolution (as it is in m. 5 of “Blues for Alice”) and B♭7 as V^7 of E♭ (as in m. 8 of “Solar”, for example).

This sort of interpretive analysis lies at the heart of Steven Rings's work in *Tonality and Transformation*; the GISes he develops there are designed to capture the intuitions that collections of pitches can be heard (or experienced) differently in different contexts. We can adapt this work slightly to capture the intuition that tonic major-minor sevenths are experienced differently than dominant major-minor sevenths; Rings would say that the two sets have different *qualia*.[34]

The tonal GIS Rings develops in his second chapter consists of ordered pairs of the form (scale degree, acoustic signal); as he has it, "the notation ($\hat{7}$, x) . . . represents the apperception: 'scale degree seven inheres in acoustic signal x.' "[35] Rings goes on to describe sets of these ordered pairs, which we will use to capture our intuitions about the varying roles of the B♭7 chord, as shown below:[36]

$$\left\{\begin{matrix}(\hat{4},\mathrm{A}\flat)\\(\hat{7},\mathrm{D})\\(\hat{5},\mathrm{B}\flat)\end{matrix}\right\}\qquad\qquad\left\{\begin{matrix}(\flat\hat{7},\mathrm{A}\flat)\\(\hat{3},\mathrm{D})\\(\hat{1},\mathrm{B}\flat)\end{matrix}\right\}$$

B♭7 as dominant B♭7 as tonic

Here, the left figure (read bottom to top as root, third, seventh) represents B♭7 as a dominant seventh of E♭ (i.e., with $\hat{5}$, $\hat{7}$, and $\hat{4}$) while the right figure represents the same three pitch classes as a tonic major-minor seventh (with $\hat{1}$, $\hat{3}$, and $\flat\hat{7}$).[37] Rings's system of heard scale degrees allows us to distinguish between the sets S_{IMm7} and S_{dom}: the B♭7 in m. 5 of "Blues for Alice" is a member of S_{IMm7}, while the B♭7 in m. 8 of "Solar" is a member of S_{dom}.

With the distinction between tonic and dominant minor-major sevenths worked out, we can now specify the generic space of Figure 2.20 to create what we might call a "blues ii–V space"; a small portion of this space is shown in Figure 2.22. This space, though, presents another complication: the top arrow marked with a question mark represents a transformation from B♭7 as

---

[34] Rings, *Tonality and Transformation*, 41–43 (and throughout).

[35] Ibid., 44.

[36] Ibid., 55.

[37] In fact, we could make similar statements for all of the sets developed in this chapter: S_{min} would then become (speaking loosely) "the set of minor-minor seventh chords acting as $\hat{2}$, $\hat{4}$, and $\hat{1}$ in some key." In most cases, however, this level of precision is unnecessary, since the quality of the chord uniquely identifies its function.

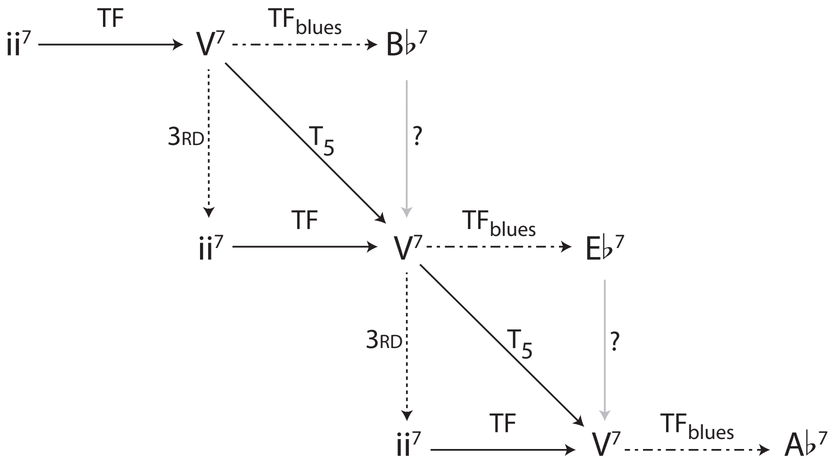

Figure 2.22. A small portion of "blues ii–V space."

tonic to B♭7 as dominant. The pitch classes remain the same, but the *quale* of the chord changes from tonic to dominant, so this transformation is not simply the identity (T_0).

Because this transformation is one of *quale*, we can turn to Rings's tonal GIS for an explanation. Intervals in this GIS are measured with ordered pairs, like the elements themselves: the first element is a scale-degree interval (measured upward), and the second is a pitch-class interval.[38] "Pivot intervals" are those intervals where the second element of the pair is 0.[39] In the situation here, we have what Rings would call a "pivot fifth" between the two B♭7 chords:

$$\left\{\begin{matrix}(\flat\hat{7}, \mathrm{A}\flat) \\ (\hat{3}, \mathrm{D}) \\ (\hat{1}, \mathrm{B}\flat)\end{matrix}\right\} \xrightarrow[\text{“pivot fifth”}]{(5\text{th},0)} \left\{\begin{matrix}(\hat{4}, \mathrm{A}\flat) \\ (\hat{7}, \mathrm{D}) \\ (\hat{5}, \mathrm{B}\flat)\end{matrix}\right\}$$

The pitch-class interval here is 0, since both chords contain B♭, D, and A♭, and the scale-degree interval is a 5th ($\hat{1}$ to $\hat{5}$, $\hat{3}$ to $\hat{7}$, and $\flat\hat{7}$ to $\hat{4}$). With this transformation defined, we can now more fully realize our intuitions about the short passage in "Blues for Alice" (mm. 4–6):

$$\ldots\ \mathrm{Cm7} \xrightarrow{\mathrm{TF}} \mathrm{F7} \xrightarrow{\mathrm{TF_{blues}}} \mathrm{B\flat 7} \xrightarrow{\text{pivot 5th} \,\bullet\, \textsc{3rd}} \mathrm{B\flat m7} \xrightarrow{\mathrm{TF}} \mathrm{E\flat 7}\ \ldots$$

---

[38] Rings, *Tonality and Transformation*, 46–48.
[39] Ibid., 58–66.

2.3.3 Interaction with Diatonic Spaces

The preceding consideration of other kinds of tonic chords has taken us relatively far afield from the starting point of this chapter, and indeed these extensions are not necessary to understand most tonal jazz. For many purposes, the conventional space developed in Sections 2.1 and 2.2 will be sufficient. What is missing in our treatment so far, though, is the concept of a global tonic. This dissertation, after all, is interested in tonal jazz, and most of this music (and certainly all of the examples in this chapter) is in a key. To this point, we have acknowledged this fact only by mentioning the key of a particular tune in our analytical commentary, or circling the tonic chord in a representation of the space. Defining ii–V space as a fully chromatic space has many advantages: it is rare that every chord in a tune can be understood in a single key, and it is convenient not to have to switch continually between diatonic collections. Moreover, chromatic spaces are much more regular than their diatonic counterparts: chromatic step size is consistent, while diatonic step size varies between one and two half steps.[40]

Still, given the exploration of diatonic transformational systems in Section 1.5, it seems wise to consider what a diatonic ii–V space might look like. We first made the space chromatic by arranging individual ii–V–I progressions in descending fifths (recall Figure 2.6). We could instead arrange the space according to the *diatonic* circle of fifths, as shown in Figure 2.23. This space looks much like the chromatic space, with the exception of the diminished fifth between $\hat{4}$ and $\hat{7}$, where the regular transformational structure of the chromatic space breaks down. The change of the descending perfect fifth (T_5) to a diminished fifth (T_6) means that all of the transformations linking these two key areas must all be combined with T_1: C7 $\xrightarrow{\text{3RD} \bullet T_1}$ C♯m7, C7 $\xrightarrow{T_6}$ F♯7, and Fmaj7 $\xrightarrow{\text{7TH} \bullet T_1}$ F♯7.[41]

As noted in Chapter 1, diatonic space (the cyclic group C_7) can be generated by any of its members, while chromatic space (C_{12}) can only be generated by the members 1, 5, 7, or 11

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[40] For more on diatonic systems generally, see John Clough and Gerald Myerson, “Variety and Multiplicity in Diatonic Systems,” *Journal of Music Theory* 29, no. 2 (Autumn 1985): 249–70.

[41] These transformations are all relatively parsimonious, and seem in some way related to the SLIDE_7 transformation introduced above. We will delay a discussion of these parsimonious aspects of these transformations more generally until the next chapter.

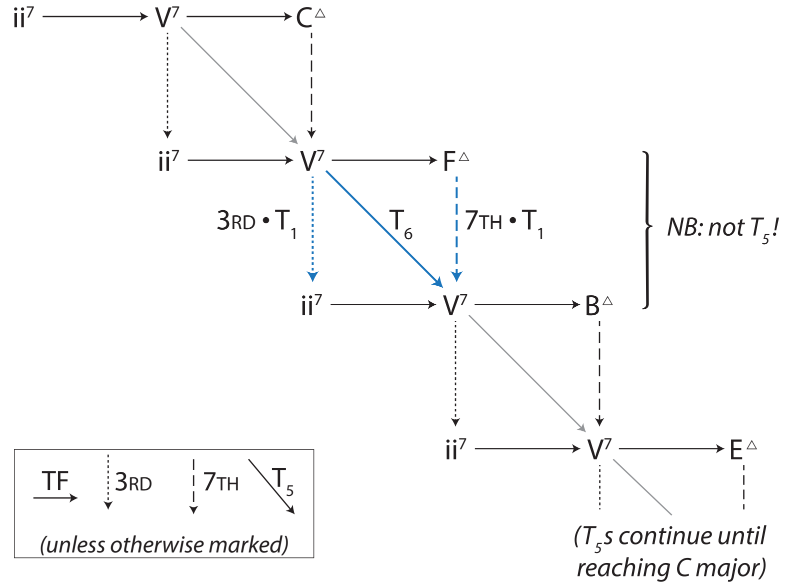

Figure 2.23. A portion of ii–V space, conformed to the white-key diatonic circle of fifths.

(half-steps and perfect fourths/fifths). Diatonic ii–V space, then, offers the interesting possibility of departing from the fifths-based space used so far in this chapter, in favor of some other organization of the space (since any interval we might choose will generate the entire space). To see how such an organization might allow us to capture different kinds of analytical insights, I want to return to briefly to Lee Morgan's "Ceora."

In the analysis of "Ceora" in Section 2.1.2, we saw that the whole tune takes place in four key areas: D♭, C, B♭, and the tonic A♭. Given this organization, we might consider arranging ii–V space in descending diatonic steps, as shown in Figure 2.24. (The entire figure could be wrapped around a circle so that the identical ii–V–I progressions in A♭ at the top and bottom of the figure line up.) This arrangement into steps means that the key areas used in the tune are adjacent in the space; in the chromatic space of Figure 2.8, they were separated by an intervening fifth.

This figure is structurally a bit different than the other spaces explored in this chapter, so it will be helpful to examine it in some detail before returning to "Ceora." The arrangement into descending steps means that we can no longer align all of the seventh chords sharing a root; only

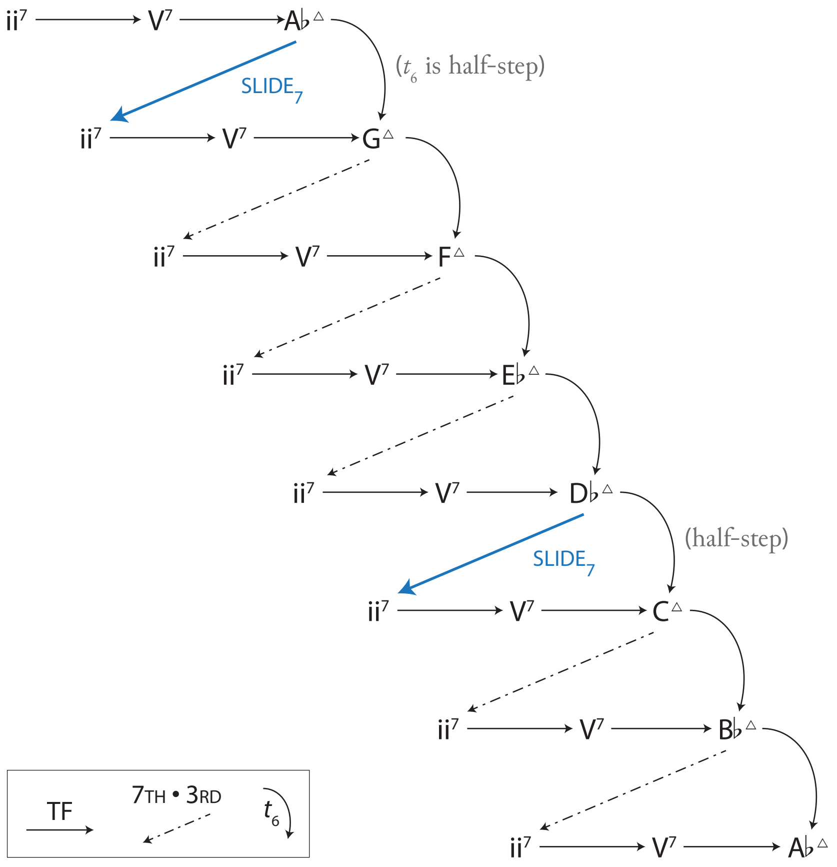
ii⁷ → V⁷ → A♭△
(t₆ is half-step)
SLIDE₇
ii⁷ → V⁷ → G△
ii⁷ → V⁷ → F△
ii⁷ → V⁷ → E♭△
ii⁷ → V⁷ → D♭△
(half-step)
SLIDE₇
ii⁷ → V⁷ → C△
ii⁷ → V⁷ → B♭△
ii⁷ → V⁷ → A♭△
TF
7TH • 3RD
t₆

Figure 2.24. An A♭-major diatonic ii–V space, arranged in descending steps.

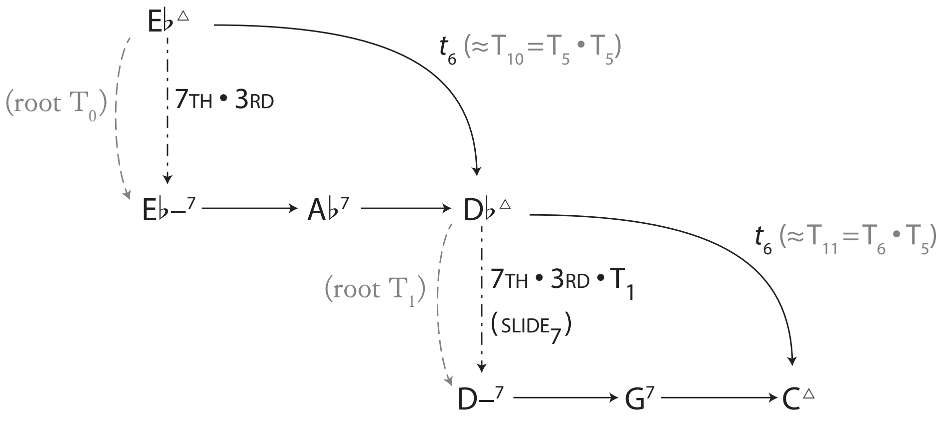
E♭△
t₆ (≈T₁₀=T₅ • T₅)
(root T₀)
7TH • 3RD
E♭–⁷ → A♭⁷ → D♭△
t₆ (≈T₁₁=T₆ • T₅)
(root T₁)
7TH • 3RD • T₁
(SLIDE₇)
D–⁷ → G⁷ → C△

Figure 2.25. Detail of diatonic ii–V space, showing the SLIDE$_7$ transformation between key centers related by half-step.

the major and minor seventh chords sharing a root are adjacent in the space.[42] The Gmaj7 and Gm7 (as ii^7/F) chords are close to one another, for example, but G7 (V^7/C) is farther removed. The key areas in this figure are not connected by T_5, but instead by t_6: all of the roots of the major seventh chords (reading down the right side of the figure) are members of the 4-flat diatonic collection. This t_6 operation affects only chord roots, unlike the t_k operator in Section 1.5.1; all of the resulting chords are major sevenths. Though the diatonic distance between key areas is consistent, the chromatic distance varies: there are two points in the space connected by half steps rather than whole steps (compare, for example, Fmaj7 $\xrightarrow{t_6}$ E♭maj7 and D♭maj7 $\xrightarrow{t_6}$ Cmaj7).

As we saw above, the tritone appearing in diatonic space alters the transformational structure somewhat: transformations spanning this tritone must must be combined with T_1. Here, the relationship between most I^7 and ii^7 chords is the transformation 7TH • 3RD, but between the keys of D♭ and C (as well as A♭ and G), it is 7TH • 3RD • T_1. This transformation is in fact equivalent to the transformation SLIDE$_7$ (see the detail in Figure 2.25); this diatonic origin is one of the reasons the SLIDE$_7$ transformation is so common in tonal jazz.

All of "Ceora" takes place in a relatively small portion of diatonic ii–V space; Figure 2.26 gives an analysis of the A section in this space (the changes can be found in Figure 2.7).[43] The analysis of course looks very similar to our analysis in Section 2.1.2, but the stepwise arrangement of the space helps us to show different analytical insights. Moves in the space that are relatively close in the chromatic fifths arrangement in Figure 2.8 appear much larger in this arrangement (the move from A♭maj7 to E♭m7, marked "4" in this figure), and vice versa (the SLIDE$_7$ from D♭maj7 to Dm7, marked "7").

It is worth noting at this point that although we have adapted ii–V space to show aspects of diatonicism, ii–V space is still chromatic. The transformations are still defined on ordered triples

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[42] This figure has been skewed somewhat to conserve space on the page. If it were drawn in a manner parallel with standard ii–V space, a ii^7 chord would be directly below the I^7 chord with the same root. As is the case throughout this study, the particular visual representation chosen for a given space does not affect the abstract structure of the space itself.

[43] "Ceora" is perhaps even more diatonic than this space implies, since Cmaj7 and B♭maj7 never appear in the music, while D♭maj7 does. Thus the chord qualities strongly suggest A♭ major: I and IV (A♭ and D♭) both appear as major sevenths, while ii and iii (B♭ and C) appear only as minor seventh chords (unstable ii^7 chords).

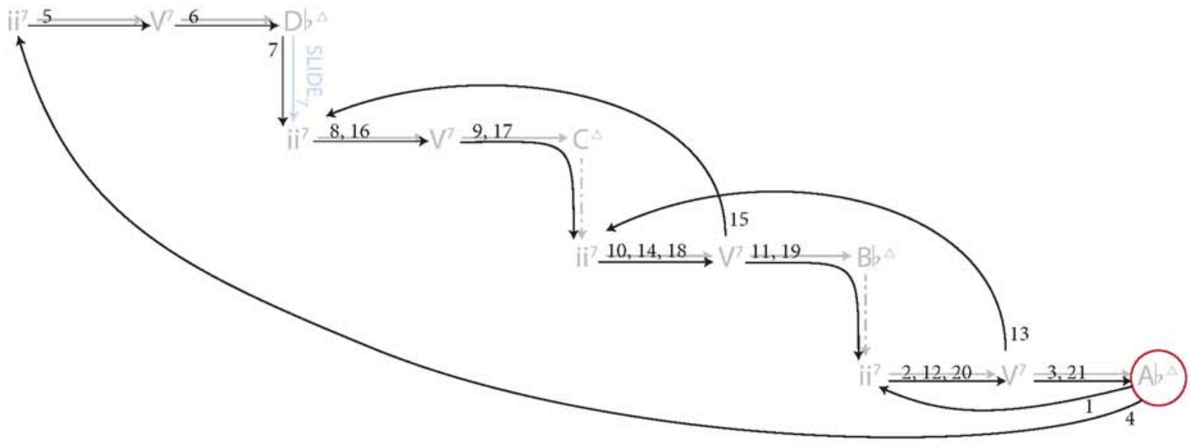

Figure 2.26. An analysis of "Ceora" in diatonic ii–V space.

of mod-12 (not mod-7) integers, and nothing in the ii–V–I progressions themselves has changed. We used the guiding influence of a diatonic collection in this section only to choose the key centers we showed in particular representation of the space. This use reflects the construction of jazz itself; tunes are often globally diatonic (in a key), while locally chromatic, using ii–V–I progressions to tonicize other key areas to a much greater extent than is usually seen in classical music. The reason for this is largely practical. The head-solos-head form of most jazz means that we hear the same progression repeated many times (Morgan's recording of "Ceora" runs about 6 ½ minutes, for example), and using only pitches from the A♭-major diatonic collection would quickly become boring.

The arrangement of ii–V space in Figure 2.24, combining chromatic and diatonic operations, is mathematically complicated. As Steven Rings notes, transformation networks involving both chromatic and diatonic operations violate Lewin's formal definition of a transformation network, since they act on different sets.[44] The underlying transformation graph is not path consistent, since the transformation $\textsc{slide}_7 \bullet \mathrm{TF} \bullet \mathrm{TF}$ is in general not equal to the transformation t_6.[45] Put another way, putting Gmaj7 in the top row of Figure 2.24 while leaving the transformational labels unchanged does not work: obeying the t_6 arrow requires Fmaj7 to occupy the row below, but following the other path would give F♯maj7. The graph is, however, realizable: it is possible, as the figure itself attests, to fill in the nodes such that the arrows *do* make sense.[46] Both Rings and Hook have shown that transformation networks that are not path consistent—like the diatonic ii–V space developed in this section—can nevertheless be analytically productive.

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[44] Rings, *Tonality and Transformation*, 98–99.

[45] Path consistency is described in Hook, "Cross-Type Transformations," 25–28.

2.3.4 Summary

By using the ii–V–I as the basis of the transformational spaces developed in this chapter, we can now understand a large swath of tonal jazz harmony. Because this progression is so ubiquitous, many jazz tunes can be well understood using only the spaces here (perhaps with some adaptations, as suggested in this final section). Treating chords as ordered triples of root, third, and seventh allowed us to define transformations in a way that is still valid when the actual form of a chord might differ greatly from instance to instance. This is a useful abstraction, and we will continue to use it in the next chapter, where we will also consider relationships among our sets of ordered triples more generally.

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[46] Hook, "Cross-Type Transformations," 29.

Chapter 3
Thirds Spaces

The space developed in the last chapter was organized primarily in descending fifths, and works well for most tonal jazz. Motion by thirds, both major and minor, is also a common (though less frequent) occurrence, and will be the focus of this chapter. Harmonic motion by thirds is one of the main emphases of non-jazz transformational theorists; this chapter will allow the opportunity to explore connections between our approach to jazz harmony and the existing neo-Riemannian and transformational literature.

3.1 Minor-Third Substitutions

3.1.1 Formalism

The most common substitution for the dominant in jazz is undoubtedly the tritone substitution (discussed in Section 2.2), but the minor-third substitution is also relatively common, especially in the bebop era. By way of example, Figure 3.1 gives the changes to the opening five bars of Tadd Dameron's "Lady Bird." Normally the progression in mm. 3–4 would function as a ii–V in the key of E♭, but in m. 5 it resolves instead to C. What might have been a ii–V–I in C (Dm7–G7) does not appear; the ii–V is transposed up a minor third to become Fm7–B♭7.

The identical tendency tones shared by tritone-substituted dominants makes them relatively easy to explain, but minor-third substitution is more difficult. Jazz harmony textbooks often do not provide an explanation for the phenomenon: Jerry Coker, for example, simply states that "the I

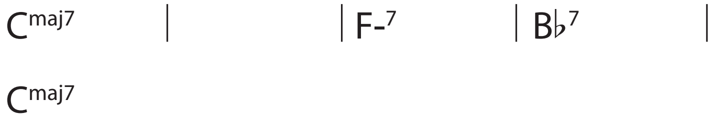

Figure 3.1. Changes to "Lady Bird" (Tadd Dameron), mm. 1–5.

chord . . . is often preceded by IV-7 to ♭VII7, instead of the usual V7 chord."[1] The *Berklee Book of Jazz Harmony* places the ♭VII7 chord in its chapter on "modal interchange" (what might also be called modal mixture), and notes that its function is ambiguous: the chord has dominant quality but not dominant function, since it lacks the leading tone (of the following tonic).[2] Unlike the tritone substitution, there is no strong voice-leading rationale for the minor-third substitution; it is simply a progression that bebop players often used, and that we as analysts must now contend with.[3]

There is, though, a certain similarity between the tritone and minor-third substitutions: just as the tritone evenly divides the octave, so too the minor third evenly divides the tritone. In the previous chapter, introducing tritone substitutes to ii–V space effectively divided the space in half, splitting the complete space into "front" and "back" sides (recall Figure 2.10). Repeating this process again results in a space that looks something like Figure 3.2 (which we will call "m3 space"). The introduction of minor thirds once again changes the topology of the space. This is somewhat easier to see by focusing again only on the centrally-located dominant seventh chords; while the tritone version of ii–V space is topologically equivalent to a Möbius strip, the minor-third version of the space is equivalent to a torus (see Figure 3.3).[4]

Figure 3.3 looks remarkably similar to a more familiar toroidal figure common in music theory: the neo-Riemannian Tonnetz. Despite the surface similarity though, the two are quite different. Like the Tonnetz, the dominant sevenths at the center of m3 space are arranged into axes of perfect fifths (verticals), minor thirds (horizontals), and major thirds (northwest–southeast diagonals, not shown in Figure 3.3).[5] Crucially though, the vertices in the m3-torus are dominant

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[1] Jerry Coker, *Elements of the Jazz Language for the Developing Improvisor* (Miami: Belwin, 1991), 82.

[2] Joe Mulholland and Tom Hojnacki, *The Berklee Book of Jazz Harmony* (Boston: Berklee Press, 2013), 123–24.

[3] That is, there is no strong voice-leading from ♭VII^7 to I^7 in the same way as there is in the tritone substitution, ♭II^7–I^7. We will return to the voice leading of minor third substitutions below.

[4] Turning Figure 3.3 into a torus involves gluing the top and bottom edges together and the left and right edges together; the dotted line representing perfect fifths then wraps around the surface of the torus in a continuous line (as though you had wrapped a barber's pole around a doughnut).

[5] In fact, the graphs of the note-based Tonnetz and the m3-torus here are isomorphic. This fact introduces some tantalizing possibilities, but none turn out to be terribly interesting since, as Richard Cohn has shown, the consonant triad is unique among trichords in its capability for parsimonious voice leading; "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," *Journal of Music Theory* 41, no. 1 (Spring 1997): 1–7. As such, the m3-torus—made of (026) trichords—does not show common-tone relations.

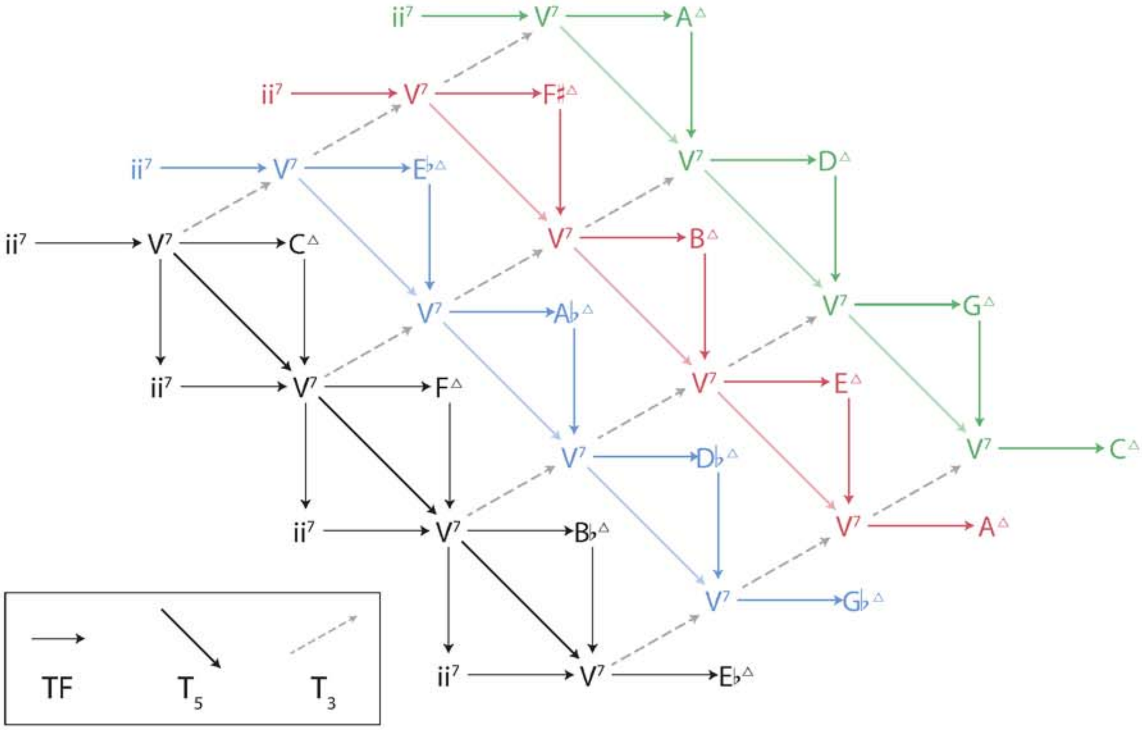
ii^7 V^7 A$^\triangle$
ii^7 V^7 F♯$^\triangle$
ii^7 V^7 E♭$^\triangle$
V^7 D$^\triangle$
ii^7 V^7 C$^\triangle$
V^7 B$^\triangle$
V^7 A♭$^\triangle$
V^7 G$^\triangle$
ii^7 V^7 F$^\triangle$
V^7 E$^\triangle$
V^7 D♭$^\triangle$
V^7 C$^\triangle$
ii^7 V^7 B♭$^\triangle$
V^7 A$^\triangle$
V^7 G♭$^\triangle$
ii^7 V^7 E♭$^\triangle$
TF
T$_5$
T$_3$

Figure 3.2. The complete minor-third representation of ii–V space (ii^7 chords omitted on rear levels for clarity), or m3 space.

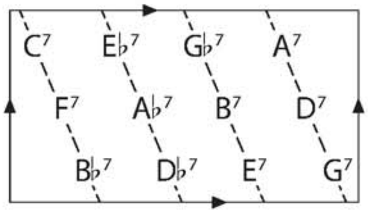
C^7 E♭^{7} G♭^{7} A^7
F^7 A♭^{7} B^7 D^7
B♭^{7} D♭^{7} E^7 G^7

Figure 3.3. The toroidal center of minor-third space (the m3-torus).

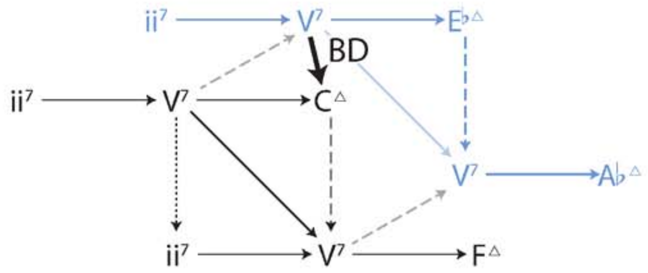

Figure 3.4. The BD transformation from B♭7 to Cmaj7 in minor-third space.

seventh chords (ordered triples), not individual notes; the m3-torus does not represent a parsimonious voice-leading space. The neo-Riemannian Tonnetz can also be drawn to represent triads instead of individual notes, but the resulting graph (Douthett and Steinbach's "chicken-wire torus") no longer resembles the m3-torus here.[6]

The minor-third arrangement of ii–V space makes it easy to define a transformation to represent the minor-third substitution. Because jazz musicians often refer to the minor-third substitution as the "backdoor substitution," we will call this transformation BD:

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{dom}}, \text{ then } \text{BD}(X) = Y = (y_r, y_t, y_s) = (x_r + 2, x_s - 4, x_t - 3) \in S_{\text{maj}}$$

Musically, there is no compelling reason to define BD such that the third is calculated from the previous chord's seventh and vice versa; there is no voice-leading connection between them as there is in the TF transformation. Defining them this way, however, allows the same function to model both the transformation from minor to dominant seventh and from dominant to major seventh.[7] In the space, BD is represented as a diagonal line moving "frontward" between two layers; see Figure 3.4. With this definition, we can easily understand the progression in mm. 3–5 of "Lady Bird": Fm7 $\xrightarrow{\text{TF}}$ B♭7 $\xrightarrow{\text{BD}}$ Cmaj7.

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[6] Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (1998): 246–48. The chicken-wire torus is the dual graph of the more common note-based Tonnetz; both are shown in Dmitri Tymoczko, "The Generalized Tonnetz," *Journal of Music Theory* 56, no. 1 (2012): Figure 1.

[7] The definition of $S_{\text{min}} \xrightarrow{\text{BD}} S_{\text{dom}}$ is not given, since it is relatively rare in jazz. It models a progression like iv^7–V^7, which is much more common in classical music.

As noted above, the minor-third substitution is not explained in terms of its voice leading in the jazz pedagogical literature, since it does not contain the tonic-defining $\hat{4}$–$\hat{7}$ tritone. There is precedent, however, for the backdoor substitution in classical music. Dmitri Tymoczko has written about third substitution more generally, noting that it is often derived by replacing a root-position chord with one in first inversion (e.g., replacing IV with ii^6).[8] Though he does not mention minor third substitution in his chapter on jazz, he identifies the voice-leading proximity of third-related chords as fundamental, since "they can substitute for one another without much disrupting the music's contrapuntal or harmonic fabric."[9] The minor-third substitution in jazz, then, does have some voice-leading rationale: the V^7 and ♭VII^7 chords are connected to *each other* by a minimal amount of voice-leading work (a point to which we will return in the final section of this chapter), but this voice-leading proximity does not manifest in the surface motion from, for example, B♭7–Cmaj7. This logic, though, would seem to suggest that a chord a minor third *below* the dominant can be substituted just as well (e.g., E7–Cmaj7 standing in for G7–Cmaj7), but this progression is extremely rare in jazz.

3.1.2 Analytical Interlude: Joe Henderson, "Isotope"

Minor-third space, as it has been developed so far, may seem like merely another arrangement of ii–V space. One might reasonably ask why it merits a section in this chapter, instead of being merely an extension of the space like those explored in Section 2.3. In a situation parallel to that of 19th-century chromatic tonality, jazz after 1960 began to use more chromatic progressions—especially those built on thirds—in what still might be called tonal jazz. In these compositions, there is still a prevailing sense of key, but local harmonic progressions depart from the descending-fifths norm of earlier jazz.[10] Exploring minor-third space, and understanding why it

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[8] Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 232.

[9] Ibid., 283.

[10] Keith Waters has called this period "jazz's second practice" ("Chick Corea, Postbop Harmony, and Jazz's Second Practice" [paper presented at the annual meeting of the Society for Music Theory, Charlotte, NC, November 2013]). Waters, along with J. Kent Williams, has explored post-tonal jazz harmony using the Tonnetz and hyper-hexatonic systems familiar from classical theory; see "Modeling Diatonic, Acoustic, Hexatonic, and Octatonic

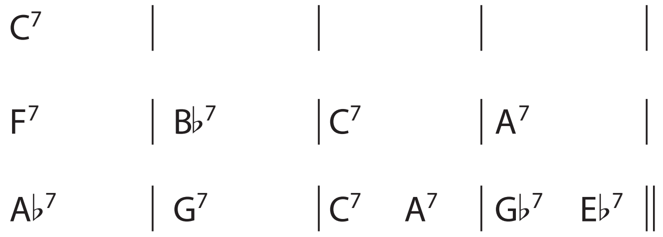

Figure 3.5. Solo changes for "Isotope" (Joe Henderson).

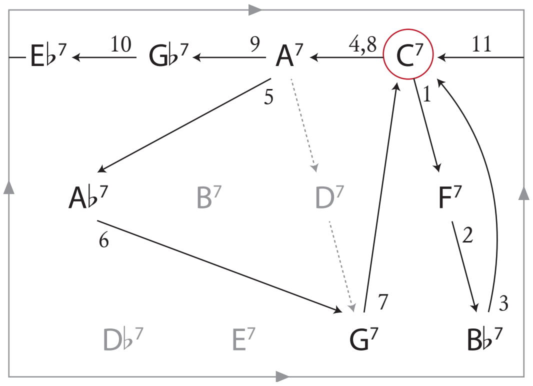

Figure 3.6. An analysis of "Isotope"(solo changes) in the m3-torus.

merits special discussion, is easiest to do by using an example: Joe Henderson's composition "Isotope."

The solo changes for "Isotope" are given in Figure 3.5.[11] The tune is a modified 12-bar blues, and contains the typical lowered seventh of the blues dialect: all of the chords (even the tonic C chords) are major-minor sevenths. While we could analyze this set of changes in the "blues ii–V space" of Section 2.3.2, using the m3-torus of Figure 3.3 will help to highlight how minor-third space can be used in analysis.

An analysis of "Isotope" in the m3-torus is given in Figure 3.6. The solo changes begin with a tonic major-minor seventh chord for four bars, before moving to the IV chord in m. 5. Instead of

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Harmonies and Progressions in Two- and Three-Dimensional Pitch Spaces; or Jazz Harmony after 1960," *Music Theory Online* 16, no. 3 (August 2010).

[11] "Isotope" was first recorded on Henderson's album *Inner Urge*, released in 1965. It uses a slightly different set of changes for the solos than it does for the head (often referred to simply as the "solo changes" and "head changes"). This is often the case when the head changes are complex, fast-moving, or contain unusual extensions to account for specific melody notes. These changes are taken from *The Real Book*; the C chord in m. 7 is played as either a major or a dominant seventh on the *Inner Urge* recording, so analyzing it as C7 here seems reasonable.

moving directly from IV to I in m. 7 (which would be typical for a blues), Henderson moves first to B♭, resulting in a variant of the backdoor progression B♭7–C7 in mm. 6–7.[12] The tonic function of C7 is extended by moving to A7 in m. 8, a minor third away. What would be a string of T_5 operations, A7–D7–G7–C7, is disrupted slightly by the tritone substitution of A♭7 for D7 (represented in the m3-torus by replacing a chord with the chord two spaces to its left or right). Once the progression returns to the tonic C7 in m. 11, Henderson uses a complete minor-third cycle as a turnaround, maintaining tonic function for two measures before beginning the next chorus.

The logic of the minor-third substitution means that the B♭7 chord in m. 6 is understood as a substitute for true dominant G7. Likewise, the A7 chord in m. 8 seems to substitute for the tonic in the same way; in an ordinary 12-bar blues, mm. 7–8 would both contain tonic. This correspondence suggests an interesting possibility: the top row of harmonies in Figure 3.6 all seem to have tonic function, while the two chords used in the bottom row both act as dominants. The harmonies in the middle row, then, seem to serve as predominants (or subdominants), appearing in "Isotope" just before the G7 and B♭7 chords.[13]

This functional analysis helps to make sense of the unusual turnaround in the last two bars of the tune. Turnarounds are inherently prolongational structures, a way of providing harmonic interest between a chorus-concluding tonic and the next chorus-beginning tonic.[14] While most turnarounds use functional harmony (a ii–V–I progression or some variant), Henderson uses a non-functional minor-third cycle. Coming as it does at the end of the chorus, which makes liberal

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[12] We could easily define a BD_{blues} transformation, similar to TF_{blues} from the last chapter; for now, we can simply understand this variant as BD • 7TH.

[13] The question of the meaning of "function" is is a difficult one. Here I mean the term as Brian Hyer does: "it is not what a chord *does* that matters, but what it *is*: a functional designation names a chord's *being*" ("What Is a Function?," in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Rehding [New York: Oxford University Press, 2011], 109, emphasis original).

[14] The word "prolongational" in this sentence is admittedly problematic, given its Schenkerian implications. By using it here I mean only that at some deeper level the turnaround is harmonically superfluous, as it occurs after the main tonal conclusion of the chorus (an observation confirmed by the fact that the turnaround is usually omitted in the last head). I do not mean to imply that the turnaround in the last two bars of "Isotope" is harmonically uninteresting; indeed, it is the most distinctive feature of the piece.

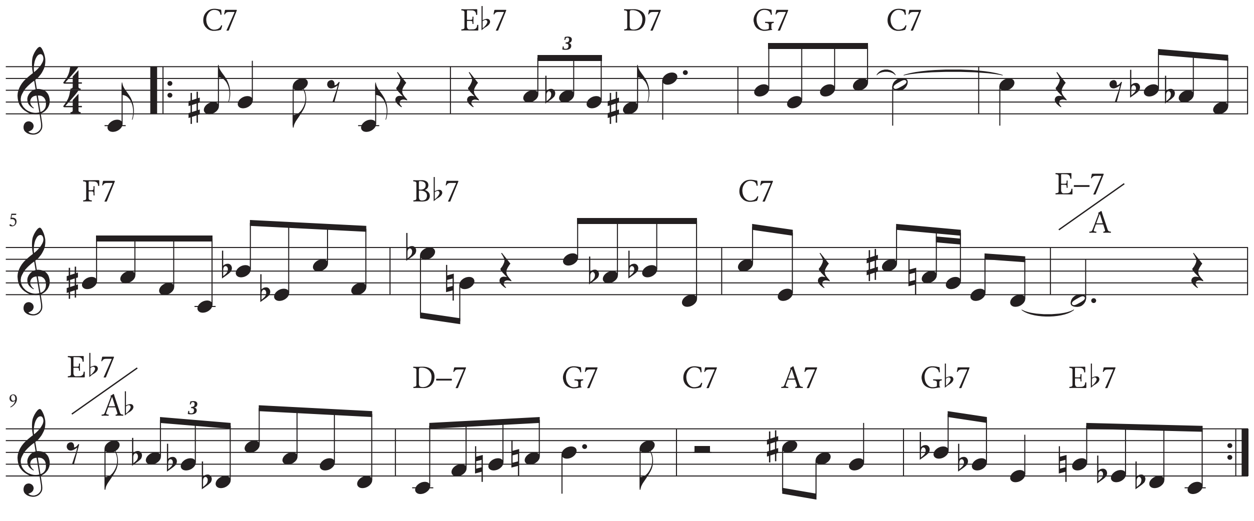

Figure 3.7. "Isotope," head (Joe Henderson).

use of minor-third substitutions, we are primed to hear this cycle as a unique way of maintaining tonic function while avoiding the use of a functional harmonic progression.

This minor-third motion is seen first in the head changes, given in Figure 3.7. (Because the changes are altered in the head in order to fit the melody, this figure gives the melody as well.) Most of the alterations between the head and solo changes occur in the first four bars of the tune; the remaining differences are relatively insignificant.[15] While the solo changes give only C7 in the first four bars, the head changes elaborate this harmony with an alteration of a I–ii–V–I progression: the II^7 chord in m. 2 is preceded by E♭7. While it would be easy to write off this chord as an upper-neighbor harmony to the more structural II chord, doing so would minimize the important role of the minor-third substitution in the rest of the tune. What appears at first to be an inconsequential embellishment (substituting E♭7 for C7) gains in significance throughout the tune, first becoming realized in the backdoor progression in mm. 5–7 and reaching its fullest expression in the turnaround that ends the chorus.

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[15] The most obvious differences in mm. 5–12 of the tune are the "slash chords" in mm. 8–9. The chord symbol Em7/A indicates an E minor seventh chord played with an A in the bass; the resulting sound is an A7 chord with a suspended fourth (D replaces C♯). The older *Real Book* gives the same change as A7sus4. The only other slight alteration is the addition of the ii chord, Dm7, in m. 10.

Henderson's tour of the m3-torus in "Isotope" is interesting for a number of reasons. First, he is able to create a sense of tonal function and harmonic progression while using almost entirely major-minor ("dominant") seventh chords. This is a feature common to many blues tunes, but is stronger in "Isotope" given the pervasive use of minor-third substitutions. Second, this is a tune that does not seem to make much sense in the descending-fifths ii–V space of the previous chapter. While certainly some of the tune makes use of harmonic motion in fifths, the backdoor progression in mm. 6–7 and the final turnaround would appear as seemingly random, nonsensical jumps in ii–V space.

Finally, the progression of "Isotope" is not one that is easily explained using neo-Riemannian theory as it is usually applied to classical music. Constructing a Tonnetz usually relies on having two varieties of musical objects under consideration (commonly major and minor triads, or half-diminished and dominant seventh chords), while our m3-torus only uses major-minor sevenths. Neo-Riemannian theories often focus on smooth voice leading; measured in these terms, the B♭7 and C7 chords of mm. 5–6 are quite distant from one another, despite their functional equivalence to a V–I motion.[16] The turnaround, most unusual from a tonal perspective, is actually quite typical of patterns usually analyzed in neo-Riemannian terms: the major-minor sevenths found there are all minimal perturbations of a single diminished seventh chord, and each can be connected to the next with a minimal amount of voice-leading work (two semitones moving in opposite directions).[17]

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[16] Exactly how far apart B♭7 and C7 depends on how one chooses to measure voice-leading distance, and whether we consider major-minor sevenths in the usual way, as four-note chords, or in the way we have been doing so here, as ordered triples of root, third, and seventh. In Jack Douthett's Four-Cube Trio, for example, B♭7 and C7 are maximally far apart—4 semitones; see Richard Cohn, *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature* (New York: Oxford University Press, 2012), 157–58.

[17] The notes in these four dominant sevenths form an octatonic collection, and have been studied fairly extensively in the literature. See, for example, ibid., 152–58; Douthett and Steinbach, "Parsimonious Graphs," 245–46; and Tymoczko, *A Geometry of Music*, 371.

3.2 Major-Third Spaces

3.2.1 INTRODUCTION: COLTRANE CHANGES

Root motion by major third is one of the most difficult harmonic motions to explain using traditional tonal methods; it is with this kind of music that transformational methods have proven to be most useful.[18] The increasing use of these progressions in nineteenth-century harmony has a parallel in jazz, as Keith Waters has shown; in both, the harmonic vocabulary is familiar, but harmonic progressions are often unfamiliar.[19] Nonfunctional harmony is a defining feature of some post-bop jazz, including much of the music of Chick Corea, Herbie Hancock, Wayne Shorter, and others.[20] This dissertation, though, is interested specifically in tonal jazz, and we will remain careful during this discussion to avoid straying too far afield from this topic.

The locus classicus for root motion by major third in jazz is of course John Coltrane's "Giant Steps," first recorded in 1959 on the album of the same name.[21] Much of the use of nonfunctional harmony in jazz that develops after 1960 can be traced back to "Giant Steps"; Keith Waters has outlined this lineage in selected compositions of Wayne Shorter, Bill Evans, and Herbie Hancock.[22] Given this influence, "Giant Steps" will serve here as a useful foil for major-third cycles in jazz more generally. Though we will delay a proper analysis of the tune until we have developed some formalism, a short overview will be useful at this point.

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[18] The examples are too numerous to list here, but for an overview, see Matthew Bribitzer-Stull, "The A♭–C–E Complex: The Origin and Function of Chromatic Major Third Collections in Nineteenth-Century Music," *Music Theory Spectrum* 28, no. 2 (Fall 2006): 167–90.

[19] Waters, "Chick Corea, Postbop Harmony, and Jazz's Second Practice."

[20] Waters's work in particular has focused extensively on this music, though he is hardly alone. See, for example, Patricia Julien, "The Structural Function of Harmonic Relations in Wayne Shorter's Early Compositions: 1959–1963" (PhD diss., University of Maryland, 2003); Steven Strunk, "Notes on Harmony in Wayne Shorter's Compositions, 1964–67," *Journal of Music Theory* 49, no. 2 (Autumn 2005): 301–32; Keith Waters, "Modes, Scales, Functional Harmony, and Nonfunctional Harmony in the Compositions of Herbie Hancock," *Journal of Music Theory* 49, no. 2 (Fall 2005): 333–57; and Waters and Williams, "Jazz Harmony after 1960."

[21] Many people have discussed "Giant Steps" in the literature; the most substantial work in this area is Matthew Santa's "Nonatonic Progressions in the Music of John Coltrane," *Annual Review of Jazz Studies* 13 (2003): 13–25. Guy Capuzzo compares Santa's analysis to one done by Pat Martino in "Pat Martino's *The Nature of the Guitar*: An Intersection of Jazz Theory and Neo-Riemannian Theory," *Music Theory Online* 12, no. 1 (February 2006). See also David Demsey, "Chromatic Third Relations in the Music of John Coltrane," *Annual Review of Jazz Studies* 5 (1991): 145–80; and Matthew Goodheart, "The 'Giant Steps' Fragment," *Perspectives of New Music* 39, no. 2 (July 2001): 63–95.

[22] Keith Waters, "'Giant Steps' and the ic4 Legacy," *Intégral* 24 (2010): 135–62.

B^{maj7} D^{7}	G^{maj7} $B\flat^{7}$	$E\flat^{maj7}$	$A\text{-}^{7}$ D^{7}
G^{maj7} $B\flat^{7}$	$E\flat^{maj7}$ $F\sharp^{7}$	B^{maj7}	$F\text{-}^{7}$ $B\flat^{7}$
$E\flat^{maj7}$	$A\text{-}^{7}$ D^{7}	G^{maj7}	$C\sharp\text{-}^{7}$ $F\sharp^{7}$
B^{maj7}	$F\text{-}^{7}$ $B\flat^{7}$	$E\flat^{maj7}$	$C\sharp\text{-}^{7}$ $F\sharp^{7}$

Figure 3.8. Changes to "Giant Steps" (John Coltrane).

The changes to "Giant Steps" are given in Figure 3.8. The major-third construction of the tune is readily apparent: the three tonal centers are B, G, and E♭, as evidenced by the major seventh chords. These local tonics are all preceded by their dominants (mm. 1–3, 5–7) or by complete ii–V progressions (all other locations). Though the distance between the key centers is unusual, the individual progressions are not.[23] "Giant Steps" is not exactly tonal, but neither is it really atonal. When I listen to the piece, at least, the impression is not one of nonfunctional harmony, but rather of tonal harmony used in an unconventional way. This distinction is easiest to understand with a counterexample: Figure 3.9 gives the changes to Wayne Shorter's ballad "Infant Eyes."[24] Here there are no ii–Vs, and the only V–I progressions occur across formal boundaries. Shorter's use of harmony *does* seem nonfunctional, and gives the piece a floating quality that "Giant Steps" does not have. Rather, "Giant Steps" is strongly forward-directed: all of the dominant chords push toward their respective tonics, and although the global tonic may be in question, local tonic chords are crystal clear.[25]

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[23] Frank Samarotto has suggested to me in connection with an unpublished paper of his that "Giant Steps" is chromatically coherent, while locally diatonic. As such, it represents an example of his "hypothetical" Type 4 coherence, "in which areas of diatony occur only in local isolation and in which some other (presumably post-tonal) coherence might be in effect" ("Treading the Limits of Tonal Coherence: Transformation vs. Prolongation in Selected Works by Brahms" [paper presented at the annual meeting of the Society for Music Theory, Madison, WI, November 2003]).

[24] These changes are from Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 30. "Infant Eyes" appears on Shorter's album *Speak No Evil* (1964).

[25] Logical arguments could be made for both B and E♭ as the prevailing key of "Giant Steps." Given the organizing influence of the major-third cycle, I am not sure the question is so important; the tune uses tonal progressions, but may not be *in* a key.

G-^{7}	F-^{7}	$\text{E}\flat^{\text{maj7}}$	$\text{A}^{7\flat 9}$		
$\text{G}\flat^{\text{maj7}}$	F^{sus}	$\text{E}\flat\text{-}^{7}$	$\text{B}\flat^{\text{sus}}$	$\text{B}\flat^{7}$	
$\text{E}\flat^{\text{maj7}}$	$\text{E}\flat^{\text{sus}\flat 9}$	$\text{E}\flat^{\text{maj}\sharp 11}$	E^{maj7}		
B^{maj7}	$\text{B}\flat^{\text{sus}}$	$\text{A}\flat\text{-}^{7}$	$\text{E}\flat^{\text{sus}}$	$\text{D}^{7\flat 9}$	*D.C.*

Figure 3.9. Changes to “Infant Eyes” (Wayne Shorter).

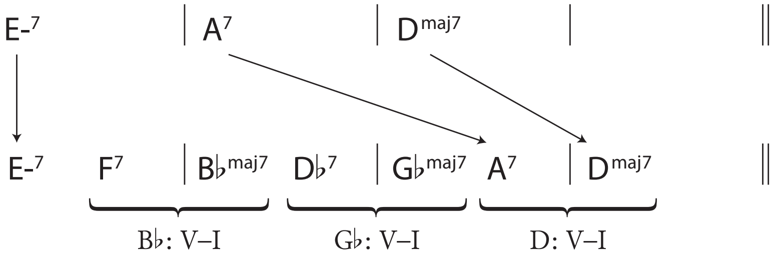

Figure 3.10. Coltrane’s major-third cycle as a substitution for a ii–V–I progression. (Adapted from Levine, *The Jazz Theory Book*, 359.)

“Tune Up”	E-^{7}	A^{7}	D^{maj7}	
“Countdown”	E-^{7} F^{7}	$\text{B}\flat^{\text{maj7}}$ $\text{D}\flat^{7}$	$\text{G}\flat^{\text{maj7}}$ A^{7}	D^{maj7}
	D-^{7}	G^{7}	C^{maj7}	
	D-^{7} $\text{E}\flat^{7}$	$\text{A}\flat^{\text{maj7}}$ B^{7}	E^{maj7} G^{7}	C^{maj7}
	C-^{7}	F^{7}	$\text{B}\flat^{\text{maj7}}$	
	C-^{7} $\text{D}\flat^{7}$	$\text{G}\flat^{\text{maj7}}$ A^{7}	D^{maj7} F^{7}	$\text{B}\flat^{\text{maj7}}$
	E-^{7}	F^{7}	$\text{B}\flat^{\text{maj7}}$	$\text{E}\flat^{7}$
	E-^{7}	F^{7}	$\text{B}\flat^{\text{maj7}}$	$\text{E}\flat^{7}$

Figure 3.11. The changes to “Countdown” (Coltrane), compared with “Tune Up” (Miles Davis).

The major-third cycle of "Giant Steps" is the most well-known example, but Coltrane first developed the progression as an elaborate substitution over a standard ii–V–I progression; this particular set of substitutions is often referred to as "Coltrane changes."[26] Figure 3.10 shows how this process works: the goal of the progression (in this case, Dmaj7) is shifted to the fourth bar; then, major seventh chords related by major third are placed on the downbeats (B♭maj7 and G♭maj7); finally, all of the major sevenths are preceded by their own dominants. This process can clearly be seen in Coltrane's composition "Countdown," which is based on the changes to Miles Davis's "Tune Up" (see Figure 3.11).[27] Coltrane changes can be superimposed over any four-measure ii–V–I progression, so they can be found not only in Coltrane's own compositions, but also in his improvisations on other tunes and his reharmonizations of standards like "Body and Soul."[28]

3.2.2 Developing a Transformational System

Because Coltrane changes can be considered a ii–V variant, it is logical to include them in this study, even though we may be slightly pushing the limits of "tonal jazz." We now have a preliminary understanding of how the substitution works, but it still remains to incorporate it into the transformational system under development here. First, though, there is a bit of unfinished business to take care of: in Section 1.3, we touched on several transformational approaches only briefly, promising to return to them at a point when they would be more relevant. Given the central role of harmonic motion in thirds in many neo-Riemannian theories, it seems appropriate to fulfill that promise at this point.

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[26] Exactly how Coltrane devised this substitution set is difficult to say: authors have at various times pointed to classical sources—especially Nicolas Slonimsky's *Thesaurus of Scales and Melodic Patterns*—as well as the music of Thelonious Monk, Dizzy Gillespie, and Tadd Dameron, among others. One source that is nearly always cited is the tune "Have You Met Miss Jones?" (Richard Rodgers/Lorenz Hart), which is analyzed in Section 3.2.3 below. For a review of these possible origins, see Demsey, "Chromatic Third Relations," 148–57; and Lewis Porter, *John Coltrane: His Life and Music* (Ann Arbor: University of Michigan Press, 1998), 145–47.

[27] Nearly every discussion of Coltrane changes includes the "Countdown"/"Tune Up" pairing. See, for example, Demsey, "Chromatic Third Relations," 159–62; Mark Levine, *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 359–60; or many others. "Countdown" was also recorded on *Giant Steps*, and "Tune Up" can be heard on *Cookin' with the Miles Davis Quintet* (1957).

[28] Demsey provides a list of third-relations in jazz tunes in an appendix to "Chromatic Third Relations," 179–80. Coltrane's famous take on "Body and Soul" is found on the album *Coltrane's Sound* (1960), which also features two original tunes that make prominent use of the major-third cycle: "Central Park West" and "Satellite."

While not quite neo-Riemannian, Tymoczko's general notion of third substitution (discussed above in connection with the backdoor substitution) also includes major-third substitution, and thus we might consider all of the local tonics in a Coltrane-changes progression as third substitutions for the true tonic. These substitutions, according to Tymoczko, are explained by the voice-leading proximity of third-related chords—a factor that plays a critical part in the work of Richard Cohn. In both of Cohn's models of triadic space in *Audacious Euphony*, hexatonic cycles and Weitzmann regions, three M3-related major triads combine with three minor triads to form a six-chord system.[29] Thus, the three tonal centers of "Giant Steps" can be contained in a single hexatonic cycle or a single Weitzmann region.

Understanding the rest of "Giant Steps" in terms of one of these systems, though, leaves much to be desired. Cohn's systems, and others like them, are fundamentally triadic, which is clearly a problem for understanding jazz, with its saturation of seventh chords (and beyond). To account for this incongruity, we must either adapt the music to fit our analytical system, or adapt our analytical system to fit the music. The first option is clearly a nonstarter: Figure 3.12 gives a non-example of "Giant Steps" analyzed in Jack Douthett's "Cube Dance."[30] While this analysis makes the major-third cycle of the tonic chords clear, reducing the chords to triads loses the detail of the chord qualities (both major-seventh and dominant-seventh chords become major triads), as well as their functional relationships. Despite the prominent emphasis on M3-cycles in these theories, then, using these triadic systems as an analytical basis for our work here would not seem to be the answer.

There are of course neo-Riemannian theories involving seventh chords, but these turn out to be not so helpful for our purposes here either. Theories that include only the (0258) tetrachords—half-diminished and dominant sevenths—are obviously not suitable for analyzing the

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[29] Cohn, *Audacious Euphony*. Hexatonic cycles are discussed primarily in Cohn's chapter 2, and Weitzmann regions in chapter 4; chapter 5 combines these models into a single system which is then used throughout the rest of the book. Lewis Porter mentions Weitzmann's treatise on the augmented triad as a possible influence on Coltrane (*John Coltrane: His Life and Music*, 146).

[30] In this figure "+" indicates major triads and "−" indicates minor triads. "Cube Dance" appears in Douthett and Steinbach, "Parsimonious Graphs," 254, and is one of Cohn's primary models of triadic space; see *Audacious Euphony*, 86–109 and following.

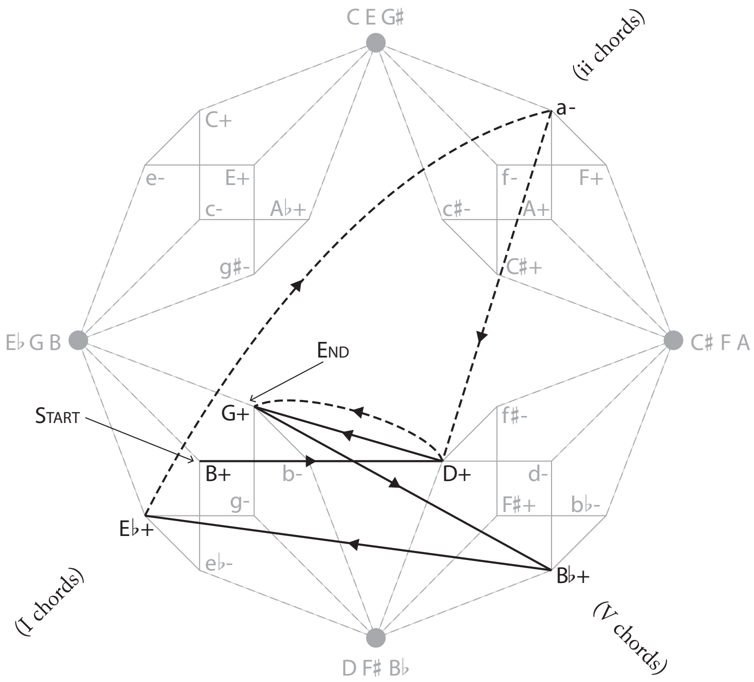
C E G♯
(ii chords)
a-
C+
e-
E+
c-
A♭+
g♯-
f-
F+
c♯-
A+
C♯+
E♭ G B
C♯ F A
End
Start
G+
f♯-
B+
b-
D+
d-
g-
F♯+
b♭-
E♭+
e♭-
B♭+
(I chords)
(V chords)
D F♯ B♭

Figure 3.12. “Giant Steps,” mm. 1–5, analyzed in Douthett’s Cube Dance. Begin by following the solid arrows, then continue with the dashed arrows.

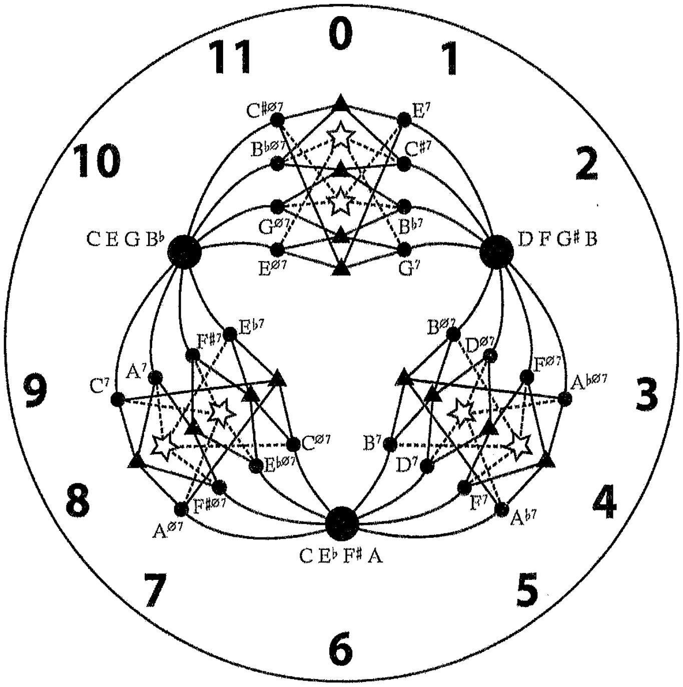
0
1
2
3
4
5
6
7
8
9
10
11
C♯ø7
E7
B♭ø7
C♯7
Gø7
B♭7
Eø7
G7
C E G B♭
D F G♯ B
E♭7
F♯7
A7
C7
Cø7
E♭ø7
F♯ø7
Aø7
Bø7
Dø7
Fø7
A♭ø7
B7
D7
F7
A♭7
C E♭ F♯ A

Figure 3.13. Cohn’s “Four-Cube Trio” (*Audacious Euphony*, Fig. 7.16, 158.) The black triangles indicate minor seventh chords, while the hollow stars indicate French sixth chords.

multiple seventh-chord types in jazz. The seventh-chord analogy of "Cube Dance" is what Cohn calls the "Four-Cube Trio," which is shown in Figure 3.13.[31] As is readily apparent, Four-Cube Trio does not contain any major seventh chords, so it would also create problems if pressed into use for analyzing "Giant Steps."

There is, though, a more fundamental problem with these neo-Riemannian theories of seventh chords, at least when approaching M3-cycles in jazz. As we have mentioned, many neo-Riemannian theories focus on efficient voice-leading, and parsimonious relationships among seventh chords can be understood as minimal perturbations of fully-diminished seventh chords.[32] This does not generate major-third cycles (as in the triadic case), but rather partitions the octave into *minor* thirds. The three dominant sevenths appearing in "Giant Steps," for example, are in three different "towers" in the Four-Cube Trio, which does not reflect the organizing influence of major thirds in the way that the triadic Cube Dance does.

Some theorists have turned to neo-Riemannian theory to explain jazz progressions, though; closest to our intent here is Matthew Santa's nonatonic system for analyzing Coltrane.[33] In a parallel with Cohn's hexatonic systems, Santa draws from the nonatonic (or enneatonic) collection in order to explain "Giant Steps" in terms of parsimonious voice leading. Figure 3.14 shows one of Santa's cycles, along with three-voice parsimonious realization. (In this figure, note that the triangle indicates a major *triad*, not a major seventh chord.) This nonatonic system seems to be a convincing analysis of the opening of "Giant Steps," but it comes up a bit short as a general theoretical system. First, Santa considers only major triads and incomplete dominant seventh chords; all major seventh chords are reduced to triads, and minor sevenths—like the ii^7 chords of "Giant Steps"—are simply ignored.[34] The cycle in Figure 3.14 is generated by the collection {D,

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[31] This figure is taken from Cohn's book, and has several errors, the most important of which is that the C at the 10 o'clock position should be a C♯, forming a fully-diminished seventh chord. "4-Cube Trio" was originally devised by Jack Douthett, and is very similar to the "Power Towers" graphic in Douthett and Steinbach, "Parsimonious Graphs," 256 (which omits the French sixth chords). For more on its history, see Cohn, *Audacious Euphony*, 157n15.

[32] This is a central thesis of Cohn's chapter 7 (see especially *Audacious Euphony*, 148–58), and figures prominently in Tymoczko's geometric theory (*A Geometry of Music*, 97–112).

[33] Santa, "Nonatonic Progressions in Coltrane."

[34] Santa omits the fifth of dominant seventh chords, as we have been doing here. The reasons, though, are different: the stated reason is to keep the cardinalities of the chords the same, but he does not mention why he chooses not to use major seventh chords (rather than triads) and complete dominant sevenths, for example. Santa notes later that

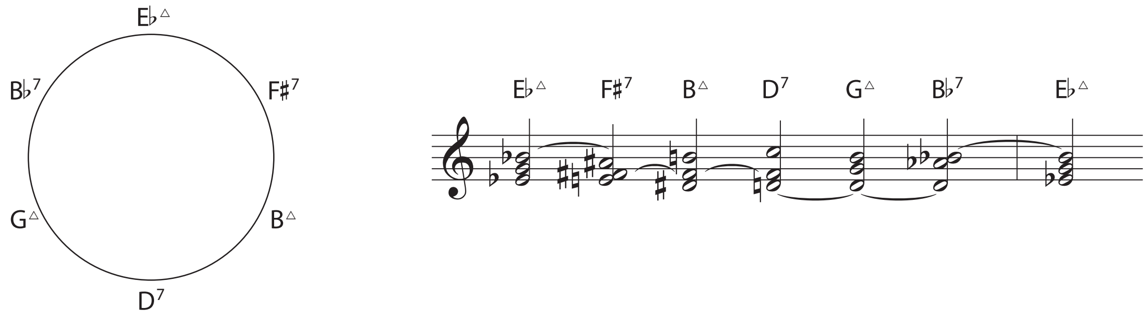

Figure 3.14. Matthew Santa's nonatonic cycles: the "Western" nonatonic cycle (left), and a three-voice parsimonious realization (right). (Adapted from "Nonatonic Progressions in Coltrane," 14.)

Major triads	Minor triads	Incomplete V^7
CM	Cm	C7
E♭M*	E♭m	B♭7*
EM	Em	E7
GM*	Gm	D7*
A♭M	A♭m	A♭7
BM*	Bm	F♯7*

Table 3.1. All possible consonant triads and incomplete dominant sevenths in the nonatonic collection {D, E♭, E, F♯, G, A♭, B♭, B, C}. Members of Santa's Western system are marked with a star.

E♭, E, F♯, G, A♭, B♭, B, C}, but it does not contain all of the triads or incomplete dominant sevenths in that collection (see Table 3.1). Santa's 4-cycle system, then, is somewhat misleading, since any triad or dominant seventh can be located in two different nonatonic collections.

Having brought up all of these approaches only to show their shortcomings, though, the question remains: what should a transformational system that includes major-third relations look like? Although the relationship of M3-cycles and smooth voice leading is valuable, so far in this study we have focused primarily on functional relationships, and it would seem foolish to abandon that approach here. As we noted above, "Giant Steps" does contain a major-third cycle, but within that cycle the progressions are functional: it is locally diatonic, but globally chromatic.

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including the fifth involves one of the three missing notes from the nonatonic collection, which is acceptable because "the fourth voice is not essential to the voice leading of the cycle" (Santa, "Nonatonic Progressions in Coltrane," 15).

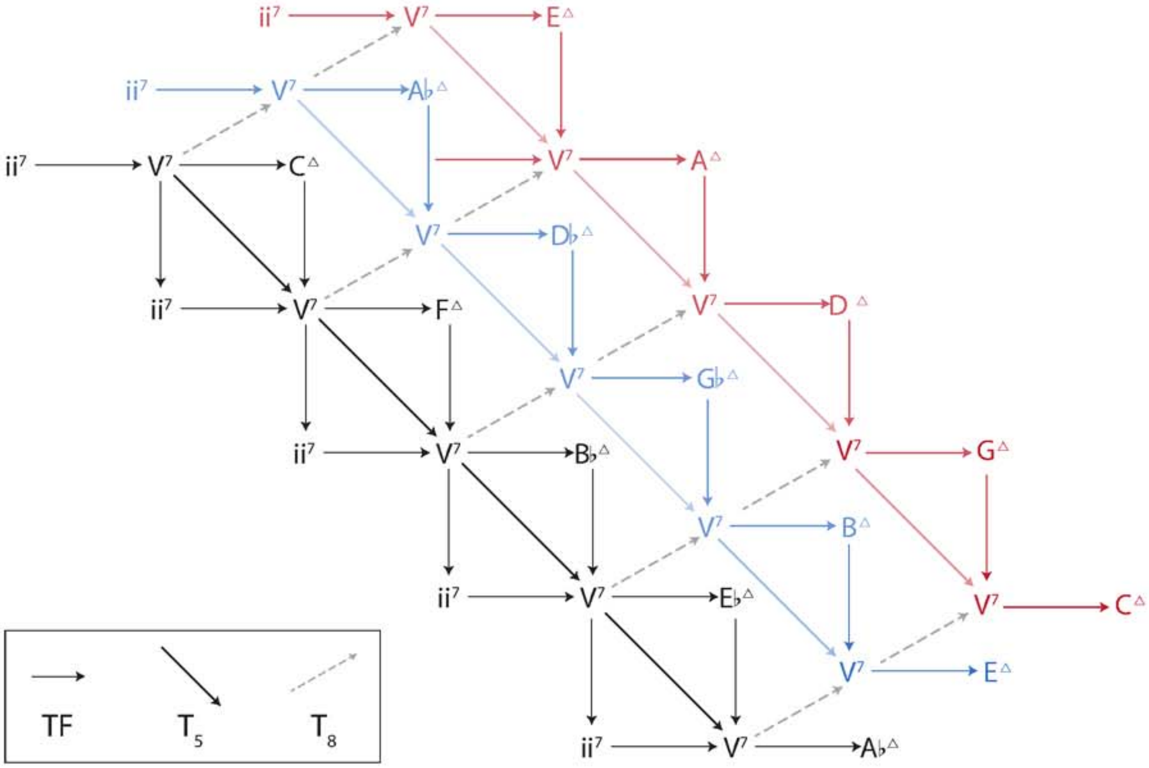
ii^7
V^7
E$^{\triangle}$
ii^7
V^7
A$\flat^{\triangle}$
ii^7
V^7
C$^{\triangle}$
V^7
A$^{\triangle}$
V^7
D$\flat^{\triangle}$
ii^7
V^7
F$^{\triangle}$
V^7
D$^{\triangle}$
V^7
G$\flat^{\triangle}$
ii^7
V^7
B$\flat^{\triangle}$
V^7
G$^{\triangle}$
V^7
B$^{\triangle}$
ii^7
V^7
E$\flat^{\triangle}$
V^7
C$^{\triangle}$
V^7
E$^{\triangle}$
ii^7
V^7
A$\flat^{\triangle}$
TF
T$_5$
T$_8$

Figure 3.15. A major-third organization of ii–V space (ii^7 chords omitted on rear levels for clarity).

As it turns out, we can once again adapt ii–V space in order to show organization by major third, as shown in Figure 3.15. This figure looks very similar to the minor-third organization in Figure 3.2, but the relationships between layers have changed.[35] Here, the "layers" of the space are arranged in descending major thirds (T_8), while the descending fifths arrangement is otherwise unchanged. This arrangement means that all of "Giant Steps" happens in a single horizontal slice of the space.[36] This organization of ii–V space reflects our intuitions about the organization of this tune and others like it: by maintaining the integrity of the ii–V–I progressions and instead altering the relationships between them, we can keep both the local functional progressions important to improvising musicians *and* reflect the unusual chromatic organization of the tune itself.

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[35] The major-third figure represents a kind of cross-section of the minor-third torus: to see this clearly, locate the key areas C, A♭, and E on both Figure 3.2 and Figure 3.15.

[36] In fact, the rest of the figure is unnecessary for "Giant Steps"; the piece is easier to understand using a subgraph of the complete M3-space that contains only the ii–V–I progressions in B, G, and E♭.

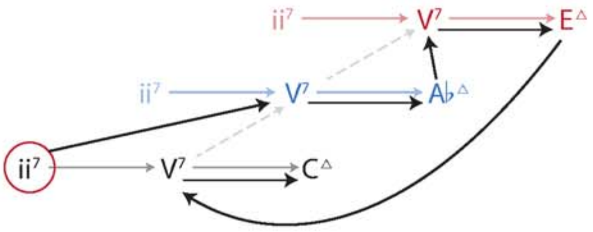

Figure 3.16. Coltrane changes as a ii–V–I substitution, shown in M3-space.

The arrangement into T_8-related "layers" also helps to clarify the function of Coltrane changes as a substitution for a ii–V–I progression (see Figure 3.16). As usual, we could define a transformation to help explain this substitution. Though the T_8 between major seventh chords is certainly important, the most unusual surface feature in the substitution is the jump from a major-seventh chord to the dominant seventh whose root is a minor third higher; it is this harmonic move that gives the progression its forward momentum. We might call this transformation CS (for "Coltrane Substitution"):

$$\text{If } X = (x_r, x_t, x_s) \in S_{\text{maj}}, \text{ then } \text{CS}(X) = Y = (y_r, y_t, y_s) = (x_r + 3, x_t + 3, x_s + 2) \in S_{\text{dom}}$$

With this transformation, we can understand the entire ii–V–I substitution as follows:

$$\text{Dm7} \xrightarrow{\hspace{6cm}\text{TF}\hspace{6cm}} \text{G7} \xrightarrow{\text{TF}} \text{Cmaj7}$$

becomes

$$\text{Dm7} \xrightarrow{\text{TF} \bullet T_8} \text{E}\flat 7 \xrightarrow{\text{TF}} \text{A}\flat\text{maj7} \xrightarrow{\text{CS}} \text{B7} \xrightarrow{\text{TF}} \text{Emaj7} \xrightarrow{\text{CS}} \text{G7} \xrightarrow{\text{TF}} \text{Cmaj7}$$

This M3-space is well equipped to show the logic of major-third cycles, though other kinds of tonal relationships are more difficult to see: both tritone substitutes and minor-third substitutions are maximally far away in M3-space, for example. While this is indeed a limitation, the fact that ii–V space and its variants share the same essential features means that each can be substituted for another as needed for a given analytical situation. The spaces developed so far—ii–V space, its tritone-substituted variant, minor-third space, and now major-third space—all reflect the basic descending-fifths orientation of tonal jazz, but each prioritizes a particular secondary relationship.[37] Maintaining the same basic structure in our analytical apparatus allows us to understand a wide range of music as variations on a basic, functionally harmonic theme. There is no need for a great switching of context from the logic of tritone substitutions to the logic of Coltrane changes, nor is there a need to invoke set classes of different cardinalities to justify dividing the octave into three or four equal parts. This presentation is developed in a rough parallel with the music itself: jazz musicians did not (and do not) discard everything they learned from bebop when approaching a progression like that of "Giant Steps"; rather, each is part of a single, coherent through line of tonal jazz.

3.2.3 Analytical Interlude: Richard Rodgers/Lorenz Hart, "Have You Met Miss Jones?"

Though we could turn to any number of Coltrane's middle-period compositions as analytical examples to illustrate major-third spaces, we will instead opt for the tune that is always cited as one of his influences: the Richard Rodgers and Lorenz Hart standard, "Have You Met Miss Jones?" While some of Coltrane's tunes (like "Giant Steps") use M3-cycles almost exclusively, "Miss Jones" will allow us the opportunity to see how organization by major thirds can participate in more typical, fifths-based jazz harmony. The changes for the tune are given in Figure 3.17; the analysis here will proceed in sections.[38]

The analysis of "Miss Jones" in M3-space is given in Figures 3.18a–3.18c. The A section (Figure 3.18a) is fairly typical, though it does include a fully-diminished seventh chord, which we have not yet seen in this study. This F♯°7 clearly functions as a passing chord, harmonizing the bass line F–F♯–G. The analysis in M3-space interprets this chord, as jazz musicians often do, as a D7♭9 without a root: functionally, the two chords are identical, with each leading to the following

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[37] These spaces represent all but one of the equal partitions of the octave: while we could easily construct a "whole-tone space," it would not have very many applications to tonal jazz. In cases where whole-tone relationships seem important, it is usually not problematic to consider a whole tone as a combination of two perfect fifths, which are readily shown in all of the other spaces.

[38] These changes are again taken from *The Real Book*, and are the standard changes for the tune; nearly all recordings agree with this set of changes.

A	F^{maj7}	F♯$^{\circ7}$	G-^{7}	C^{7}
	A-^{7}	D-^{7}	G-^{7}	C^{7}
A′	F^{maj7}	F♯$^{\circ7}$	G-^{7}	C^{7}
	A-^{7}	D-^{7}	C-^{7}	F^{7}
B	B♭^{maj7}	A♭-^{7} D♭^{7}	G♭^{maj7}	E-^{7} A^{7}
	D^{maj7}	A♭-^{7} D♭^{7}	G♭^{maj7}	G-^{7} C^{7}
A″	F^{maj7}	F♯$^{\circ7}$	G-^{7}	C^{7} B♭^{7}
	A-^{7} D^{7}	G-^{7} C^{7}	F^{maj7}	

Figure 3.17. Changes to “Have You Met Miss Jones?” (Richard Rodgers/Lorenz Hart).

Gm7.[39] This Gm7 initiates a home-key ii–V in mm. 3–4 that resolves deceptively to Am7 in m. 5, at which point the piece begins a iii–vi–ii–V turnaround to return to Fmaj7 for the repeat of the A section.

The second A section begins like the first, but the end is altered so that the bridge can begin on the subdominant, B♭ (Figure 3.18b). The bridge of this tune is its most well-known aspect, and contains the major-third cycle. This organization by major third is readily apparent in the space, as the music seems to break free of the descending fifths to elaborate the subdominant with a sequence that moves into the rear layers of the space and then returns. After arriving on B♭ in the first bar of the bridge, the tune moves to a ii–V–I in G♭ (a major third lower), followed by a ii–V–I in D (yet another major third lower). After making its way to the rear of the space, it begins to work its way back up the chain of thirds, finishing the bridge with a return to a ii–V–I in G♭. This G♭maj7 chord moves via a SLIDE$_7$ transformation to a home-key ii–V–I, which returns to Fmaj7 to begin the final A section. This final section (Figure 3.18c) is nearly the same as the first, but altered slightly in the last four bars to arrive more strongly on tonic in the penultimate bar of the form.

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[39] Levine, *The Jazz Theory Book*, 85.

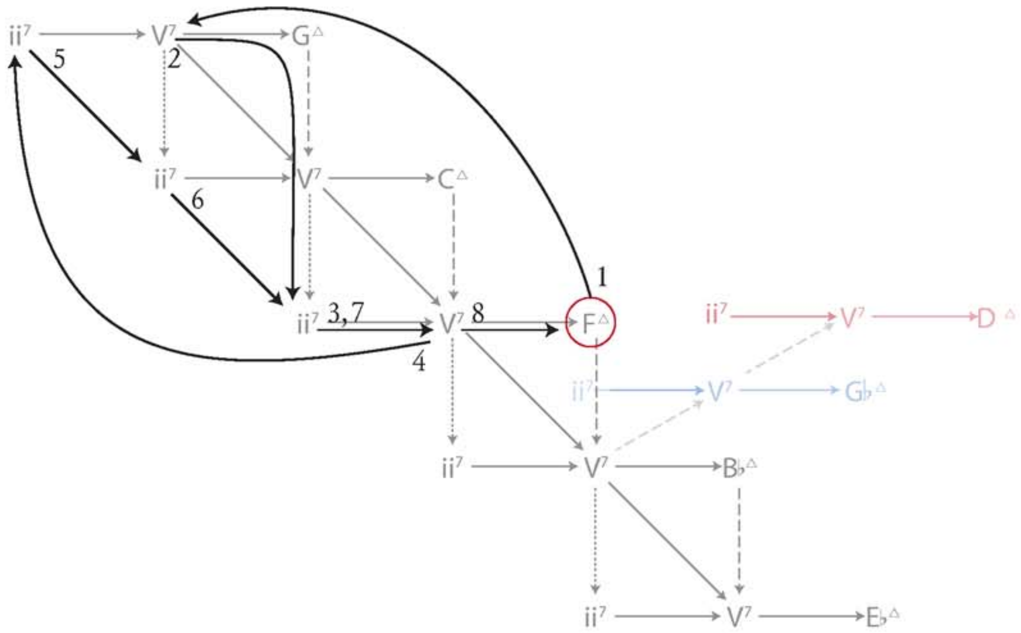
ii⁷
V⁷
G△
5
2
ii⁷
V⁷
C△
6
1
ii⁷
3,7
V⁷
8
F△
ii⁷
V⁷
D△
4
ii⁷
V⁷
G♭△
ii⁷
V⁷
B♭△
ii⁷
V⁷
E♭△

Figure 3.18a. "Miss Jones," A section (mm. 1–8), analyzed in M3-space.

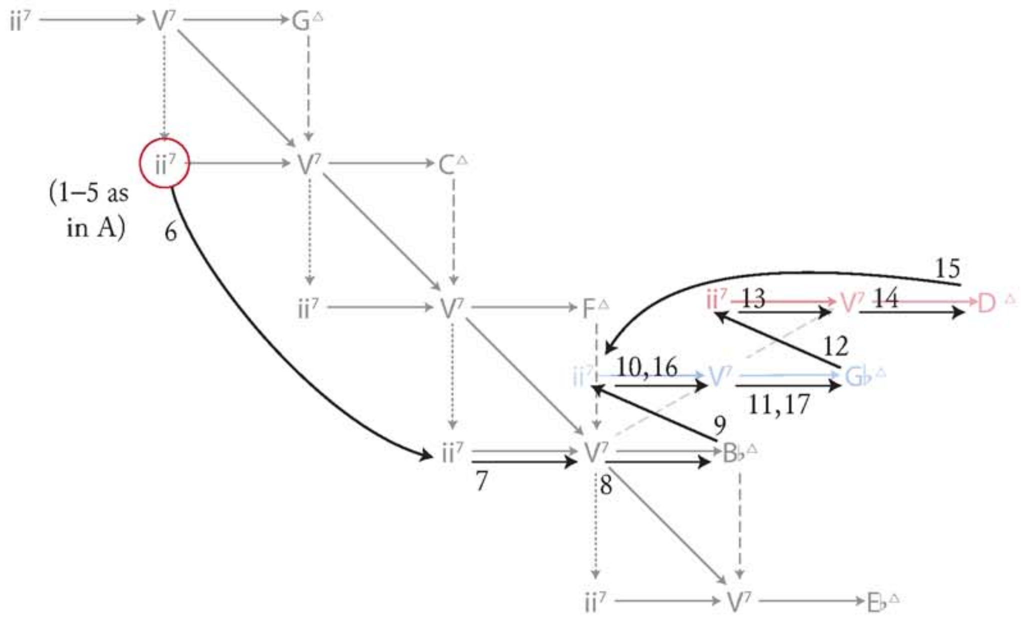
ii⁷
V⁷
G△
ii⁷
V⁷
C△
(1–5 as in A)
6
15
ii⁷
V⁷
F△
ii⁷
13
V⁷
14
D△
12
ii⁷
10,16
V⁷
G♭△
11,17
9
ii⁷
V⁷
B♭△
7
8
ii⁷
V⁷
E♭△

Figure 3.18b. "Miss Jones," second A section and bridge (mm. 9–23).

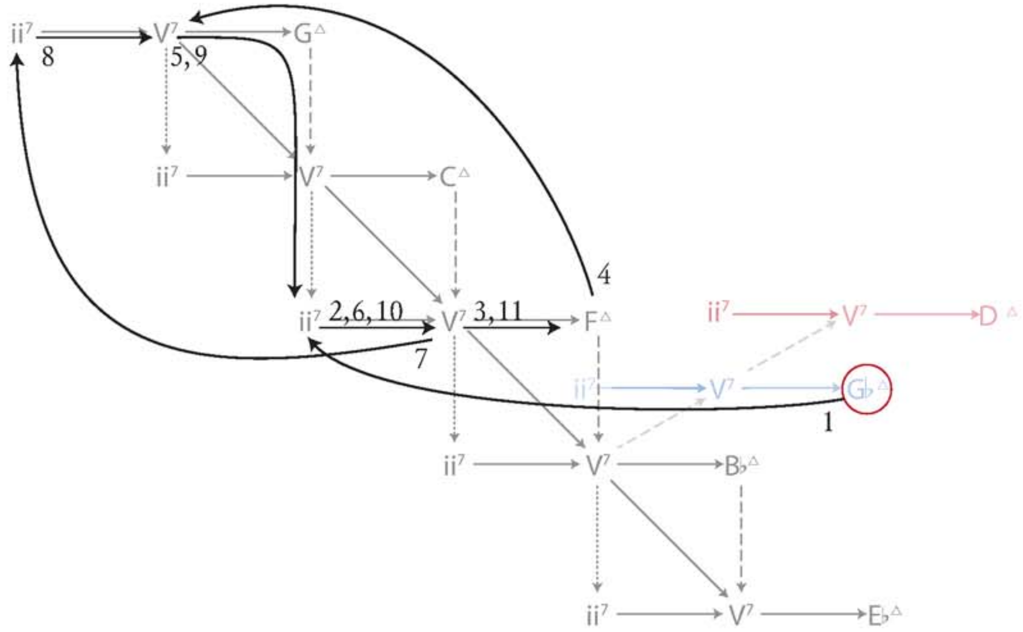
ii⁷
V⁷
G△
8
5, 9
ii⁷
V⁷
C△
4
ii⁷
2, 6, 10
V⁷
3, 11
F△
7
ii⁷
V⁷
D△
ii⁷
V⁷
G♭△
1
ii⁷
V⁷
B♭△
ii⁷
V⁷
E♭△

Figure 3.18c. "Miss Jones," last two bars of bridge and final A section (mm. 23–32).

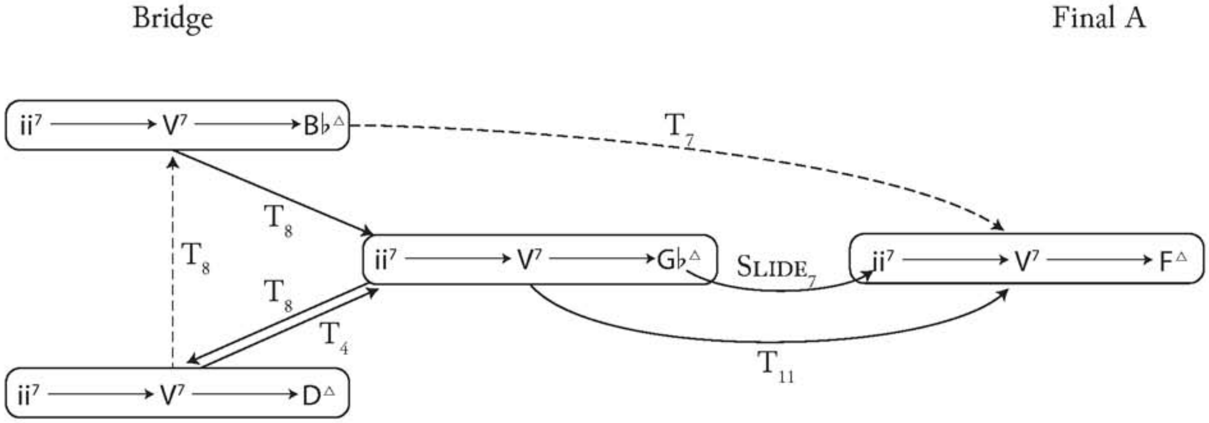
Bridge
Final A
ii⁷
V⁷
B♭△
T7
T8
T8
T8
T4
ii⁷
V⁷
G♭△
SLIDE7
ii⁷
V⁷
F△
T11
ii⁷
V⁷
D△

Figure 3.19. A transformation network for the bridge of "Miss Jones."

A more detailed transformation network for the bridge of "Miss Jones" is given in Figure 3.19.[40] In this network, transformations actualized in the music are shown as solid arrows, while others are shown with dotted arrows. (As usual, the unlabeled arrows are TF transformations.) A complete T_8-cycle is thwarted when the Dmaj7 chord moves instead back to G♭, but the dotted arrow shows that another T_8 move would have completed the cycle. It also clarifies the return to F major in the last A section: a larger-scale T_{11} from G♭ to F is accomplished via the SLIDE$_7$ transformation from G♭maj7 to Gm7.

The analysis of "Have You Met Miss Jones" in M3-space demonstrates how the logic of major-third cycles in jazz is not independent from that of standard fifths-based harmony, but instead an extension of it. Any of our spaces would illustrate the A sections of "Miss Jones" equally well, but the construction of M3-space allows us to better understand the bridge. In ordinary ii–V space (Figure 2.10 on p. 50), the ii–V–I progressions related by major third are maximally far apart; analyzing the bridge in that space would make the T_8-related ii–V–Is seem like nonsensical harmonic motions. Rearranging the basic space as we have done in this section shows that the M3-cycle participates in a coherent way within the logic of the otherwise typical harmony of the tune.

3.3 Parsimonious Voice-Leading

Given the importance of parsimonious voice-leading in many current neo-Riemannian and transformational theories of harmony, it seems prudent to examine the transformational system we have been developing in the last two chapters in that light. Several of the transformations we have defined are indeed parsimonious, moving individual voices efficiently (the 3RD and 7TH transformations, among others) while others are less so (the transformation CS from this chapter, for example). Though there is indeed a great deal of literature on parsimonious voice-leading, all of

---

[40] The "bubble notation" used in Figure 3.19 is first used in *GMIT*, 205–6. It is, as Lewin describes it, a "network-of-networks": each bubble here represents a single ii–V–I network (the unlabeled arrows are again TF transformations), and these networks are connected by larger-scale transpositions. In this figure, the SLIDE$_7$ transformation breaks through the bubble itself, and describes a transformation directly from G♭maj7 to Gm7.

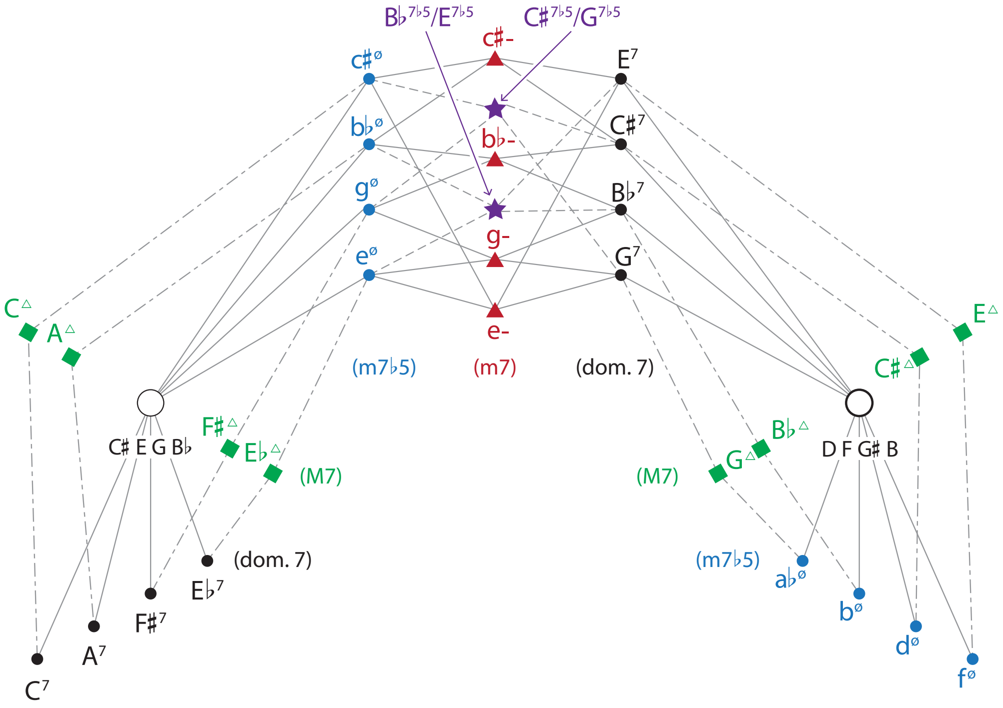
$B\flat^{7\flat5}/E^{7\flat5}$
$C\sharp^{7\flat5}/G^{7\flat5}$
c♯-
E^7
$c\sharp^{ø}$
$b\flat^{ø}$
b♭-
$C\sharp^7$
$g^{ø}$
$B\flat^7$
g-
$e^{ø}$
G^7
e-
$C^{\triangle}$
$A^{\triangle}$
$E^{\triangle}$
$C\sharp^{\triangle}$
(m7♭5)
(m7)
(dom. 7)
C♯ E G B♭
$F\sharp^{\triangle}$
$E\flat^{\triangle}$
(M7)
(M7)
$G^{\triangle}$
$B\flat^{\triangle}$
D F G♯ B
(dom. 7)
$E\flat^7$
$F\sharp^7$
A^7
C^7
(m7♭5)
$a\flat^{ø}$
$b^{ø}$
$d^{ø}$
$f^{ø}$

Figure 3.20. A portion of Four-Cube Trio, redrawn to include major seventh chords. All lines, regardless of style, represent a single voice moving by half-step.

our transformations are defined on ordered triples of the form (root, third, seventh), *not* on triads or seventh chords. As such, we will need to take a few steps in order to connect our work here with the literature that deals with these basic chord types directly.

For the moment, let us set aside the ordered-triple representation we have been using here and return to four-note seventh chords proper. Parsimonious relationships among seventh chords are shown in Douthett's "Four-Cube Trio" (recall Figure 3.13); Figure 3.20 redraws a portion this figure to include major seventh chords.[41] Because we are interested in this figure's application to jazz, the minor seventh chords have been labeled with root names, and the French sixth chords are labeled as dominant seventh chords with flatted fifths (a favorite chord of Thelonious Monk).[42]

---

[41] In addition to including the major seventh chords, this figure corrects some errors in Cohn's version (*Audacious Euphony*, Fig. 7.16, 158): in his version, the voice leading to and from the French sixth chords and minor seventh chords is incorrect. My thanks to Thomas Cooke-Dickens for helping to find many of these errors.

[42] The French sixth chord, set class (0268), is of course invariant when transposed by tritone, so both roots are given in this figure.

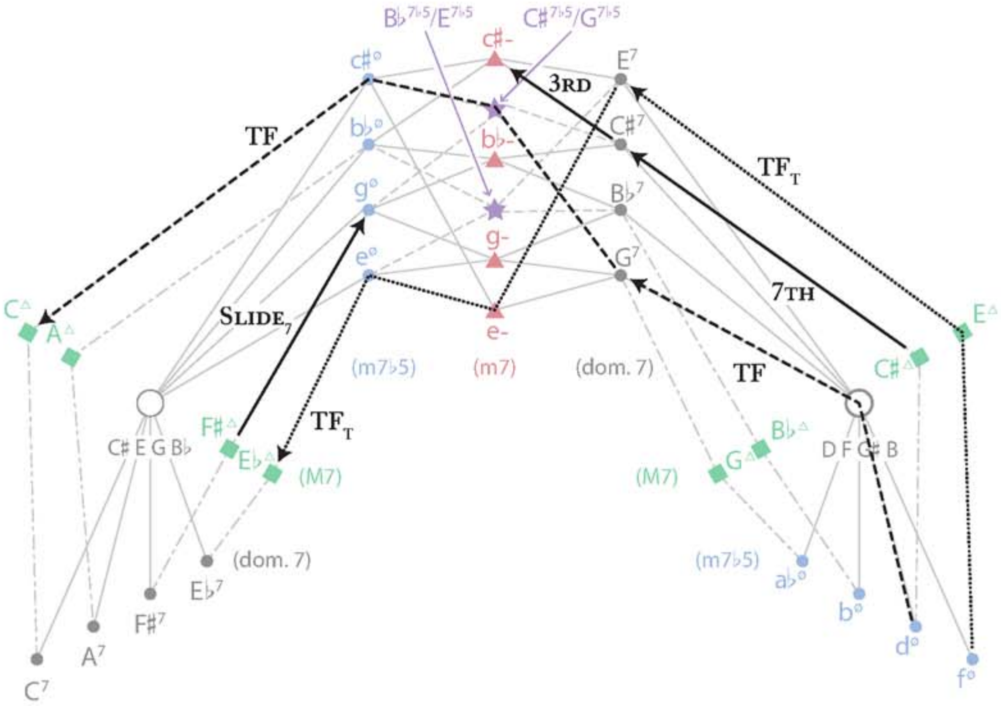
$B\flat^{7\flat5}/E^{7\flat5}$
$C\sharp^{7\flat5}/G^{7\flat5}$
c♯-
c♯°
E^7
3RD
TF
b♭°
b♭-
$C\sharp^7$
TF$_T$
g°
$B\flat^7$
g-
e°
G^7
7TH
C△
A△
SLIDE$_7$
e-
E△
(m7♭5)
(m7)
(dom. 7)
TF
C♯△
F♯△
TF$_T$
C♯ E G B♭
E♭△
(M7)
(M7)
G△
B♭△
D F G♯ B
(dom. 7)
(m7♭5)
$E\flat^7$
a♭°
$F\sharp^7$
b°
A^7
d°
C^7
f°

Figure 3.21. The transformations 3RD, 7TH, SLIDE$_7$, TF, and TF$_T$ in the Four-Cube Trio.

By way of illustration, Figure 3.21 shows some of the transformations we have defined in the last two chapters in the Four-Cube Trio. The most important thing to note about this figure is that it is *not* a voice-leading graph of the transformations as they have been defined. Because all of our transformations are defined on ordered triples (ignoring the fifth of a seventh chord), mapping them into four-note space can be misleading. In our ordered triple representations, for example, there is no distinction between half-diminished and minor seventh chords, nor between the French sixths and the dominant sevenths (i.e., B♭7 is indistinguishable from B♭7♭5). This figure is included only to demonstrate that some of the transformations do represent single voice-leadings (the 3RD, 7TH, and SLIDE$_7$ transformations), while others do not. It is also worth noting that the SLIDE$_7$ transformation is the only one we have defined in which the voice-leading ascends (indicated by a clockwise motion in the figure).

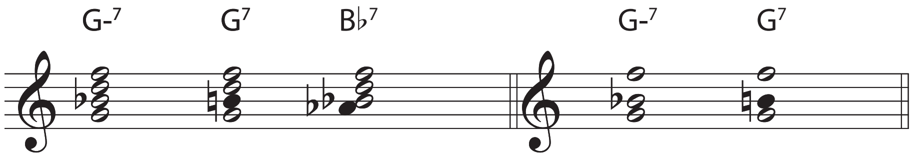

Figure 3.22. Parsimonious voice leading of the minor seventh chord, with its fifth (left) and without (right). Common tones are shown with hollow noteheads.

Starting Chord	Common Tones	Voice leading	Result	Transformation
Cmaj7	r, t	$s-1$	C7	7TH
Cmaj7	r, s	$t-1$	[CmM7]	[3RD]
Cmaj7	t, s	$r+1$	C♯m7	SLIDE_7
C7	r, t	$s+1$	Cmaj7	7TH^{-1}
C7	r, s	$t-1$	Cm7	3RD
C7	t, s	–	–	–
Cm7	r, t	$s+1$	[CmM7]	[7TH^{-1}]
Cm7	r, s	$t+1$	C7	3RD^{-1}
Cm7	t, s	$r-1$	Bmaj7	SLIDE_7^{-1}

Table 3.2. Parsimonious voice-leading among members of S_{min}, S_{dom}, and S_{maj}. In this table, r, t, and s indicate the root, third, and seventh of a chord, respectively.

The parsimonious picture that emerges after we collapse the chords which have the same ordered-triple representation is much less interesting.[43] Most damaging to the structure of the Four-Cube Trio is that the minor seventh chords become more discriminating: considered as an ordered triple, a minor seventh chord is only connected to one dominant seventh chord, not two (see Figure 3.22). This, plus the collapse of the French sixths into dominant sevenths, means that every chord is connected to exactly one other by single-half-step voice-leading. (For reference, a complete voice-leading roster for the ordered-triple representation is given in Table 3.2.)

---

[43] This is not terribly surprising; our ordered triples are trichords of set classes (015), (016), and (026). Richard Cohn has shown ("Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations") that the consonant triad, (037), is unique among trichords in its ability to form parsimonious relationships. Although his work there does not examine parsimonious relationships among members of different set classes, the fact that our three types are not nearly even means that we should not expect to find very many of these relationships.

We could generate the entire set of 36 ordered triples using the single-voice half-step transformations 7TH, 3RD, and SLIDE_7^{-1}:[44]

$$\text{Cmaj7} \xrightarrow{\text{7TH}} \text{C7} \xrightarrow{\text{3RD}} \text{Cm7} \xrightarrow{\text{SLIDE}_7^{-1}} \text{Bmaj7} \xrightarrow{\text{7TH}} \text{B7} \ldots\ \text{D}\flat\text{m7} \xrightarrow{\text{SLIDE}_7^{-1}} \text{Cmaj7}$$

The resulting graph, however, is simply a circle, and does not mirror the rich voice-leading network of the Four-Cube Trio. We could redefine all of the other transformations in terms of these single voice-leadings ($\text{Dm7} \xrightarrow{\text{TF}} \text{G7}$ becomes $\text{Dm7} \xrightarrow{\text{SLIDE}_7^{-1} \bullet (\text{7TH} \bullet \text{3RD} \bullet \text{SLIDE}_7^{-1})^6 \bullet \text{7TH}} \text{G7}$), but these decompositions would not seem to give much insight into tonal jazz—which is, after all, the aim of this study.

Throughout the course of the last two chapters, we have developed a fairly complete transformational system for jazz harmony. Taking ii–V space as our starting point, we have seen how it can be altered in various ways to show different aspects of standard tonal jazz harmony. To this point, though, we have focused almost exclusively on chord symbols (via their abstraction into ordered triples). Of course, jazz harmony involves quite a bit more than relationships among three-note chords: these basic structures are altered in various ways by rhythm section members, and we have yet to say anything at all about the role harmony plays for an improvising musician. To do so, we will need to expand our harmonic universe somewhat; the next chapter begins to take steps in that direction.

---

[44] The inverse is needed for SLIDE_7 because it was defined as a transformation from a major seventh chord to the minor seventh whose root is a half-step higher. A cycle that uses the non-inverse SLIDE_7 transformation would require inverses on both the 3RD and 7TH transformations.

CHAPTER 4
Chord-Scale Transformations

For jazz musicians, a chord symbol implies more than just the root, third, and seventh; its fifth, ninth, and potentially other chord members are implied as well. Instead of considering the extended harmonies common in jazz as stacks of thirds, musicians often describe harmony in a more linear fashion, using a scale to stand in for a chord symbol. What is often called "chord-scale theory" is a major part of jazz pedagogy, and cannot be ignored as we try to approach a general theory of jazz harmony. This chapter begins with an introduction to chord-scale theory, both in its original form and its later pedagogical adaptations, and then continues to incorporate it into the transformational system developed thus far; finally, we will see how the theory allows us to make analytical insights that can go beyond the chord-symbol-based analysis of previous chapters.

4.1 George Russell's *Lydian Chromatic Concept*

The ultimate origin of chord-scale theory is George Russell's *Lydian Chromatic Concept of Tonal Organization*, first published in 1953 but revised several times throughout Russell's life.[1] Russell was a jazz pianist, drummer, and a well-known composer and arranger; he devised the majority of the *Lydian Chromatic Concept* while hospitalized for tuberculosis in 1945–46.[2] The influence of the *Concept* (as it is often called) is difficult to overstate. Joachim-Ernst Berendt and Günther Heusmann describe it as "the first work deriving a theory of jazz harmony from the immanent laws of jazz, not from the laws of European music," and the blurbs on the back cover contain praises from musicians including Gil Evans, Ornette Coleman, Eric Dolphy, and Toru Takemitsu.[3]

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[1] George Russell, *The Lydian Chromatic Concept of Tonal Organization*, 4th ed., vol. 1, *The Art and Science of Tonal Gravity* (Brookline, MA: Concept, 2001); hereafter, simply *LCC*. References to the book in this dissertation will be to this final edition unless otherwise noted. Though a full reception history of the *Lydian Chromatic Concept* is beyond the scope of this project, it is worth noting that the later editions focus more heavily on the theory than the earlier editions, which were more practical in nature. In the original edition, the section on theoretical foundations appears after eight initial "lessons"; in the 2001 edition, this material has been moved front and center to Chapter 1. Furthermore, the last edition was reframed as the first volume in what was to be a multi-volume set; at the time of Russell's death in 2009, only the first volume had been published.

[2] *Encyclopedia of Popular Music*, s.v. "George Russell," last modified July 4, 2006, http://www.oxfordmusiconline.com/subscriber/article/epm/48476.

Despite its importance to jazz theory, though, Russell's work has not received much attention in music-theoretical scholarship on jazz. Dmitri Tymoczko, for example, does not mention Russell at all in his survey on the pedagogical use of chord-scales in jazz, and the only mention of Russell in his book is in a footnote unrelated to Russell's contributions to chord-scale theory.[4] There may be many reasons for this—Russell's serpentine and hard-to-follow prose are probably not least among them—but regardless, an introduction to Russell's theories as he conceived them will be in order here. The *Lydian Chromatic Concept* can be divided into two main components, which we will treat separately in the following sections: Lydian tonal organization and chord/scale equivalence.

4.1.1 Lydian Tonal Organization

Russell's central insight—indeed, the Concept itself—is that the Lydian scale, rather than the major scale, serves a fundamental role in equal-tempered music. He offers many explanations, but this central idea is easiest to demonstrate, as he does, with an example. Figure 4.1a reproduces Russell's first example; he provides the following instructions and explanation:

> Sound both of the following chords separately. Try to detect the one which sounds a greater degree of unity and finality with its tonical [*sic*] C major triad. . . . In tests performed over the years in various parts of the world, the majority of people have repeatedly chosen the second chord—the C Lydian Scale in its tertian order. (*LCC* 1)

The lowest note of a stack of six perfect fifths is what Russell calls the "Lydian tonic"; the B–F tritone in the C major scale "disrupts the perfect symmetry of the fifths" (*LCC* 4; see Figure 4.1b).

---

[3] Joachim-Ernst Berendt and Günther Huesmann, *The Jazz Book: From Ragtime to the 21st Century*, 7th ed., trans. H. and B. Bredigkeit et al. (Chicago: Lawrence Hill, 2009), 602. Takemitsu's fascination with the *Concept* is discussed at length in Peter Burt, "Takemitsu and the Lydian Chromatic Concept of George Russell," *Contemporary Music Review* 21, no. 4 (2002): 79–109.

[4] Dmitri Tymoczko, "The Consecutive-Semitone Constraint on Scalar Structure: A Link Between Impressionism and Jazz," *Intégral* 11 (1997): 135–79; and *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 366n13. My intent here is not to single out Tymoczko, but only to note that modern theorists who engage directly with Russell's ideas do not always mention his work. The most complete treatments of the *Concept* in modern scholarship are Burt's previously-cited article on Takemitsu and Brett Clement's work on Frank Zappa (who, while influenced by jazz, is not part of the jazz mainstream that is the focus of this study); see "A New Lydian Theory for Frank Zappa's Modal Music," *Music Theory Spectrum* 36, no. 1 (Spring 2014).

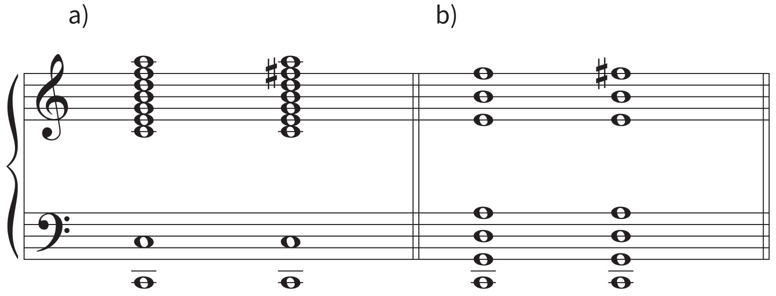

Figure 4.1. The major and Lydian scales, as stacks of thirds and fifths (Russell's Examples I:1, I:2a, and I:5a).

He does concede that C is also understood as tonic in the major scale arrangement, but that it does not sound resolved, since "the presence of the Lydian *do* on the major scale's fourth degree permanently denies [it] that possibility" (*LCC* 4). For Russell, the major scale is always in a state of tension, wanting to "resolve" to the Lydian.[5]

Closely allied with the Lydian tonic is the concept of tonal gravity, which Russell describes as the fundamental principle of the *Concept*. In a stack of fifths, tonal gravity flows downward: "the tone F♯ yields to B as its tonic—F♯ and B surrender 'tonical' authority to E, and so on down the ladder of fifths—the entire stack conferring ultimate tonical authority on its lowermost tone, C" (*LCC* 3). The concept of tonal gravity provides the justification for the primacy of the Lydian scale, since the major scale cannot be constructed by generating perfect fifths from its tonic.

Its theoretical justifications aside, the Lydian scale has an almost mystical quality to Russell, which can sometimes be off-putting. A somewhat longer passage from the *Concept* will help to illustrate Russell's fascination with the scale, as well as his usual circuitous mode of presentation (the emphasis and non-bracketed ellipses are original):

> The LYDIAN TONIC, as the musical "Star-Sun," is the seminal source of tonal gravity and organization of a Lydian Chromatic scale. [. . .] UNITY is the state in which the Lydian Scale exists in relation to its I major and VI minor tonic station chords, as well as

---

[5] I am using the term "scale" here as Russell does: he always refers to the Lydian as a scale rather than a mode. Dmitri Tymoczko has argued against this usage, saying that there is a "widespread tendency to elide the difference between scale and mode" (*A Geometry of Music*, 366n14). To call the Lydian a mode would imply that it is simply a reordering of a major scale, though, and for Russell the two are fundamentally different objects.

> those on other scale degrees. Unity *is* . . . instantaneous completeness and oneness in the *Absolute Here and Now* . . . above linear time.
>
> The Lydian Scale is the musical *passive* force. Its unified tonal gravity field, ordained by the ladder of fifths, serves as a theoretical basis for tonal organization within the Lydian Chromatic Scale and, ultimately, for the entire Lydian Chromatic Concept. There is no "goal pressure" within the tonal gravity field of a Lydian Scale. The Lydian Scale exists as a self-organized *Unity* in relations to its tonic tone and tonic major chord. The Lydian Scale implies an evolution to higher levels of tonal organization. The Lydian Scale is the true scale of tonal unity and the scale which clearly represents the phenomenon of tonal gravity itself. (*LCC* 8–9)

Russell's logic is, of course, circular: the Lydian tonic is by definition the note that is the bottom of a stack of six perfect fifths, and the principle of tonal gravity confers a special status on the bottom of a stack of six fifths (conveniently, the Lydian tonic). Partly for this reason, this part of Russell's theory has not really been taken seriously by modern scholars. He never gives a reason, for example, that the stack should not be extended further: would the lowest note of a stack of *seven* fifths not be imbued with even more tonal gravity? This complication reappears when Russell later presents the complete "Lydian Chromatic Order of Tonal Gravity," given here starting on F (*LCC* 12):

F C G D A E B C♯ A♭ E♭ B♭ G♭

What should be a perfect fifth from B to F♯ is replaced by a whole step (B–C♯), so that the pitch forming a minor ninth with the Lydian tonic (G♭) does not appear until the last note. This sleight of hand also prevents there from being more than one succession of six perfect fifths in the series. If it had continued in perfect fifths, there would be by definition seven Lydian tonics!

Considering these inconsistencies, we might ask if there is anything worth saving in Russell's ideas. He is probably right that most people prefer the sound of the stack of thirds on the right of Figure 4.1a, with the F♯.[6] And the Lydian scale *does* have some practical advantages over the major scale. As Russell points out, $\sharp\hat{4}$ appears before $\natural\hat{4}$ in the harmonic series (he does not mention that $\flat\hat{7}$ appears before $\natural\hat{7}$).[7] The Lydian scale is also unique in that it is possible to form all twelve

---

[6] Though Russell attributes this preference to the Lydian scale, we could probably point to another reason: the second chord does not contain the dissonant minor ninth between F♮ and E, the third of the chord.

[7] Russell does recognize that $\sharp\hat{4}$ in the harmonic series is not pure. Describing this potential problem, he notes that in relation to a C fundamental, "the eleventh overtone, represented as F♯ has a frequency of 551 cents: 1/100th of a

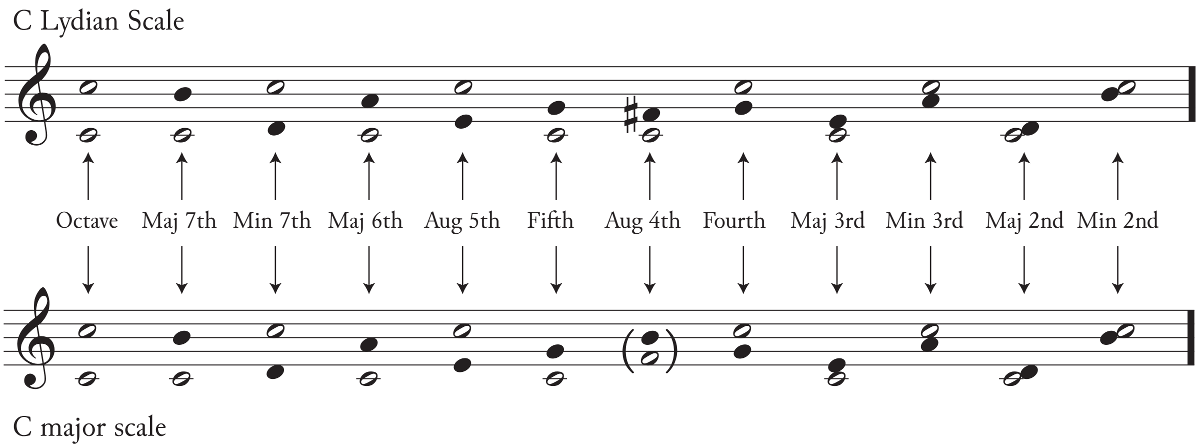

Figure 4.2. Russell's "Interval Tonic Justification" for the Lydian scale (his Example I:9).

interval types with the tonic; Russell's explanation of this fact is reproduced in Figure 4.2 (the F in the major scale is shown with a hollow note because it is the true Lydian tonic for the scale).

Though we may not share Russell's preoccupation with the Lydian scale, we should not let his idiosyncratic views distract us from the more important points of the theory. Theorists are, of course, used to adopting worthwhile theoretical ideas from authors without assuming their entire worldview. We regularly practice Schenkerian analysis without adopting Schenker's views on the superiority of German music, and even 18th-century authors like Johann Phillip Kirnberger accepted Jean-Phillipe Rameau's fundamental bass without necessarily espousing his more contentious thoughts on harmonic generation or *subposition*.[8] And yet, with the *Concept*, this does not seem to have taken place.[9] If we grant Russell these inconsistencies, though, his ideas prove to be remarkably useful, as we shall see.

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semitone closer to F♯ than F♮. In fact, the overtone series fails to list the F♮ tone even when carried out to twenty harmonics" (*LCC* 3).

[8] Joel Lester, *Compositional Theory in the Eighteenth Century* (Cambridge, MA: Harvard University Press, 1992), 240–41.

[9] My own suspicion is that Russell's work is too new to be considered historically, but not new enough to be taken seriously as modern scholarship. I discuss this idea at length in "Reconceptualizing the *Lydian Chromatic Concept*: George Russell as Historical Theorist" (paper presented at the annual meeting of the Society for Music Theory, St. Louis, MO, October 2015); see also Kyle Adams, "When Does the Present Become the Past? A Re-examination of 'Presentism' and 'Historicism'" (paper presented at the annual meeting of the Society for Music Theory, Charlotte, NC, November 2013).

Before moving on to chord-scales proper, we should first examine the scales themselves, as their initial presentation in the *Concept* is entangled with the discussion of the nature of the Lydian scale. The scales are generated (more or less) from the chromatic order of tonal gravity, which is given again here in its generic form:

I V II VI III VII +IV +V ♭III ♭VII IV ♭II

When taken together, the entire series represents the Lydian Chromatic Scale, the foundation of the titular Concept.

The Lydian Chromatic (or LC) scale contains eleven "member scales," each of which is chosen, Russell says, for three reasons:

> a. a scale's capacity to parent chords considered important in the development of Western harmony
> b. a scale as being most representative of a tonal level of the Lydian Chromatic scale
> c. the historical and/or sociological significance of a scale. (*LCC* 12)

These eleven scales are further divided into seven principal scales and four horizontal scales. The seven principal scales are derived from the Lydian Chromatic scale, and are shown in Figure 4.3. These scales are given what Russell calls their "ingoing-to-outgoing" order in regards to the F Lydian tonic; "ingoing" and "outgoing" may be read as "consonant" and "dissonant," respectively.[10] The principal scales are probably more familiar under different names, as shown in Table 4.1.[11]

The means by which the LC scale generates the seven principal scales is explained the diagram reproduced in Figure 4.4.[12] Russell's explanation of this diagram is somewhat confusing. The term "tone order" is never defined, except to say that the LC scale has five of them (it is unclear why there is no 8-tone order). The shaded "consonant nucleus" describes the fact that all of the standard chord types—major, minor, seventh, augmented, and diminished—are contained within

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[10] Russell defines "ingoing" only in passing, saying that "all music conceived within the equal tempered system maintains a closer (more INGOING) relationship to one tone than to all others, regardless of the music's style or genre" (*LCC* 9).

[11] Russell's idiosyncratic names have mostly fallen out of use, since they are long and difficult to remember. It is easier for an improvising musician, for example, to recall the "half–whole diminished scale" than it is to remember the "auxiliary diminished blues scale" (named, incidentally, for the fact that it shares $\flat\hat{3}$, $\natural\hat{3}$, and $\flat\hat{7}$ with the blues scale). The "Lydian dominant" scale is an exception: though Russell does not use the term himself, it is clearly derived from his work.

[12] The remainder of this paragraph is a gloss on the material in *LCC* 12–17.

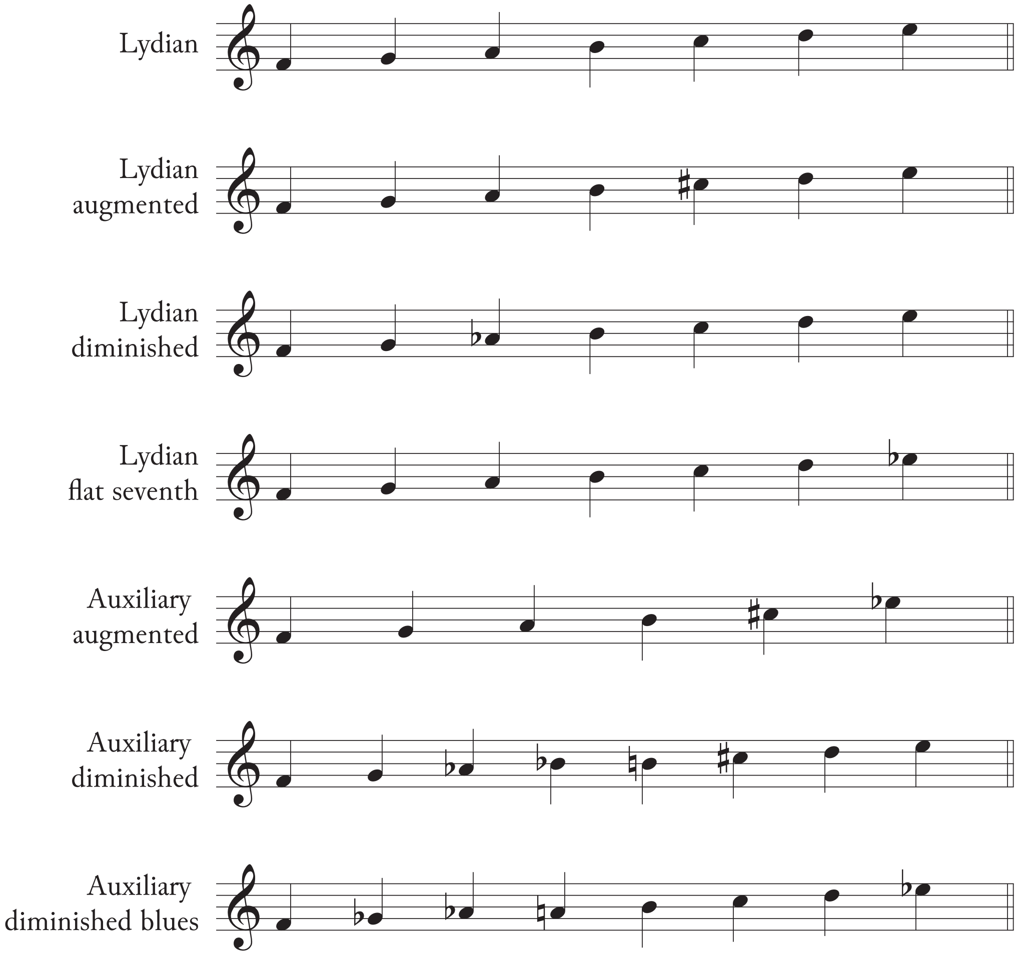
Lydian
Lydian augmented
Lydian diminished
Lydian flat seventh
Auxiliary augmented
Auxiliary diminished
Auxiliary diminished blues

Figure 4.3. The seven principal scales of the F Lydian Chromatic scale.

Russell's name	Other common names
Lydian	–
Lydian augmented	3rd mode of melodic minor, 7th mode of acoustic
Lydian diminished	4th mode of harmonic major
Lydian flat seventh	Lydian dominant, acoustic, 4th mode of melodic minor
Auxiliary augmented	whole-tone
Auxiliary diminished	octatonic, diminished (whole–half)
Auxiliary diminished blues	octatonic, diminished (half–whole)

Table 4.1. Russell's principal scale names and their other common names.

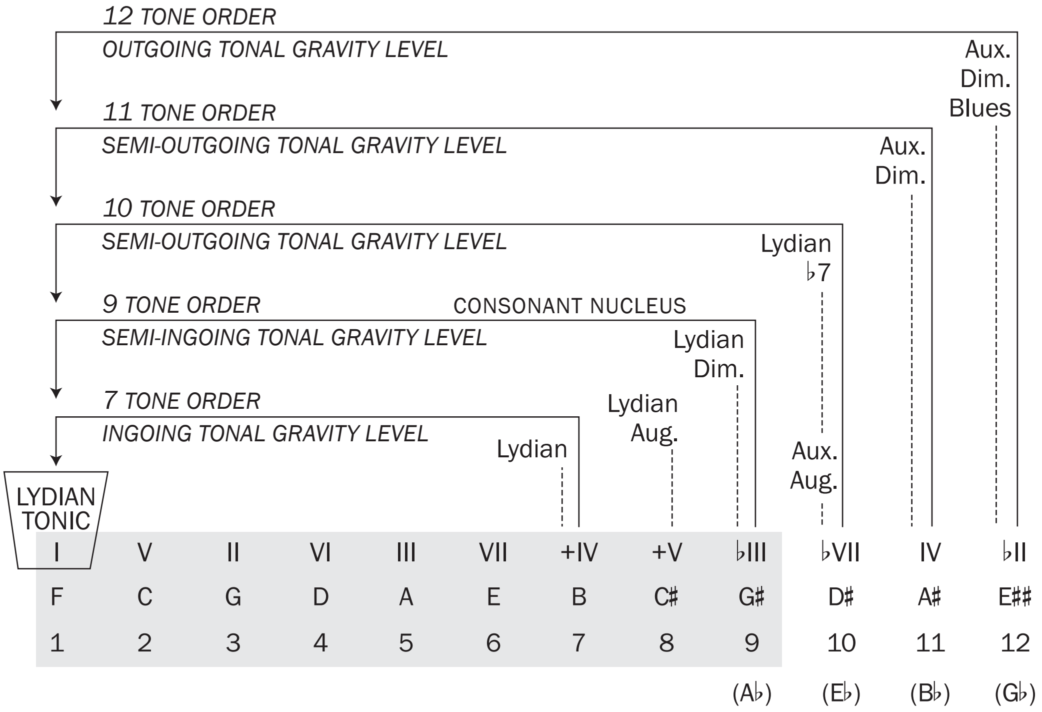
12 TONE ORDER
OUTGOING TONAL GRAVITY LEVEL
Aux. Dim. Blues
11 TONE ORDER
SEMI-OUTGOING TONAL GRAVITY LEVEL
Aux. Dim.
10 TONE ORDER
SEMI-OUTGOING TONAL GRAVITY LEVEL
Lydian ♭7
9 TONE ORDER
CONSONANT NUCLEUS
SEMI-INGOING TONAL GRAVITY LEVEL
Lydian Dim.
7 TONE ORDER
INGOING TONAL GRAVITY LEVEL
Lydian
Lydian Aug.
Aux. Aug.
LYDIAN TONIC
I V II VI III VII +IV +V ♭III ♭VII IV ♭II
F C G D A E B C♯ G♯ D♯ A♯ E♯♯
1 2 3 4 5 6 7 8 9 10 11 12
(A♭) (E♭) (B♭) (G♭)

Figure 4.4. Russell's Lydian Chromatic Order of Tonal Gravity (his example II:3).

it.[13] The consonant nucleus also provides a (tautological) explanation for the missing fifth in the order of tonal gravity: "the skipping of the interval of a fifth between the seventh and eighth tones of the Lydian Chromatic Scale allows the five basic chord categories of Western Harmony to be assimilated by its Nine-Tone Order, Semi-Ingoing Level, in the logical order of their development in Western Harmony and the Lydian Chromatic Scale" (*LCC* 16).

The other four of the eleven member scales are known as the "horizontal scales," and are shown in Figure 4.5. For Russell, "horizontal" is used in opposition to the "vertical" generation of the Lydian scale. Because the major scale is not a stack of perfect fifths, he considers it to be generated in a different direction. All of the horizontal scales have $\natural\hat{4}$; Russell only includes them because of their "historical and/or sociological significance." The horizontal scales do not, as we shall see, generate chords in the same way as the vertical scales, and for Russell they exist in a constant state of tension between the "false" tonic and the true Lydian tonic.

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[13] Exactly what Russell means by "seventh" is unclear; he likely means the dominant seventh, though all four standard seventh chord types (major, minor, diminished, and half-diminished) appear in the consonant nucleus.

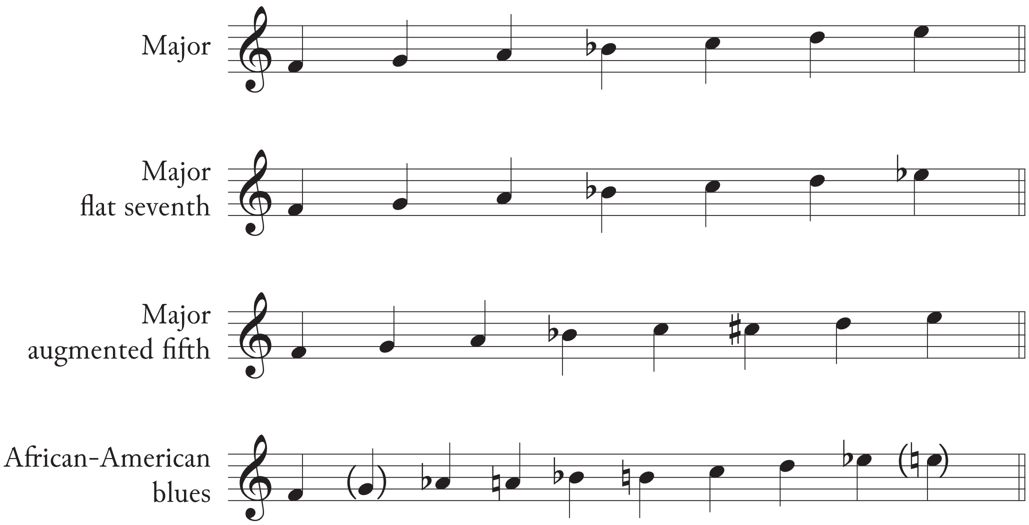

Figure 4.5. The four horizontal scales of the F Lydian Chromatic scale.

Given that most of Russell's ideas on Lydian tonal organization have disappeared from modern chord-scale theory, it is reasonable to ask why so much space has been devoted to them here. There are two reasons. First, much modern scholarship does not seriously engage with Russell's ideas, and as a result most theorists are not familiar with its first incarnation. Because the original presentation of chord-scale theory is tied up with that of Lydian tonal organization, understanding the former is important in order to make sense of the latter. Second, and more importantly, one of the goals of this dissertation is to take jazz musicians' conceptions of harmony seriously: chord-scale theory is an integral part of the way jazz is taught, and therefore many practicing musicians understand harmony in terms of this theory. Later in this chapter, we will seek to revive some of Russell's initial formulation of the *Concept* as we develop a transformational system of chord-scales.[14]

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[14] The counterargument is perhaps obvious: if the ideas about Lydian tonal organization have fallen by the wayside, why bother trying to resuscitate them? As I hope to show in the following sections, Russell's systematic approach is useful as a means of formalizing what has since become implicit knowledge.

4.1.2 Chord/Scale Equivalence

While Russell may have understood Lydian tonal organization to be the most important part of his new theory, the part that has survived—flourished, even—is his novel conception of chord/scale equivalence.[15] Russell's first mention of the concept explains its inception:

> In a conversation I had with Miles Davis in 1945, I asked, "Miles, what's your musical aim?" His answer, "to learn all the changes (chords)," was somewhat puzzling to me since I felt—and I was hardly alone in the feeling—that Miles played like he already knew all the chords. After dwelling on his statement for some months, I became mindful that Miles's answer may have implied the need to relate to chords in a new way. This motivated my quest to expand the tonal environment of the chord beyond the immediate tones of its basic structure, leading to the irrevocable conclusion that every traditionally definable chord of Western music theory has its origin in a PARENT SCALE. In this vertical sense, the term refers to that scale which is ordained—by the nature of tonal gravity—to be a chord's source of arising, and ultimate vertical completeness; the chord and its parent scale existing in a state of complete and indestructible chord/scale unity—a CHORDMODE. (*LCC* 10)

What Davis was looking for is essentially a way of determining what notes he could play over a given chord. Simply knowing the chord tones no longer seemed to be enough, since various extensions and alterations can change the sound of the chord. Chord-scale theory is ultimately, then, an improvisational expedient: a single scale stands in for a chord symbol. Chord symbols with alterations are represented by different scales, and thus musicians do not necessarily have to keep track of all of the individual chord tones.

Russell's later explanation of the concept is uncharacteristically clear:

> The chord and its parent scale are an inseparable entity—the reciprocal sound of one another. . . . In other words, the complete sound of a chord is its corresponding mode within its parent scale. Therefore, the broader term CHORDMODE is substituted for what is generally referred to as "the chord." (*LCC* 20–21)

---

[15] Dmitri Tymoczko makes the argument that something like chord-scale theory existed in the music of the Impressionists, and suggests that jazz musicians may have discovered it by listening to Debussy and Ravel ("The Consective-Semitone Constraint," 152 and 173). Given the initial reception of the *Concept* as the first real theory of jazz, I am skeptical of this claim, and treat Russell's ideas as their first appearance. Certainly, though, the tenets of chord-scale theory can be applied to other musics: Russell himself examines passages of Bach, Beethoven, and Ravel, among others.

It is important to understand that for Russell, the two terms—chord and scale—are truly equivalent: one does not substitute for the other, rather one *is* the other.[16] Here, we arrive at the reason for the inclusion of this material in a dissertation about jazz harmony. If we take Russell seriously (and I am arguing that we should), a harmony is a scale, and vice versa. The two ideas are inseparable, and a study of harmony in jazz would be incomplete without a commensurate discussion about scales.

At this point a brief overview of Russell's brand of chord-scale theory is appropriate. This material accounts for the majority of the length of the *Concept*, so we will be careful here to avoid going into all of its painstaking detail. Determining the scale that belongs with a particular chord is a multi-step process: first, identify the parent Lydian scale; then, determine the harmonic genre based on the characteristic modes of the Lydian scale.[17]

Russell goes through all seven modes of the Lydian scale, identifying the "principal chords" of each mode. These chords represent the purest form of the mode, and the basis for the chord/scale matching process. An overview of these is given in Table 4.2, which merits a bit of commentary.[18] First, the order of modes in the table follows Russell's order of presentation, which (though he does not explain it) roughly coincides with the frequency of each principal chordmode in jazz practice. Second, the "sub-principal chords" are those which are also representative of a given mode; they "do not contain all the tones of [the] relative Principal Chordmode," but they "still exist in a state of unity with [the] parent Principal scale" (*LCC* 23). Last, those modes with B in their names refer to bass notes: the III Major (IIIB) group refers to major chords with the third in the bass.

The treatment of Mode V here bears special mention. The fifth mode of the Lydian scale is of course the ordinary major scale, which Russell took great pains to show earlier was *not* a

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[16] In order to avoid confusion, we will generally respect the distinction between chord and scale from this point on. Where we have occasion to refer to the explicit chord/scale equivalence, we will use the term "chord-scale" rather than Russell's "chordmode." Russell's interchangeable use of "chord," "chordmode," and "mode" tends to confuse more than it helps, and "chord-scale" is the generally accepted term in modern scholarship and pedagogy.

[17] This process is simplified somewhat by the inclusion of a foldout chart in the book (in the first published edition, it was referred to as the "Lydian slide rule"). The chart is somewhat difficult to understand, though, and including it here would likely confuse matters.

[18] This table summarizes the material in *LCC* 23–29.

Mode	Spelling	Principal chordmode	Sub-principal chords
I Major	C D E F♯ G A B	Cmaj13♯11	CM (triad), Cmaj6, Cmaj7, Cmaj7♭5
II Seventh	D E F♯ G A B C	D13	D7, D9, D11
VI Minor	A B C D E F♯ G	Am13	Am (triad), Am6, Am7, Am9, Am11
III Major (IIIB/Minor +5)	E F♯ G A B C D	Cmaj13♯11/E	C/E, Cmaj7/E, etc.
+IV Minor Seventh ♭5	F♯ G A B C D E	F♯m11$^{\substack{\flat 5\\ \flat 9\\ \flat 13}}$	F♯m7♭5, F♯m7♭5♭9, F♯m11♭5♭9
V Major (VB)	G A B C D E F♯	Cmaj13♯11/G	C/G, Cmaj7/G, etc.
VII Eleventh ♭9 (VIIB)	B C D E F♯ G A	Cmaj13♯11/B	B11♭9, C/B, Cmaj9/B, Cmaj9♯11/B

Table 4.2. Modes of the C Lydian scale.

chord-generating scale.[19] Its role as the fifth mode of the Lydian scale is only to act as support for the consonant Lydian harmony. The principal chordmode for this scale is the same as that of the Lydian proper, with the fifth in the bass. This chord and its relatives are by nature unstable (cf. the cadential $^6_4$ chord), and this instability allows Russell to avoid a potential complication of his theory.

After the explanation of the modes of the Lydian scale, Russell goes on to show how his seven vertical scales give rise to other kinds of chords. To do so, he introduces another bit of terminology, the Primary Modal Genre (PMG):

> A PMG is an assemblage of Principal Chord Families of similar type: a Principal Chord Family mansion housing the spectrum of variously colored Principal Chord Families of the same essential harmonic genre. (*LCC* 29)

All of the principal chordmodes in Table 4.2 are PMGs, and the six other vertical scales—Lyd. augmented, Lyd. diminished, Lyd. flat seventh, Aux. augmented, Aux. diminished, and Aux. dim. blues—generate similar assemblages of chordal types.

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[19] His treatment of Mode II is similarly problematic, as Lydian Mode II (a vertical scale) is distinct from the Major flat seventh scale (a horizontal one).

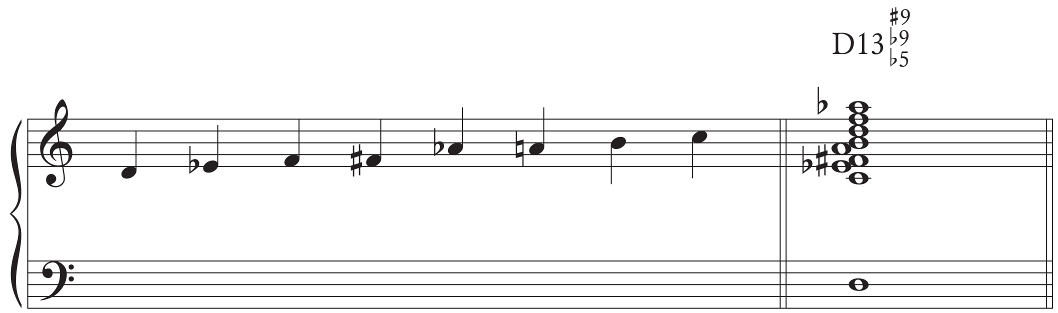

Figure 4.6. The second mode of the C auxiliary diminished scale, in scalar and tertian formations.

Primary Modal Tonic	Primary Modal Genre
I	major and altered major chords
II	seventh and altered seventh chords
III	[I] major and altered [I] major 3B (minor +5) chords
+IV	minor seventh ♭5 / [I] major +4B chords
V	[I] major and altered [I]5B chords
VI	minor and altered minor chords
VII	eleventh ♭9 / [I] major 7B chords
+V	seventh +5 chords

Table 4.3. The eight principal modal tonics and their associated modal genres (Russell's example III:30).

Figure 4.6 gives an example of how this works in practice. The left-hand side of the figure shows the second mode of the C auxiliary diminished scale, and the right-side gives its vertical expression as an altered dominant chord: D13♯9♭9♭5. Russell works through all of the modes of the six other vertical scales, and the chart included with the book lists almost all of these. The eight PMGs fall into general categories, which are shown in Table 4.3 and will be sufficient for our purposes here.[20]

It is by now, I hope, apparent how the *Concept* can simplify matters somewhat for an improvising jazz musician. Once the process of matching chord symbols with scales is learned, it is a relatively simple matter to determine what notes will sound good over, for example, a D13♯9♭9♭5 chord. Russell seems to have taken Davis's wish to "learn all the changes" to heart; he takes care to note that indeed *all* of the harmonies of Western music can be found somewhere in the chart, and notes that many "non-traditional harmonic colors" can be found as well (*LCC* 29).

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[20] There are eight PMGs rather than seven because Russell needs to account for the sharp fifth in the Lydian augmented scale. Other altered scale degrees (♭II, ♭III) are seen simply as alterations and are not counted among the principal genres. Russell does not explain why, but the reason is probably related to the fact that the +V genre gives rise to an important class of chords (7+5), while the others do not.

At the same time, though, it is probably also apparent that Russell's system is somewhat more complicated than it needs to be. His ideas about Lydian tonal organization are in fact the source of much of this complication. To see the extent to which the two are entangled, take Russell's explanation of how to find the parent scale for an unadorned E♭7 chord (a relatively straightforward example):

> Over the roman numerals of the scales of Chart A are listed different chord families. For example, over roman numeral II of the Lydian Scale are listed 7th, 9th, 11th, and 13th chords. They belong to the same family: the (II) seventh chord family of a Lydian Chromatic Scale.
>
> The E♭7 chord is found in this family above roman numeral II of the Lydian Scale in the right column of Chart A. The Lydian Scale is therefore the parent scale of the E♭7 chord.
>
> Place the root of the E♭7 chord on roman numeral II, and E♭ becomes the second degree of that chord's parent scale.
>
> Think down a major 2nd interval; if E♭ is the second degree of the parent scale, D♭ is the first degree. Therefore D♭ is the tonic (root) of the E♭7 chord's parent scale. This tonic is called the Lydian tonic. For the E♭7 chord, D♭ is the Lydian Tonic and the parent scale is D♭ Lydian. (*LCC* 59)

This is quite a long process to determine that the most ingoing (consonant) scale for an E♭7 chord is the second mode of the D♭ Lydian scale. The equivalence of chords and scales is genuinely useful for improvising musicians, but the Lydian organization is more abstract. Faced with this situation, jazz musicians made the obvious simplification: over an E♭7 chord, play the E♭ Mixolydian scale.

Russell's theory becomes interesting, though, when we realize that any of the member scales of the Lydian tonic can stand in for the ordinary Lydian. Once you have determined that the parent scale of an E♭7 chord is D♭ Lydian, then it becomes easy to substitute more complicated scales built on the same Lydian tonic. If you wanted to create a more dissonant (outgoing) sound, you might instead play the second mode of the D♭ Lydian flat seventh scale; the second mode of the D♭ auxiliary augmented blues scale would be more dissonant still. Because the seven principal scales form a spectrum of consonance to dissonance—as Russell frames it, there is a progression of unity from ingoing to outgoing—the *Concept* provides a means of measuring how closely a particular progression or improvisation stays to a particular Lydian tonic. This idea is the core of what we might recover from Russell, and we will return to it when we begin to develop a transformational system in the next section.

4.1.3 Chord-Scale Theory after Russell

Russell's fundamental insight about the nature of chords and scales was revolutionary, and now forms the basis for much of modern jazz pedagogy. Most of the later sources for chord-scale theory, as it has come to be called, do not contain any mention of the Lydian generation of the tonal system, or go through the fuss of finding a parent Lydian scale and its associated PMG. Some of these texts do not mention Russell at all, which we might take as evidence that (not unlike Rameau's fundamental bass) chord/scale equivalence is such a natural way of thinking about music that it was taken for granted and no longer associated with its original author. Given the influence of this theory in jazz pedagogy, it will be worthwhile to sketch a brief outline of the literature here, if only to show how it differs from Russell's conception.

Each text uses slightly different variations on the theory, but Mark Levine's *Jazz Theory Book* will serve here as a surrogate for the theory in general.[21] Levine divides his chapter on chord-scale theory into four parts: major scale harmony, melodic minor scale harmony, diminished scale harmony, and whole-tone scale harmony. For each of these families, he describes the modes of the given scale and the harmonies (chord symbols) associated with them.

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[21] Mark Levine, *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 31–94. Dmitri Tymoczko summarizes several textbooks' views on chord-scale theory in "The Consective-Semitone Constraint," 174–79. Other texts that discuss chord-scale theory at length include Andy Jaffe, *Jazz Harmony* (Tübingen: Advance Music, 1996); Joe Mulholland and Tom Hojnacki, *The Berklee Book of Jazz Harmony* (Boston: Berklee Press, 2013); Jamey Aebersold, *Jazz Handbook* (New Albany, IN: Jamey Aebersold Jazz, 2010), http://www.jazzbooks.com/mm5/download/FQBK-handbook.pdf; and Richard Graf and Barrie Nettles, *The Chord Scale Theory and Jazz Harmony* (Advance Music, 1997). I am eliding the minor differences in these discussions, since they will not affect the transformational system in the next section. John Bishop provides a good overview of the distinctions in "A Permutational Triadic Approach to Jazz Harmony and the Chord/Scale Relationship" (PhD diss., Louisiana State University, 2012), 77–81.

Mode	Chord symbol
Ionian	Cmaj7 (avoid $\hat{4}$)
Dorian	Dm7
Phrygian	Esus♭9
Lydian	Fmaj7♯4
Mixolydian	G7 (avoid $\hat{4}$); Gsus
Aeolian	Am♭6
Locrian	Bm7♭5

Table 4.4. Levine's chord-scale description of C major scale harmony (*Jazz Theory Book*, 34).

Mode	Chord symbol	Mode name
I	CmM7	minor-major
II	Dsus♭9	–
III	E♭maj7♯5	Lydian augmented
IV	F7♯11	Lydian dominant
V	CmM7/G	–
VI	Am7♭5	half-diminished, Locrian ♯2
VII	B7alt.	altered, diminished whole-tone

Table 4.5. Levine's chord-scale description of C melodic minor scale harmony (*Jazz Theory Book*, 56).

Many later chord-scale theorists describe "avoid notes" in scales; these are notes that are dissonant with the underlying harmony and should generally be avoided in improvisations except as non-harmonic tones. This is an idea that is not explicit in the *Concept*, but many of the notes that are described as "avoid" notes can be traced back to the fact that later theorists do not take the Lydian scale as their starting point ($\hat{4}$ is usually described as an avoid note on a major seventh chord). Levine's first-choice chord-scales for major scale harmony and melodic minor scale harmony are shown in Tables 4.4 and 4.5, respectively, with avoid notes listed as needed.[22] Levine does not give the modes of the diminished and whole-tone scales (for obvious reasons), and notes that the diminished scale represents 7♭9 and fully-diminished harmonies, while the whole-tone scale can be played over 7♯5 and 7alt. harmonies.

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[22] Levine and others often describe only the first choice (Russell would say "most ingoing") for matching a scale with a chord; Russell's system of substituting other member scales sharing the same Lydian tonic is generally not present in the later texts. For jazz musicians the melodic minor scale refers only to its ascending form, which is played both ascending and descending in jazz.

From this brief description, we can see how Russell's theory is more-or-less stripped of its philosophical underpinnings and used simply as a pedagogical and performance tool. Though there are vestiges of the Lydian conception of tonal space (Levine names the Lydian augmented and dominant scales), what remains is only the idea of chord/scale equivalence. On the one hand, this is certainly simpler: gone is the complicated derivation of Lydian parent scales, in its place a simple one-to-one matching of scales with chord symbols.

At the same time, though, something seems lost. For Russell, Lydian organization of tonal space was not incidental, but in fact the most important idea in the book (its title, after all, is *The Lydian Chromatic Concept of Tonal Organization*, and not *Chord/Scale Equivalence and Jazz Improvisation* or the like). Rather than simply writing off Russell's more unusual ideas as eccentric ramblings, we will aim in the next section to reincorporate some of them, in an effort to assimilate some of the "first jazz theory" back into modern scholarship.

4.2 A Chord-Scale Transformational System

Now that we have explored Russell's theory in some detail, we can take it as a basis on which to construct a transformational system. Focusing on chord-scales as first-class objects will allow us to take seriously the idea that scales *are* harmony, and will enable analytical observations about the way improvising musicians might understand harmonic structure.

4.2.1 Introduction: Scale Theory

First, though, it will be useful to take a brief tour through other analytical approaches that incorporate scales. Dmitri Tymoczko dedicates much of *A Geometry of Music* to the study of scales, and applies them analytically to both twentieth-century music and jazz.[23] He is interested primarily in voice-leading among scales, and constructs voice-leading spaces among the diatonic, acoustic, harmonic major and minor, hexatonic, octatonic, and whole-tone scales.[24] His conception of scales is somewhat different than Russell's, though; for Tymoczko, "a scale is a ruler," and provides a way of measuring musical distance.[25] The *Concept*'s view of scales, on the other hand, overlaps somewhat with Tymoczko's notion of "macroharmony": for Russell, a scale acts more like a set of notes that are all available for improvisation. In general, Russell is not interested in common-tone connections between chords or their abstract structure, and accordingly we will not have much occasion to draw on Tymoczko's work here.[26]

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[23] Tymoczko, *A Geometry of Music*, Chapter 4 and throughout. This work is a culmination of much of his earlier work that incorporates scales; see, for example, "The Consective-Semitone Constraint" and "Scale Networks and Debussy," *Journal of Music Theory* 48, no. 2 (October 2004): 219–94.

Other authors have applied chord-scale theory to jazz, though in somewhat different ways than we will do here. Garrett Michaelsen, for example, draws on Tymoczko's work on scalar voice leading to construct networks for the music of Wayne Shorter.[27] Michaelsen does take seriously the notion that chords and scales are equivalent, but his work is more interested in determining how scalar structure can bring structure to harmony that is not obviously functional. Stefan Love's work on parsimonious connections is valuable for teaching students about chord-scales, but falls somewhat short for our purposes here, since it does not include all of the scales Russell identifies.[28]

The work that intersects most closely with our work here is John Bishop's dissertation, which incorporates chord-scales into a triadic transformational system.[29] Bishop is influenced by chord-scale theory as it is taught at the Berklee College of Music, which is different in some ways from Russell's theory outlined above.[30] He is also interested in triadic approaches to improvisation;

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[24] Tymoczko, *A Geometry of Music*, 135.

[25] Ibid., 116.

[26] Nor will we have need of the extensive literature on the abstract structure of scales in general. For an introduction, see John Clough, Nora Engebretsen, and Jonathan Kochavi, "Scales, Sets, and Interval Cycles: A Taxonomy," *Music Theory Spectrum* 21, no. 1 (Spring 1999): 47–101.

[27] Garrett Michaelsen, "Chord-Scale Networks in the Music of Wayne Shorter" (paper presented at the West Coast Conference of Music Theory and Analysis, Eugene, OR, March 2012).

[28] Stefan Love, "A Model of Common-Tone Connections Among Jazz Scales," *Journal of Music Theory Pedagogy* 23 (2009): 155–69.

[29] Bishop, "A Permutational Triadic Approach."

[30] Bishop's primary source is Graf and Nettles's *Chord Scale Theory and Jazz Harmony*; Mulholland and Hojnacki's *Berklee Book of Jazz Harmony* covers much of the same material but in a more modern way (neither book mentions Russell at all). The Berklee method systematizes much of Russell's method in a way suitable for teaching undergraduate jazz musicians. Every tone in a scale, for example, is either a chord tone, an avoid note, or a "tension": an upper extension that colors the basic sound of a chord (*Berklee Book of Jazz Harmony*, xi).

in his theory, chord-scales exist as a means of generating these triads.[31] In this section we will not restrict our focus to triads, but will instead consider chord-scales as objects unto themselves.

One of the problems facing any scale theory is the need to account for scales of different cardinalities. Tymoczko's common-tone theory provides a means of connecting a means of relating the whole-tone (6 notes), diatonic, acoustic, harmonic major (all 7), and octatonic (8) scales via "split" and "merge" operations, and indeed these scales account for six of Russell's seven vertical scales.[32] Tymoczko does mention the ascending melodic minor scale, but it does not merit a place in his diagram since it is simply a mode of the acoustic scale. His system does not, though, account for Russell's "African-American blues scale" (hereafter, simply the "blues scale"), which has either 8 or 10 notes, depending on whether $\hat{2}$ and $\natural\hat{7}$ are included.[33] While we could incorporate this scale into the common-tone system—it is two splits and a semitone displacement from an octatonic collection—chord-scale theory as it is usually taught does not focus on common tones between chord-scales, but rather on determining what scale captures the sound of a particular chord.[34]

Russell's Lydian tonic system, despite all of its seemingly unnecessary complexity, provides a simple solution to the cardinality problem. Because all non-diatonic scales have a Lydian scale as their ultimate source (their parent scale), this means we can understand these other scales as alterations of some diatonic collection. All of the scales in Figure 4.3, for example, are derived from the F Lydian diatonic collection: the D melodic minor collection (Lyd. augmented), F harmonic major (Lyd. diminished), F acoustic (Lyd. flat seventh), whole-tone ($\textsc{wt}_{1}$, aux. augmented), and two octatonic scales ($\textsc{oct}_{02}$ and $\textsc{oct}_{12}$, the auxiliary diminished scales). Russell's

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31. These triadic approaches also tend towards music that is less clearly tonal, and were devised partly as a means of moving beyond the standard chord-scale approach we are examining here.

32. Tymoczko, *A Geometry of Music*, 134–35.

33. Russell's blues scale contains more pitches than the blues scale as described by some others; Mark Levine gives the C blues scale as C–E♭–F–F♯–G–B♭–C (*The Jazz Theory Book*, 219). Joe Mulholland and Tom Hojnacki give Russell's version (with $\hat{2}$ and $\natural\hat{7}$) with the caption "a more complete blues scale," but note that "there is no single blues scale. Rather, there is a large variety of scales that share common blues characteristics" (*Berklee Book of Jazz Harmony*, 135). In practice, melodies derived from the blues scale focus heavily on $\flat\hat{3}$ and $\flat\hat{7}$, and are usually unambiguous, as we will see in the analyses below.

34. Indeed, this lack of focus on common-tone connections in the pedagogical literature is the main impetus for Love's "Model of Common-Tone Connections."

0.	Lydian (diatonic)
1.	Lydian augmented
2.	Lydian diminished
3.	Lydian ♭7
4.	Whole-tone
5.	Whole-half diminished
6.	Half-whole diminished
7.	Blues scale

Table 4.6. A scale index inspired by Russell, listed from most consonant to most dissonant.

four horizontal scales, of which the blues scale is the most important, also have a Lydian tonic, and can be understood as still further variations on the Lydian collection.[35]

4.2.2 A GIS Proper

This reduction to a single diatonic collection will be the first step in devising a transformational system for chord-scales. Instead of referring to a scale's parent Lydian tonic as a Lydian scale, we will instead refer to it by a key signature: the D Lydian collection is 3♯, the E♭ Lydian collection 2♭, the F Lydian collection simply ♮, and so on. This notation is in common use and, helpfully, eliminates some of the awkwardness of having to refer constantly to the Lydian mode. The second mode of the F Lydian scale is of course the same as the G Mixolydian scale, and both refer to the collection ♮.

It is not yet clear, though, how Russell's other member scales might be incorporated into this system. To do so, we will first introduce the concept of a *scale index*, shown in Table 4.6 (they are numbered from 0 to 7 for reasons that will become clear shortly). Several things are worth noting about this table that differ from Russell's presentation. First, some of the scale names have been changed to reflect their common usage; we no longer need to remember, for example, which of the diminished (octatonic) scales is the "blues" variant.[36] I have maintained Russell's names when they clarify the relationship to the parent scale: scale 1 remains "Lydian augmented" rather than "melodic minor," since the D melodic minor scale has F Lydian, not D Lydian as its parent scale.

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[35] The shift in terminology from the Lydian "scale" to the Lydian "collection" here is deliberate, but not theoretically significant. In practice, Russell treats a scale as a collection: a group of notes from which to generate chord tones or improvisations. In this section we will be more interested in scales as collections, rather than (say) their function as "musical rulers."

[36] These names have been chosen to reflect common jazz usage, so scale 3 is the Lydian ♭7 scale rather than the acoustic scale.

There are eight scales in the scale index, but Russell gives eleven member scales. The reason for this is a practical one: two of the horizontal scales are simply diatonic modes (the major scale and major flat seventh), and the third is a major scale with an additional $\sharp\hat{5}$ (major augmented fifth), which we can usually understand as a chromatic passing tone.[37] The blues scale, though, does appear frequently in jazz, and merits its own place here.[38] This scale is given last in the order because it is one of Russell's horizontal scales, which are inherently more outgoing than their vertical companions.

With some mathematical sleight of hand, we can define a chord-scale GIS using the scale index along with a diatonic collection as above. Elements of this GIS have the form ⟨*diatonic collection*, *scale name*⟩; the F Lydian collection is described by the pair ⟨♮, Lydian⟩, while ⟨2♭, Lyd. ♭7⟩ describes the E♭ acoustic collection (E♭ Lydian is in the 2♭ diatonic collection, and the Lydian ♭7 scale on E♭ adds the pitch D♭).[39] Creating an ordered-pair GIS of course requires us to show that both elements are part of a mathematical group. Though this will not be its final form, we will define this chord-scale GIS formally here, with the knowledge that it will be relaxed in the next section. It is important to realize that this GIS is designed to reflect Russell's own conception of chord/scale equivalence. There are only three distinct octatonic collections, for example, but the GIS contains 24 distinct diminished scales: whole-half and half-whole diminished scales on all twelve Lydian tonics.

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[37] Admittedly, Russell himself would likely object to this characterization since, as noted in the first section of this chapter, the Lydian and major scales are fundamentally different objects. In practice, many jazz musicians do avoid $\hat{4}$ in major scales, and so whether a given passage is Lydian or major is often ambiguous.

[38] In fact, the blues scale is probably more common than some of Russell's vertical scales.

[39] This construction shares some similarity with Julian Hook's "spelled heptachords," in which the white-key diatonic collection is inflected by any number of accidentals. Some, but not all, elements of the GIS here could be recast as spelled heptachords, since the diminished scales require a letter doubling. Julian Hook, "Spelled Heptachords," in *Mathematics and Computation in Music*, Proceedings of the Third International Conference of the Society for Mathematics and Computation in Music, Paris, June 2011, ed. Carlos Agon et al. (Heidelberg: Springer, 2011), 84–97.

The first element of the pair is a key signature, which have been studied in a transformational context by Julian Hook.[40] Because we are interested in collections in jazz (often a non-notated music), we will consider enharmonically equivalent key signatures (like 6♯ and 6♭) to be identical. There are, then, only twelve key signatures (isomorphic to the group $\mathbb{Z}_{12}$), operated on by the sharpwise and flatwise transformations, s_n and f_n, which add n sharps or flats to a key signature, respectively; we might write $1\sharp \xrightarrow{s_1} 2\sharp$, $2\flat \xrightarrow{f_2} 4\flat$, or $1\flat \xrightarrow{s_3} 2\sharp$.[41]

The scales in the scale index do not obviously form a group, but we can (temporarily) define the eight scales to be isomorphic to $\mathbb{Z}_8$, the integers mod 8. The scales do form a progression from consonance to dissonance, and for Russell it is true that the whole-tone scale is in some sense further away from the Lydian tonic than the Lydian augmented scale.[42] They are not, however, cyclic in any meaningful way: it is not as though, for example, the blues scale is the most dissonant scale and if you take one more step you arrive back at the Lydian scale. Nor are the metaphorical distances of consonance between the scales really consistent: though Russell does not define them (and we will not attempt to do so here), the consonant distance between the two diminished scales seems much less than the distance between the whole-tone and whole-half diminished scales.[43] If we accept these limitations of the scales' group structure, though, we gain all the benefits of a GIS. Intervals between scales are calculated as integers mod 8, using the scale labels from Table 4.6: the interval from a Lydian augmented scale to a whole-tone scale is 3 ($= 4 - 1$), from a whole-tone scale to a half-whole diminished scale is 2, and so on.

We will call a transformation between chord-scales R, after Russell; these transformations have the form $R\langle$*signature transformation*, *scale index interval*$\rangle$. The transformation acts on

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[40] Julian Hook, "Signature Transformations," in *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha M. Hyde, and Charles J. Smith (Rochester: University of Rochester Press, 2008), 137–60. In this section we will not apply the full power of Hook's transformations, since they operate not only on key signatures themselves but on pitches (what he calls "floating diatonic forms").

[41] Ibid., 142. We could also write f_n as $s_n{}^{-1}$, and the entire set of signatures can be generated by s_1, adjusting for enharmonic equivalence as necessary.

[42] This is apparent in his description of the tone orders of the Lydian Chromatic scale, as reproduced in Figure 4.4. We could also define a GIS using these tone orders, but such a GIS would lose some distinctions between scales (the Lydian flat seventh and auxiliary augmented are both representatives of the 10-tone order).

[43] This limitation is not as significant, and in fact is the normal state of affairs for diatonic intervals, where, for example, intervals of both 3 and 4 semitones are called "thirds."

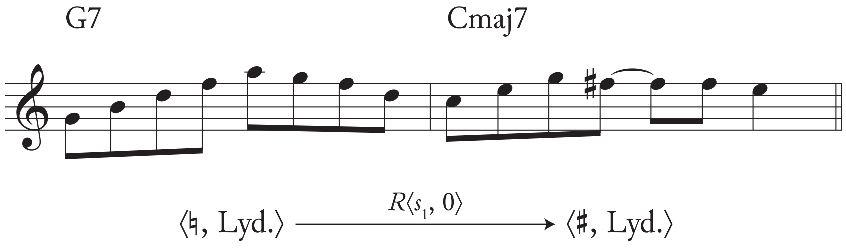

Figure 4.7. A typical V–I jazz lick, along with its chord-scale GIS analysis.

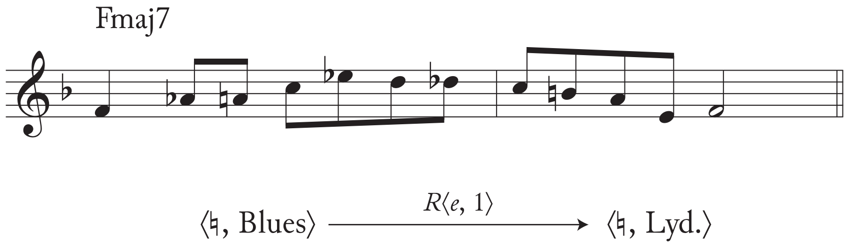

Figure 4.8. A resolution from the F blues scale to the F Lydian scale.

elements of the GIS in a pairwise fashion in the usual way. A passage like the one in Figure 4.7, for example, expresses the $R\langle s_1, 0\rangle$ transformation: the collection changes but the scale does not. Figure 4.8 shows a passage which begins with the F blues scale and resolves to the F Lydian collection, representing the transformation $R\langle e, 1\rangle$, where e indicates the identity element.

4.2.3 Relaxing the GIS

This initial pass at a chord-scale GIS is a useful first approximation, but there are some aspects of it that are somewhat unsatisfactory. How, for instance, should we determine what scales match with what diatonic collections? In some cases the answer is clear, but in others it is not. In Figure 4.7 above, for example, we labeled the G7 chord as ⟨♮, Lyd.⟩, rather than, say, ⟨2♯, Lyd. ♭7⟩; the $\hat{4}$ that would confirm either is absent. The problem seems to become even more intractable when we encounter the symmetrical scales: a diminished scale has eight possible parent diatonic collections.

Here again, George Russell provides a solution. Table 4.7 presents a portion of the foldout chart from the *Lydian Chromatic Concept* in somewhat simplified form (it may be useful to compare this table with Table 4.3 on p. 111). The top of this table gives the eight scales in the scale

	Diatonic	Lyd. augmented	Lyd. diminished	Lyd. ♭7	Whole-tone	WH diminished	HW diminished	Blues
	FGABCDE	FGABC♯DE	FGA♭BCDE	FGABCDE♭	FGABC♯E♭	FGA♭B♭BC♯DE	FG♭A♭ABCDE♭	FA♭AB♭BCDE♭
I	Fmaj7 *Lydian*	Fmaj7♯5 *Lyd. augmented*	– *Harmonic major*	F7 *Acoustic*	Faug (triad)	F°7		Fmaj7, Fm7, F7
II	G7 *Mixolydian*	G7♯11, G7♭5 *Lydian dominant*	G7♭9	G7♯5	G7♯5, G7♭5	G7♭9, G7♭5		
+IV	Bm7♭5 *Locrian*	Bm7♭5 *Locrian #2*	B°7	Bm7♭5 *Altered, dim.WT*			Bm7♭5	
+V					D♭7♯5, D♭7♭5	D♭7♯5, D♭7♯9		
VI	Dm7 *Dorian*	DmM7 *Melodic minor*	Dm7♭5	Dm7♭9			Dm7, Dm7♭5	
VII	E7♭9 *Phrygian*	E7♭9	E7alt.					

Alternate scale names are given in italics; less common chords are shown in gray.

Table 4.7. Common chords in the modes of the F Lydian Chromatic scale.

index of the previous section, while the left side lists the modal tonics. Only modes that give rise to common chords are shown in this table; notably absent are modes III and V, which are given by Russell as tonic chords with altered bass notes. The most common chords and scales (which are also the most ingoing) appear on the left side of the table, and rarer chords and scales appear nearer the right side. Alternate names for scales, when they exist, are given in italics in the appropriate box.

It is important to realize that for Russell the modal degrees (what he calls primary modal tonics) are roughly equivalent to functional categories. This matters here because it helps us to determine a scale's parent diatonic collection. All of the chords in the top row of the table are first-mode scales, and act like tonic chords. The F7 in the top row of the Lydian ♭7 column thus represents a major-minor seventh chord acting as tonic (Russell actually gives this chord symbol as "Maj ♭7" or "Maj 9th ♭7"). Likewise, dominant chords appear mostly in mode II, minor seventh chords appear in mode VI, and half-diminished sevenths in mode +IV.[44]

Of course, the pairing of modes and scales is still not unique, and is ultimately a question of analysis. Consider the scale in Figure 4.9a. This is an F acoustic scale, which can appear as an F Lydian ♭7 scale or as the second mode of an E♭ Lydian augmented scale.[45] The GIS allows us to show this, since "F acoustic as tonic" is a different GIS member than "F acoustic as dominant." Figure 4.9b places the ambiguous scale in the context of a ii–V–I progression in B♭ (the last four bars of an imaginary solo on George Gershwin's "I Got Rhythm"). Here, the acoustic scale clearly functions as a dominant, and would be labeled ⟨2♭, Lyd. aug.⟩: the parent collection is E♭ Lydian (2♭), and this is a mode of the Lydian augmented scale (number 2 in the scale index of Table 4.6). In contrast, Figure 4.9c places the fragment in an F blues ii–V–I progression (the end of Charlie

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[44] The dominant chords that appear in mode +V are related by tritone to those in mode II. The fact that these tritone-related dominants appear along with the whole-tone and diminished scales is no accident: because these scales are symmetrical at the tritone, these scales are particularly effective when soloing over dominant seventh chords. Dmitri Tymoczko discusses this practice explicitly in *A Geometry of Music*, 365–68.

[45] It is reasonable to wonder why we did not collapse the Lydian augmented and Lydian ♭7 scales into a single mode, as we did with the horizontal major scale and the vertical acoustic scale. The aim in this section is to show that the two scales do in fact function differently, which justifies their presence as separate vertical scales.

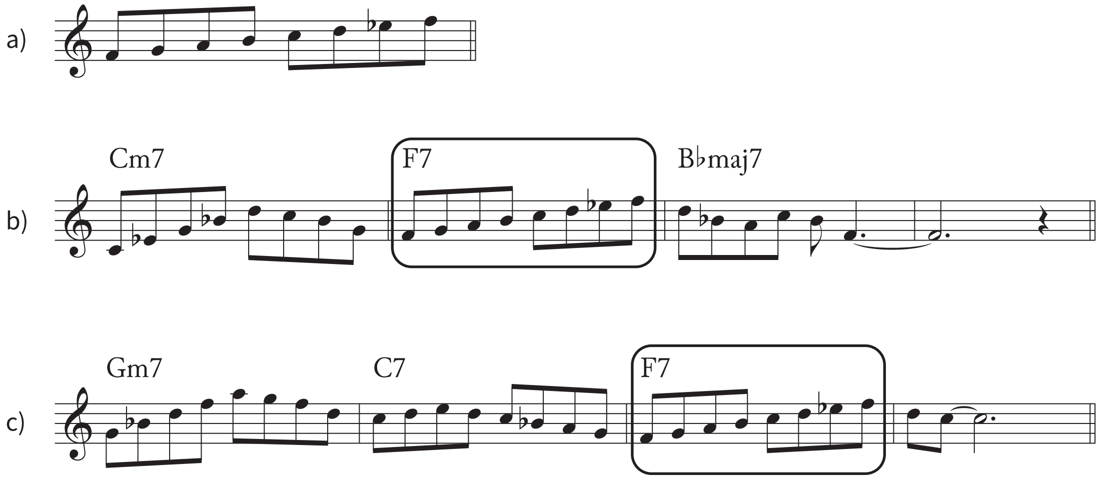

Figure 4.9. An ambiguous acoustic scale and two concrete presentations of it.

Parker's “Now's the Time,” perhaps). Because the collection now functions as a tonic chord, it represents the GIS member ⟨♮, Lyd. ♭7⟩.

The intuitions captured by the chord-scale GIS here are not quite like those represented by other theories of chord-scales. Indeed, the fact that both ⟨♮, Lyd. ♭7⟩ and ⟨2♭, Lyd. aug.⟩ refer to the same 7-element set of pitches is not immediately apparent in the GIS itself. Theories that prioritize voice-leading would likely include the F acoustic collection only once, since the voice leading from this scale to itself is maximally efficient (no voices move at all). Nor is it enough simply to label the scale as the F Lydian dominant scale, as this does not capture the difference in function between the passages in Figure 4.9b–c.

The GIS in fact is one of *functional* or *heard* chord-scales. In this way, it is more like Steven Rings's GIS for “heard scale degrees” than theories of scalar voice leading.[46] Rings argues that scale degrees are a perceived, rather than inherent, quality of music, and thus listeners have different experiences when hearing “A_4 as $\hat{1}$” and “A_4 as $\hat{7}$.”[47] In many ways, this distinction is like the one at the heart of what Russell was trying to accomplish in the *Lydian Chromatic Concept*. For Russell, the F acoustic collection played over a tonic major-minor seventh chord really *is* a different entity than the same collection played over a dominant major-minor seventh chord. This is what Russell means when he writes of "a state of complete and indestructible chord/scale unity" (*LCC* 10): if an F7 chord can function in more than one way, so too can its corresponding scale.

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[46] Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 44–50 and throughout.

[47] Ibid., 42.

Previous chapters have constructed various musical spaces in which to analyze passages, and this GIS can be turned into a space as well. Before doing that, though, we will relax its definition such that the scale indexes are no longer isomorphic to $\mathbb{Z}_8$. We noted above that this isomorphism was somewhat artificial, and the space becomes more intuitive if we simply use the non-modular integers 0–7 as elements of the space (which we will call S for now). The resulting space, though, runs afoul of the formal requirements for a GIS, which requires that IVLS form a mathematical group. The set S under addition does not form a group, since it is not closed ($6 + 5$ is not a member of S) and elements do not have inverses.

This is of course one of the well-known limitations of GISes: the musical spaces must be both symmetrical and homogeneous.[48] A GIS cannot account for musical spaces that are discontinuous or have "boundaries," which is the case here: it is not possible to conceive of a scale in the system which is more ingoing than the Lydian scale, or more outgoing than the blues scale.[49] Nevertheless, the transformations still seem to be reasonable reflections of intuitions about the nature of chord-scales. As Hook notes, "the narrative portions of Lewin's analyses [in *GMIT*] generally far transcend the logical consequences of the group structure," so the fact that a mathematical group does not underlie this no-longer-GIS should not dissuade us from exploring its analytical potential.[50]

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[48] Many authors have commented on this limitation, which Lewin first observes in *GMIT*, 27. Dmitri Tymoczko is probably the most vocal in his opposition, and proposes incorporating a distance metric into the definition of what he calls a "Lewinian interval system" ("Generalizing Musical Intervals," *Journal of Music Theory* 53, no. 2 [Fall 2009]: 245–46); in other places, he has suggested that relaxing some of the restrictions on a GIS is "anti-Lewinian" ("Lewin, Intervals, and Transformations: a Comment on Hook," *Music Theory Spectrum* 30, no. 1 [Spring 2008]: 164–68). For more on this tension, see Rachel Wells Hall's review of *GMIT* (*Journal of the American Musicological Society* 62 no. 1 [Spring 2009]: 205–22); Julian Hook, "David Lewin and the Complexity of the Beautiful," *Intégral* 21 (2007): especially 185–86; and Rings, *Tonality and Transformation*, 19–20.

[49] Admittedly, the outgoing boundary is more permeable than the ingoing: we could conceive of a scale that is more dissonant than the blues scale (the total chromatic, perhaps) and incorporate it into the system, while for Russell the Lydian boundary is absolute.

[50] Hook, "David Lewin and the Complexity of the Beautiful," 185.

Lewin does allow for semigroups of transformations, but the scale index transformations do not form a semigroup either, since a semigroup must still be closed under the group action. All possible intervals for the scale indexes (the integers 0–7) are contained in the set $\{-7, -6, \ldots, 6, 7\}$, which forms neither a group nor a semigroup. It does contain the additive identity (0), and every element has an inverse, so we can use this set under addition in practically the same way. Every interval is well-defined, but all intervals are not possible from every scale. For example: int(Lyd., Whole-tone) = 4, and int(Whole-tone, Lyd.) = -4, but there is no scale which satisfies x in the statement int(Blues, x) = 4, since there is no scale that is four levels more outgoing than the blues scale.

With these caveats, this space can be visualized using the diagram in Figure 4.10. In this figure, the diatonic (Lydian) scale is centrally located, with more outgoing scales located further toward the outside; these concentric circles combine with the ordinary circle of fifths to divide each scale into diatonic wedges.[51] The figure is inspired by Russell's description of the Lydian as a "musical 'Star-Sun'" (*LCC* 8) and the ultimate source of tonal gravity. More outgoing scales have more gravitational potential energy, as it were, and are more dissonant with the underlying diatonic collection. We can use this figure to map the two presentations of the F acoustic scale of Figure 4.9; such a mapping is given in Figure 4.11. This visualization makes clear that the second presentation (a tonic F7 chord) is more outgoing than the first (a dominant F7 chord), as well as the shift in underlying diatonic collection (E♭ Lydian vs. F Lydian).

Figure 4.11 also reveals that perhaps the objects in the system could be more informative. The right side of this figure represents a ii–V–I in F with the sequence

$$\langle\flat, \text{Dia.}\rangle \xrightarrow{R\langle e,\, 0\rangle} \langle\flat, \text{Dia.}\rangle \xrightarrow{R\langle s_1,\, 3\rangle} \langle\natural, \text{Lyd.}\ \flat 7\rangle.$$

That is, the G Dorian scale and C Mixolydian scale are both represented by the pair $\langle\flat, \text{Dia.}\rangle$. On one level, this makes sense: both scales are modes of the B♭ Lydian (or F major) scale. Still, since a chord-scale is supposed to represent a "complete and indestructible unity," it seems appropriate

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[51] The diatonic collections are shown in flatwise order traveling clockwise. This corresponds to the way the circle of fifths (or fourths) is usually presented in jazz textbooks; see, for example, Jerry Coker, *Elements of the Jazz Language for the Developing Improvisor* (Miami: Belwin, 1991), v.

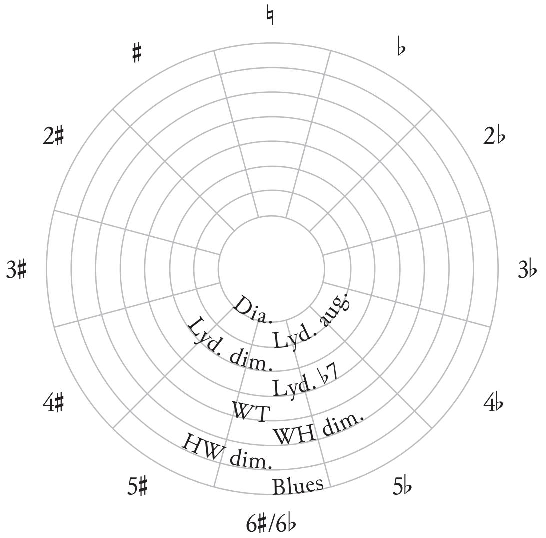
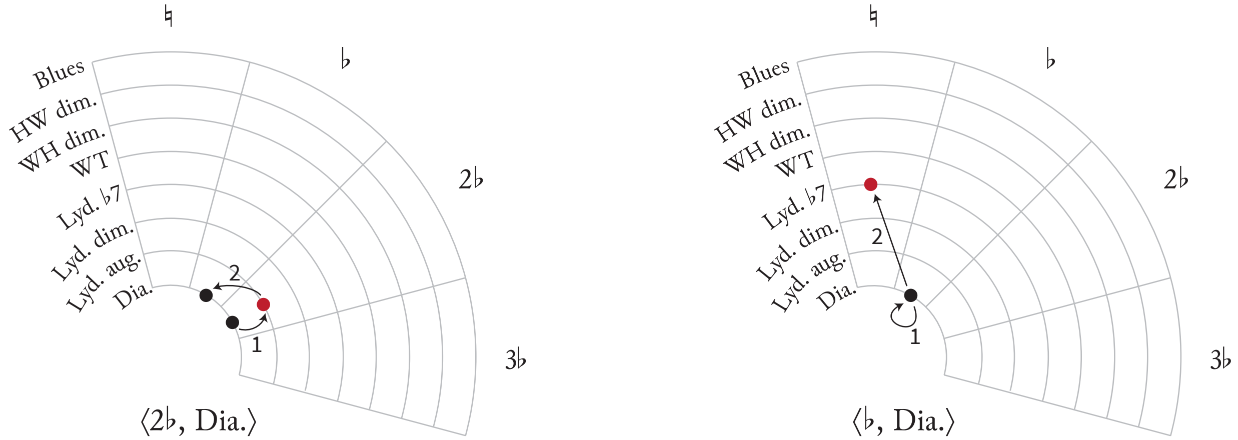

♮
♭
2♭
3♭
4♭
5♭
6♯/6♭
5♯
4♯
3♯
2♯
♯
Dia.
Lyd. aug.
Lyd. dim.
Lyd. ♭7
WT
WH dim.
HW dim.
Blues

Figure 4.10. A “planetary” model of the chord-scale GIS.


♮
♭
2♭
3♭
Blues
HW dim.
WH dim.
WT
Lyd. ♭7
Lyd. dim.
Lyd. aug.
Dia.
2
1
♮
♭
2♭
3♭
Blues
HW dim.
WH dim.
WT
Lyd. ♭7
Lyd. dim.
Lyd. aug.
Dia.
2
1

$$\begin{tikzcd}
\langle 2\flat, \text{Dia.}\rangle \arrow[d, "R\langle e, 1\rangle"] & & \langle \flat, \text{Dia.}\rangle \arrow[d, "R\langle e, 0\rangle"] \\
\langle 2\flat, \text{Lyd. aug.}\rangle \arrow[d, "R\langle s_1, -1\rangle"] \arrow[rrd, dashed, "R\langle s_2, 2\rangle"] & & \langle \flat, \text{Dia.}\rangle \arrow[d, "R\langle s_1, 3\rangle"] \\
\langle \flat, \text{Dia.}\rangle & & \langle \natural, \text{Lyd. } \flat 7\rangle
\end{tikzcd}$$

Figure 4.11. Two presentations of the F acoustic scale, shown in red, in the planetary model (compare Figure 4.9).

to add some information about the chord into the notation itself. As it stands, information about the chords themselves is separate from the transformations, and there is no way to distinguish the progression above (a ii–V–I in F) from, say, the nonsensical progression Em7♭5–Fmaj7–Bm7♭5.

We could solve this problem in several different ways, but the most obvious is to include the chord symbol itself in the chord-scale representation. This results in what we will call a chord-scale triple of the form ⟨*chord symbol*, *diatonic collection*, *scale name*⟩. This construction will allow us to draw on the work done in the previous chapters developing a system of transformations for chord symbols; we will still call the resulting transformations R, but the first element of the new triple will be a transformation between chord symbols.[52] The F-major ii–V–I of Figure 4.9c thus becomes

$$\langle \text{Gm7}, \flat, \text{Dia.} \rangle \xrightarrow{R\langle \text{TF}, e, 0 \rangle} \langle \text{C7}, \flat, \text{Dia.} \rangle \xrightarrow{R\langle \text{TF}_{\text{blues}}, s_1, 3 \rangle} \langle \text{F7}, \natural, \text{Lyd.} \flat 7 \rangle.$$

Formally, the transformations of the last two chapters act on ordered triples of chord root, third, and seventh; the additional scale information in a chord-scale triple enriches this sparse three-not representation. Recall that the chord symbol transformations are cross-type transformations, and thus even if we had not already relaxed the GIS of the previous section (removing the cyclic group $\mathbb{Z}_8$), the new version with chord symbols cannot form a GIS.

While adding chord symbols to the system does clarify matters, it also complicates them. In particular, the planetary model of Figure 4.10 no longer represents the musical space accurately. The addition of chord symbols means that a copy of the planetary model exists at every location a chord symbol appears in the previous chapters.[53] Such a visual space would be forbiddingly complex—imagine the thirds spaces of Figures 3.2 or 3.15 redrawn with the additional chord-scale models. This is a sacrifice made consciously so that the chord-scale triples and R transformations are clear in the text. In practice, we can use either the planetary model or the chord spaces of the previous chapters as the situation demands, with the knowledge that both representations exist simultaneously as part of the single conceptual chord-scale space. With the final version of the chord-scale transformational system in place, the stage is now set to turn toward actual jazz performance.

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[52] We will generally use this new form of the R transformation, but in cases where there is a need to distinguish between this version and the version used just above, "2-element R" versus "3-element R" works nicely.

[53] In fact, the situation is even more complex, since a single chord symbol can support only a subset of the 96 chord-scales. Thus every chord symbol would contain a *different* partial copy of the planetary model. Only the chords listed in Table 4.7, for example, would be able to show the scales from the ♮ diatonic wedge.

4.3 Chord-Scale Transformations in Analysis

Analyzing jazz performance is inherently more complicated than the lead-sheet analysis done in previous chapters. Because jazz is primarily an improvised music, the analyses here will rely on transcriptions, which carry with them their own set of problems.[54] As Steve Larson notes, any transcription is also a kind of analysis: in many cases it is not at all clear how a particular recorded sound should be (or indeed, whether it can be) rendered in Western musical notation.[55] In the transcriptions in this dissertation, I focus primarily on pitches and rhythms, since they are most relevant to the discussion of harmony. As such, many of the most important aspects of a performance—dynamics, articulation, timbre, intonation, and so on—are absent from the notation.[56]

The three short analyses that follow will serve as preludes to the longer analyses we will pursue in the next chapter; each introduces certain issues of analysis to be explored in more detail later. All are solos on tunes analyzed in the first three chapters, allowing us the opportunity to discover how these abstract chord progressions are realized in improvised performance. They are also solos by tenor (and soprano) saxophonists: Rahsaan Roland Kirk, Gene Ammons, Sonny Stitt, and Joe Henderson. This selection reflects some of my own preference for saxophonists, but also permits a basis for comparison: different instruments have different idiomatic patterns. Notably absent from this list are Charlie Parker and John Coltrane, undoubtedly the two most well-known jazz saxophonists. As noted in Chapter 1, this dissertation is interested in jazz harmony in the general sense; by focusing on musicians who do not commonly appear in works of music theory, we gain insight into the lingua franca of jazz, rather than the particulars of Parker's or Coltrane's style.[57]

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[54] Complete transcriptions for all solos analyzed in this chapter and the next can be found in Appendix B.

[55] Steve Larson, *Analyzing Jazz: A Schenkerian Approach* (Hillsdale, NY: Pendragon Press, 2009), 1–2.

[56] This is to say nothing of the fact that perhaps the most important aspect of a jazz solo is that it is improvised; transcriptions must necessarily only focus on a single recorded performance.

4.3.1 Rahsaan Roland Kirk, "Blues for Alice"

Multi-saxophonist Rahsaan Roland Kirk's recording of "Blues for Alice" from *We Free Kings* (1961) will act as an introduction to some of the issues of analyzing improvised performance. The complete transcription of the performance can be found on p. 210, and the analysis of the chord progression of this tune is in Section 2.2.2. Kirk often plays multiple saxophones simultaneously; each instrument is given its own staff in the transcription.

One of the main problems of analyzing chord-scales is determining exactly which notes should be taken as part of the scale, and which are simply embellishing. If one of the principal arguments of this chapter is that scales *are* harmony, then the question becomes one of determining what is non-harmonic. Sometimes it is obvious that notes are embellishing, but other cases are not so clear. The G♯ in Figure 4.12a, for example, clearly functions as a chromatic passing tone between G (the fifth of the Cm7 chord) and A (the third of F7). Figure 4.12b presents a more complicated case: is the E♭ in the scale, with E♮ serving as a chromatic passing tone, or vice versa? If we choose E♭ as the main note, the scale implied is B♭ Mixolydian, while E♮ gives a B♭ Lydian dominant scale. The choice has analytical implications, as the two scales represent two different locations in chord-scale space. It is important to note that these kinds of non-harmonic tones are not quite like ordinary non-harmonic tones. Larson (and many others) would argue that *both* the E♭ and E♮ are non-harmonic, since at some deeper level they would reduce to either D or F (neither is part of a four-voice B♭7).[58] Because we have broadened the definition of "harmony" to include chord-scales, our idea of what is "non-harmonic" must also change accordingly.

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[57] Thomas Owens (among others) refers to the bebop-inspired style as jazz's lingua franca (*Bebop: The Music and its Players* [New York: Oxford University Press, 1995], 4). To avoid focusing on Parker and Coltrane is not to say that the performers examined here are not also great musicians; all of them (but especially Stitt and Henderson) are highly regarded among jazz musicians.

[58] Larson, *Analyzing Jazz*, 5–10.

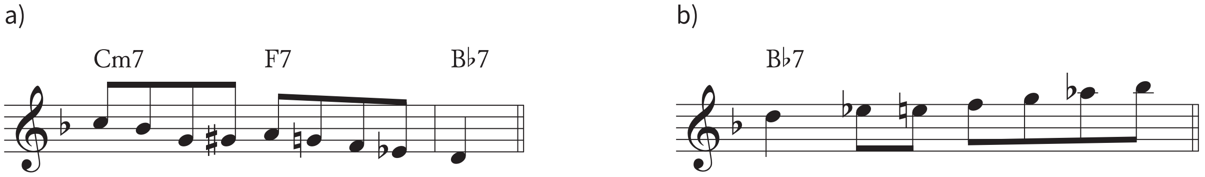

Figure 4.12. Two non-harmonic tones, from m. 112 (2:52) and m. 17 (1:00), respectively, of Kirk's solo on "Blues for Alice."

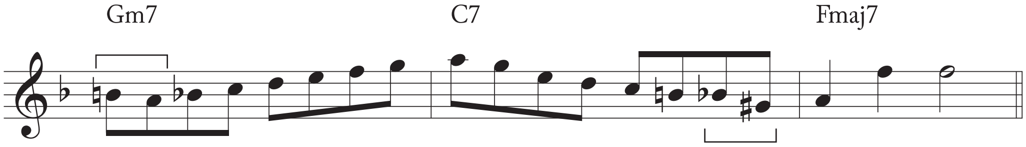

Figure 4.13. Two examples of enclosures (marked with brackets) in a ii–V–I progression, from mm. 33–35 (1:19) of Kirk's solo.

One kind of embellishing figure merits special attention, which Jerry Coker calls the "enclosure," where a pitch is approached by semitone on either side.[59] Two examples of enclosures appear in the ii–V–I progression in Figure 4.13. The first appears before the B♭ on beat 2 of the first bar, and the second before the resolution to A at the end of the passage. What is interesting about enclosures from the chord-scale perspective is that usually only one of the neighbors is non-harmonic (i.e., not part of the scale). In the first example, only the B♮ is truly non-harmonic, since the A is a part of the (very clear) G Dorian scale. Likewise, the B♮ in the next measure is a chromatic passing tone in the C Mixolydian scale between C and B♭, while G♯ is a lower neighbor to the following A.

Another issue arises when an improviser chooses to play a single scale over several chord changes; Coker calls this "harmonic generalization."[60] This is a technique often used at faster tempos or with complex chord changes (or both), since it allows the performer to slow down the effective harmonic rhythm. Often the process results in notes that clash with the underlying

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59. Coker, *Elements of the Jazz Language*, 50–54. Coker's manual is intended for student improvisers, but is analytically useful as something of an encyclopedia of idiomatic melodic devices in improvisations, since he provides many examples of each technique. He is one of the only authors I am aware of to discuss this aspect of performance; most others simply do not mention it at all, or hand-wave the problem away, saying that the knowledge will come by listening to and transcribing many recorded performances.

60. Ibid., 45–49.

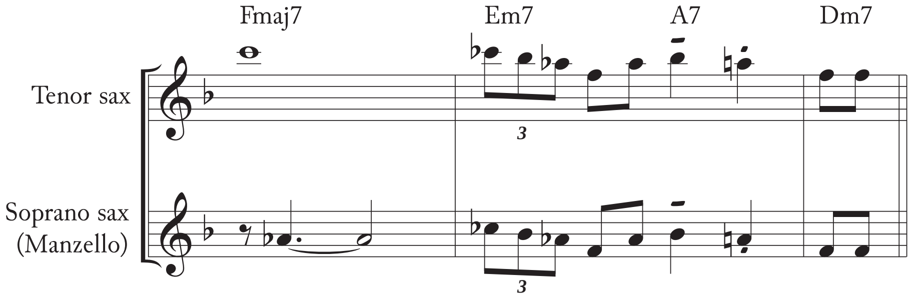

Figure 4.14. Harmonic generalization in Kirk's solo: the F blues scale appears over four different harmonies (mm. 25–27, 1:10).

harmonic progression, so Coker also notes that it is "less than ideal."[61] Kirk uses this technique at the beginning of his third chorus, where he plays the F blues scale over four chord changes (Figure 4.14).[62] B♭, A♭, and F♮ are all relatively dissonant over Em7, but here the coherence of the F blues scale allows us to focus on the figure as a single unit rather than hearing chord-to-chord. This phenomenon is easy to capture in the transformational system; all of the transformations have the form $R\langle \_\_, e, 0\rangle$, since the chord changes while the scale does not.

The final issue Kirk's solo brings to light is that of chord substitution. Previous chapters have discussed harmonic substitution at length, but only in the context of tunes. The process of substitution that happens during solos is more complicated: different substitutions can happen in different choruses, a soloist might use a substitution while the rhythm section does not (or vice versa), and so on. This is an aspect of performance that is *not* so easy to capture in chord-scale space—or at least, in any single transformational label. In general, transformational labels show only a single interpretation: a passage is either represented by one chord-scale triple or another, not both simultaneously. We can, however, examine a single passage in multiple, different ways, first exploring one interpretation then another. Steven Rings calls this process of analysis *prismatic*, where "phenomenologically rich passages are refracted and explored from multiple perspectives."[63]

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[61] Coker, *Elements of the Jazz Language*, 46.

[62] The tenor saxophone here (which sounds down an octave) is played with his left hand and the manzello—a modified soprano saxophone—is played with his right.

[63] Rings, *Tonality and Transformation*, 38.

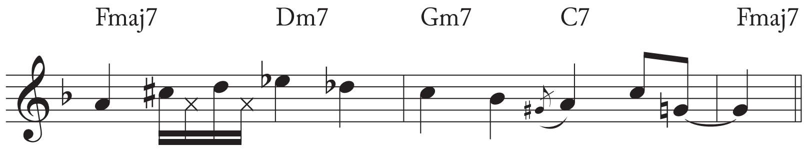

Figure 4.15. The turnaround at the end of Kirk's first chorus (mm. 11–13, 0:52).

As an example of this prismatic approach, take the turnaround from Kirk's first chorus, shown in Figure 4.15. Over the nominal Dm7 chord, Kirk plays the notes E♭ and D♭. These two notes seem incompatible with Dm7; none of Russell's scales contain a chromatic trichord (necessary if the scale is to include the chord root). One possibility is that Kirk is drawing on the total chromatic gamut, or perhaps some other scale Russell did not recognize (like the enneatonic scale). This analytical possibility seems unlikely, though; drawing on a high-cardinality scale seems excessive to explain a mere two pitches. Another, more likely, possibility is that Kirk takes the tritone substitution here, playing the notes E♭ and D♭ over an implied A♭m7 chord (or perhaps A♭7, as a tritone-substituted dominant of the following Gm7). Still another possibility is that Kirk uses a harmonic generalization, playing a diminished scale fragment (E♭–D♭–C–B♭–A–G) over a C7 harmony that is implied over the full one-and-a-half bars of the turnaround.[64]

Figure 4.16 shows these last two analytical possibilities in the planetary model of the previous section. The first involves hearing the E♭–D♭ succession as a diatonic tritone substitution for Dm7.[65] In this hearing, all of the chord-scales are diatonic, but the diatonic collection shifts greatly from the first chord to the second. (There are only three objects in this model because moving diatonically from Gm7 to C7 does not involve a scalar shift.) Figure 4.16b, on the other

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[64] The potential hearings given here are not exhaustive, of course. Experienced jazz listeners and analysts must bring along their knowledge of jazz harmonic progression and elaboration in order for this kind of analysis to make sense; the transformational system admits of many logically consistent analyses that may not effectively describe the music under consideration. It seems unlikely to me, for example, that the E♭ here is a flatted 9th over Dm7, with the D♭ acting as a chromatic passing tone, simply because ♭9 is not an extension that is commonly played over minor seventh chords.

[65] The diagram here assumes A♭7, since it is more common to substitute dominant sevenths than minor sevenths. An analysis with A♭m7 would be similar, except that the second chord would be in the 6♭ collection.

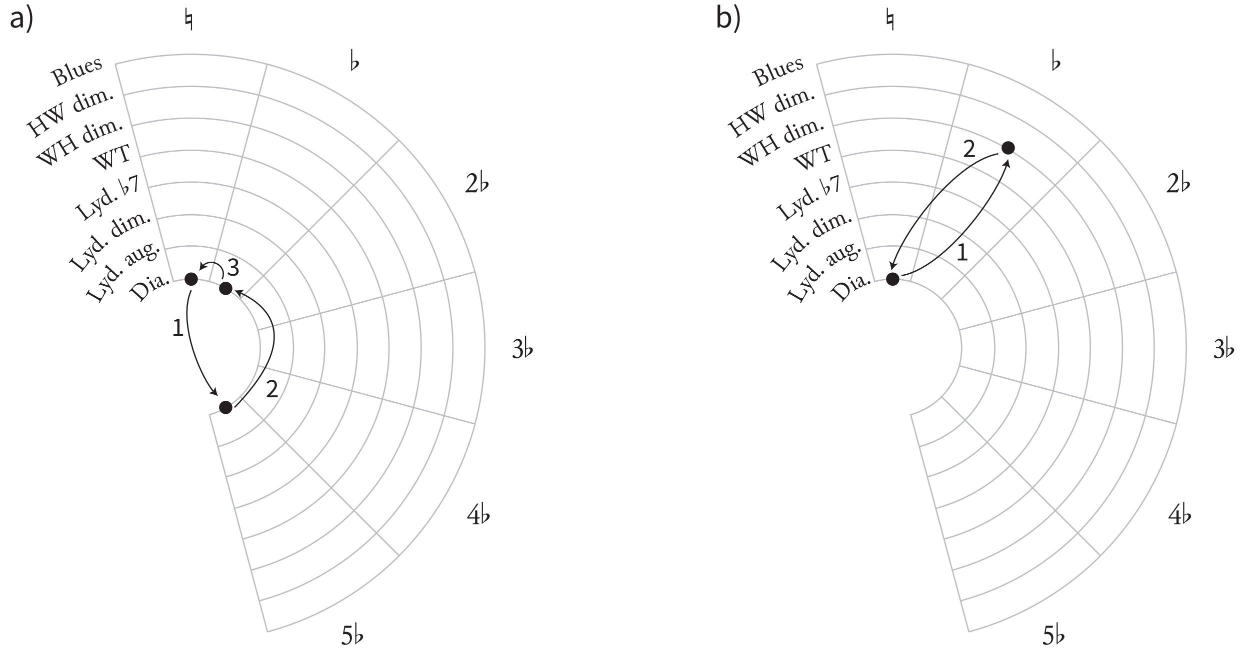

Figure 4.16. Two analytical possibilities for Kirk's turnaround, in mm. 11–13:
a) As a tritone substitution for Dm7.
b) As a harmonic generalization for C7.

hand, combines a small diatonic shift with a large leap in the scalar dimension, moving from the diatonic collection to the whole–half diminished scale.[66]

We might also construct different transformation networks for this passage, as shown in Figure 4.17.[67] Letter *a* is a strictly chronological network (time flows from left to right), which shows the basic transformations involved in the tritone-substitute hearing. Letter *b* redraws this network to reflect the organization of ii–V space: the ii–V–I is in a single horizontal line, while Dm7 is a level higher, representing its position as a ii^7 chord in C. This network also clarifies that the starting and ending Fmaj7 chord is the same point in the space. Letter *c*, on the other hand, reimagines the chord transformations as taking place in a F major diatonic space (similar to that of

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[66] Recall from Table 4.7 that a diminished scale played over a dominant chord is always the whole–half diminished scale. Conceptually, the parent scale of C7 is B♭ Lydian, so the scale in question is the second mode of the B♭ whole–half diminished scale (which is of course the same as the C half–whole diminished scale).

[67] All of the networks here are what Lewin calls "figural" networks (*Musical Form and Transformation: Four Analytic Essays* [New York: Oxford University Press, (1993) 2007], 45–53); since the *R* transformations do not form a semigroup, a "formal" network (showing all possible transformations) is not possible. Hall suggests in a footnote (Review of *GMIT* and *MFT*, 213n16) that the logical structure of the group is unnecessary in figural networks; the networks of this figure support this claim.

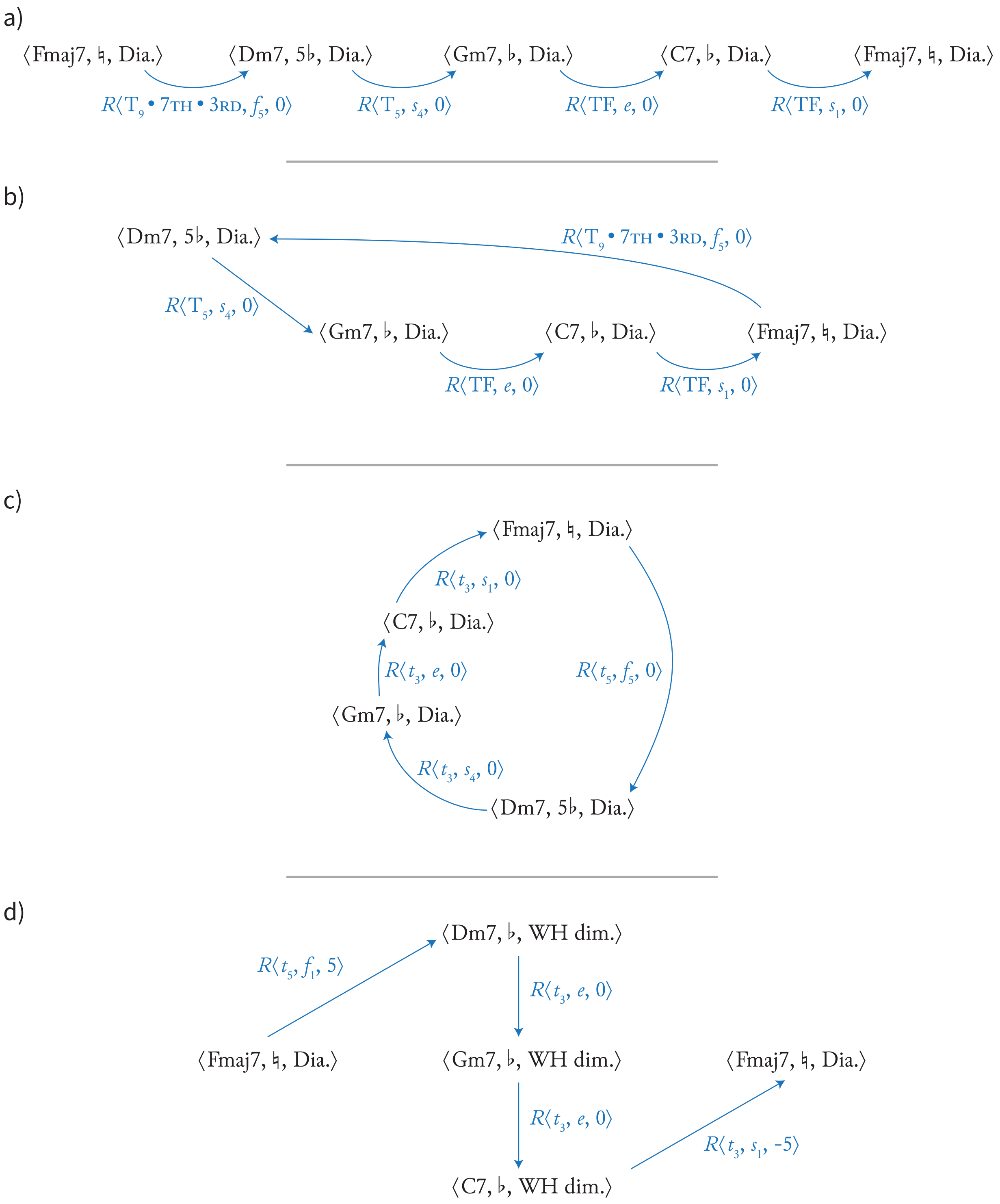
a)
⟨Fmaj7, ♮, Dia.⟩
⟨Dm7, 5♭, Dia.⟩
⟨Gm7, ♭, Dia.⟩
⟨C7, ♭, Dia.⟩
⟨Fmaj7, ♮, Dia.⟩
$R\langle T_9 \bullet \text{7TH} \bullet \text{3RD}, f_5, 0\rangle$
$R\langle T_5, s_4, 0\rangle$
$R\langle TF, e, 0\rangle$
$R\langle TF, s_1, 0\rangle$
b)
⟨Dm7, 5♭, Dia.⟩
$R\langle T_9 \bullet \text{7TH} \bullet \text{3RD}, f_5, 0\rangle$
$R\langle T_5, s_4, 0\rangle$
⟨Gm7, ♭, Dia.⟩
⟨C7, ♭, Dia.⟩
⟨Fmaj7, ♮, Dia.⟩
$R\langle TF, e, 0\rangle$
$R\langle TF, s_1, 0\rangle$
c)
⟨Fmaj7, ♮, Dia.⟩
$R\langle t_3, s_1, 0\rangle$
⟨C7, ♭, Dia.⟩
$R\langle t_3, e, 0\rangle$
$R\langle t_5, f_5, 0\rangle$
⟨Gm7, ♭, Dia.⟩
$R\langle t_3, s_4, 0\rangle$
⟨Dm7, 5♭, Dia.⟩
d)
⟨Dm7, ♭, WH dim.⟩
$R\langle t_5, f_1, 5\rangle$
$R\langle t_3, e, 0\rangle$
⟨Fmaj7, ♮, Dia.⟩
⟨Gm7, ♭, WH dim.⟩
⟨Fmaj7, ♮, Dia.⟩
$R\langle t_3, e, 0\rangle$
$R\langle t_3, s_1, -5\rangle$
⟨C7, ♭, WH dim.⟩

Figure 4.17. Four different transformation networks of the turnaround in mm. 11–13.

Section 1.5), while still hearing the tritone substitute. Finally, letter *d* represents the diminished-scale hearing: both transformations in the middle column contain $\langle e, 0\rangle$ as their second and third elements, since the scale does not change.

It may seem as though the preceding pages have reached for the pile-driver to kill the gnat, so to speak; the entire discussion was brought about by the two notes E♭–D♭.[68] These notes are not of any particular significance in Kirk's solo, and indeed the entire turnaround is relatively ordinary. As listeners and analysts, though, we can conceive of multiple ways of hearing this particular passage, and chord-scale transformations offer a way to explore these interpretations. None of the networks of Figure 4.17 is more correct than the other, and none proposes to be *the* structure of this particular passage. As Rings observes, "to the extent that [transformational] analyses reveal 'structures' at all, they are *esthesic structures* rather than immanent structures."[69] Throughout his book, he emphasizes that a particular analysis is more a record of an analytical encounter with the music than the music itself. Though we will not often focus so intently on such a small fragment of music, adopting Ring's attitude means that we *can*: the lens of transformational theory can zoom in and out as needed.

Before leaving Kirk's solo, it will be instructive to look at his improvisations on a few ii–V–I progressions, if only because they are so ubiquitous in jazz. Figure 4.18 gives three of these, each taken from the last four bars of a chorus of "Blues for Alice." [70] Letter *a* reproduces Figure 4.13, and uses the most typical ii–V–I chord-scale pattern:

$$\langle \text{Gm7}, \flat, \text{Dia.}\rangle \xrightarrow{R\langle \text{TF}, e, 0\rangle} \langle \text{C7}, \flat, \text{Dia.}\rangle \xrightarrow{R\langle \text{TF}, s_1, 0\rangle} \langle \text{Fmaj7}, \natural, \text{Dia.}\rangle.$$

The G Dorian and C Mixolydian scales are both in the ♭ collection, while the parent scale for Fmaj7 is the ♮ collection; this results in the typical s_1 V–I resolution.[71] Figure 4.18b is likewise all diatonic, and has an identical transformational structure. Its interest comes in the "double

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[68] The pile-driver/gnat metaphor comes from Richard G. Swift's response to Lewin's first published article (Letter to the editor, *Journal of Music Theory* 4, no. 1 [April 1960]: 128; quoted in Hook, "David Lewin and the Complexity of the Beautiful," 162).

[69] Rings, *Tonality and Transformation*, 37, emphasis original.

[70] Figure 4.18c is missing beats 2–4 of the third bar because this ii–V–I leads into the turnaround in Figure 4.15, which is discussed at length above and would distract from the topic at hand.

[71] Absent any additional information we will assume Russell's most ingoing scale for chords. Since there is no B (flat or natural) in any of the third bars of Figure 4.18, we assume F Lydian.

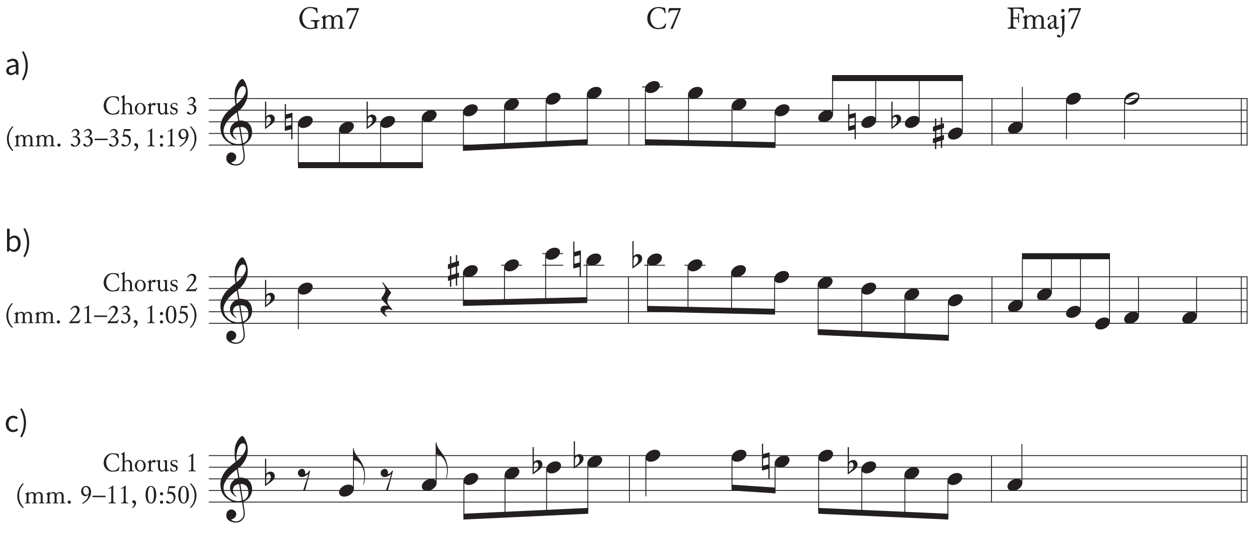

Figure 4.18. Three ii–V–I progressions from Kirk's solo on "Blues for Alice."

enclosure" figure that appears over Gm7: the downbeat B♭ in m. 22 is approached by two chromatic neighbors on each side. The A and C are both in the G Dorian scale, while G♯ and B♮ are simply embellishing, and do not take part in the chord-scale transformations. Figure 4.18c uses a diminished scale over Gm7 and includes D♭ over C7, giving a different transformation network:

$$\langle \text{Gm7}, \flat, \text{HW dim.}\rangle \xrightarrow{R\langle \text{TF}, e, -4\rangle} \langle \text{C7}, \flat, \text{Lyd. dim.}\rangle \xrightarrow{R\langle \text{TF}, s_1, -2\rangle} \langle \text{Fmaj7}, \natural, \text{Dia.}\rangle.$$

Here, the scales get more ingoing over the course of the progression from the half–whole diminished scale (implying ♭13 and ♭5) to the Lydian diminished scale (♭9) before reaching the diatonic Fmaj7.

It should be apparent that these analytical fragments of excerpts from Kirk's solo do not constitute an analysis of the solo as a whole. Nothing has been said about the overall shape of the solo, the role of register, how playing multiple saxophones limits Kirk's available pitches, or any of the countless other analytically interesting aspects. It is in many ways a typical jazz solo, and as such has served primarily as an introduction to the fundamentals of analyzing jazz improvisation, while at the same time offering a glimpse of how chord-scale transformations might be used in analysis.

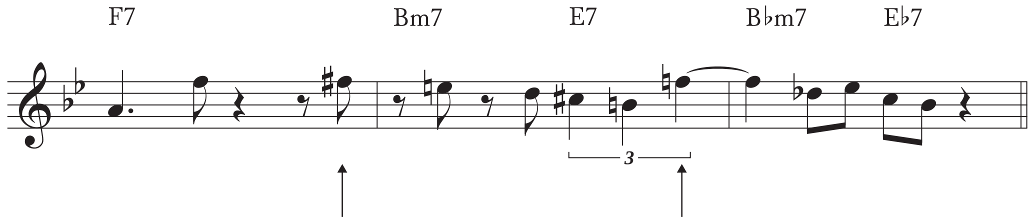

Figure 4.19. Two examples of anticipations, from Gene Ammons's solo on "Autumn Leaves" (mm. 2–4, 1:04).

4.3.2 Gene Ammons and Sonny Stitt, "Autumn Leaves"

Tenor saxophone duo Gene Ammons and Sonny Stitt recorded "Autumn Leaves" on the album *Boss Tenors* in 1961 (the complete transcription is on p. 206). "Autumn Leaves" was first analyzed in Section 1.5, in connection with diatonic chord spaces. We will return to this subject shortly, but only after a brief diversion into meter in improvisation (an aspect emphasized in this recording that did not appear in Kirk's solo on "Blues for Alice").

Both Ammons and Stitt shift barlines frequently in their solos, while Kirk almost never did. Meter is not the primary focus here, but it can affect the chord-scale analysis of a passage.[72] The most common kind of metrical shift is a slight anticipation of the following harmony; Stefan Love notes that these are "so common as to be a cliché," and they are usually obvious in analysis.[73] Figure 4.19 gives two straightforward examples from the beginning of Ammons's solo. The F♯ in m. 2 clearly belongs to the following Bm7 and not to F7, and the F♮ in the following bar functions in the same way.

Sometimes the metric shift is more dramatic; take the passage in Figure 4.20, for example. Here, Ammons seems to arrive at D7 too late, continuing the E♭7 harmony two beats into the next bar. Jerry Coker refers to these as "barline shifts," and says that while they are "not intentional,

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[72] The kind of metrical shifts discussed here are much simpler than those usually discussed in the jazz literature. The classic work on metrical dissonance in jazz is Cynthia Folio, "An Analysis of Polyrhythm in Selected Improvised Jazz Solos," in *Concert Music, Rock, and Jazz since 1945: Essays and Analytical Studies*, ed. Elizabeth West Marvin and Richard Hermann (Rochester: University of Rochester Press, 1995), 103–35; Stefan Love provides an overview of other literature in this area in "Subliminal Dissonance or 'Consonance': Two Views of Jazz Meter," *Music Theory Spectrum* 35, no. 1 (Spring 2013): 48–61.

[73] Ibid., 51.

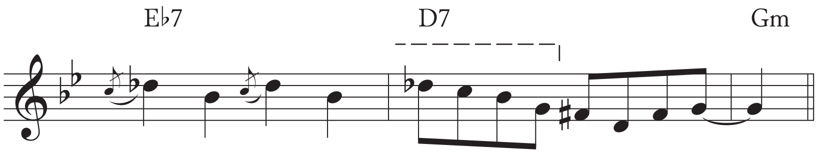

Figure 4.20. A barline shift in Ammons's solo (mm. 29–30, 1:47).

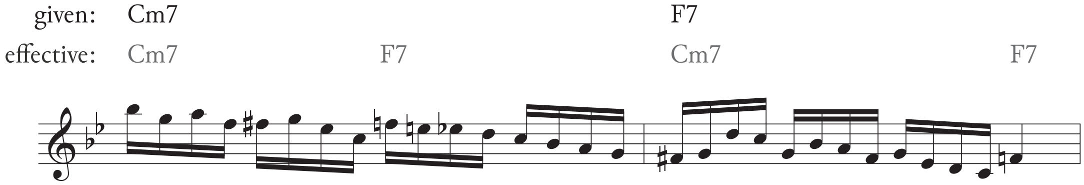

Figure 4.21. A barline shift from Sonny Stitt's solo on "Autumn Leaves" (mm. 73–74, 2:59).

necessarily, they are not errors, either, as they might be in the case of the novice who momentarily loses his/her place in the progression."[74] In this case it is relatively clear that the arrival of D7 is delayed, and that the pattern D♭–C–B♭–G should not be taken as some outgoing scale choice for D7.

In his solo, Sonny Stitt often uses barline shifts in double-time passages to increase the effective harmonic rhythm; Figure 4.21 gives one of these passages over a ii–V progression.[75] In the second half of m. 73, Stitt plays a descending F bebop scale, implying the F7 chord a half-bar early.[76] Instead of continuing to play F7 in the next bar, though, Stitt seems to return to Cm7: he uses F♯ only as a lower neighbor until the last beat, when the (strongly C-minor) figure G–E♭–D–C figure resolves to F♮. Stitt exploits the fact that there is no diatonic shift between ii^7 and V^7 chords, and alternates between the two freely in a 2♭ diatonic wash.

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[74] Coker, *Elements of the Jazz Language*, 83.

[75] In "double-time" passages, the soloist plays twice the speed of the prevailing note value, though the underlying tempo remains constant. Here there is a quarter-note pulse, and the soloists play predominantly eighth notes; the double-time passages use sixteenth notes instead. Matthew Voglewede has discussed the practice of double-time from a metrical standpoint in "Metrically Dissonant Layers of Swing: Double Time in Two of Louis Armstrong's Performances of 'Lazy River'" (paper presented at the annual meeting of the Music Theory Society of the Mid-Atlantic, Philadelphia, PA, March 2013).

[76] The dominant bebop scale is a Mixolydian scale with a chromatic note added between the chordal seventh and the root, so that when played in eighth notes the chord tones fall on the beat. David Baker (who played with George Russell early in his career) is usually credited with inventing the term; see *How to Play Bebop, Vol. 1: The Bebop Scales and Other Scales in Common Use* (Van Nuys, CA: Alfred, 1985).

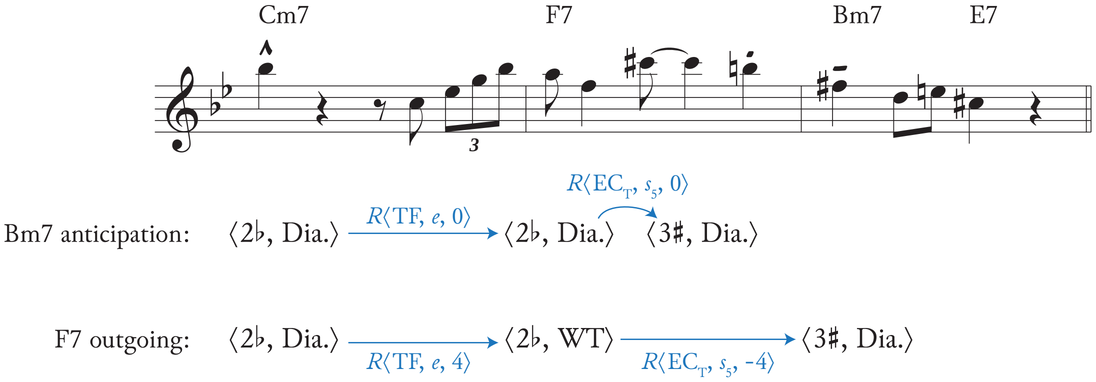

Figure 4.22. An ambiguous barline shift in Ammons's solo (mm. 41–43, 2:06), with two possible transformation networks.

Finally, there are some cases when it is unclear whether to read a passage as a barline shift or as an outgoing scale choice; Figure 4.22 gives a representative example.[77] The C♯ and B♮ in m. 42 could be heard as anticipating the Bm7 by two beats, or as part of a whole-tone scale over F7, as shown in the transformation networks. Ammons anticipates the Bm7 in the A sections of the first chorus (mm. 3 and 11, the second with the pitches C♯ and B♮), giving some credence to the anticipatory hearing. On the other hand, the first A section of the second chorus features increased chromaticism—the D7 in m. 38 uses a diminished scale—and perhaps the second A section continues the trend. Again, the prismatic approach says that it is less important to decide on a single interpretation than to realize that both are available for our perception.

Over the course of an improvised solo, a performer's choice of scale for a particular chord can change. Usually, though, some chords are more flexible in their chord-scale identity than others.[78] In the Ammons/Stitt recording, the progression Bm7–E7–B♭m7–E♭7 in the third and fourth bars of the A sections is almost always accompanied with the ⟨3♯, Dia.⟩–⟨4♭, Dia.⟩ succession.[79] The

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[77] In this figure and others following, I have omitted the chord symbol from the chord-scale triples to save space, since it is obvious from the transcription itself.

[78] By tallying up all of the appearances of a given chord/scale pairing, it would be possible to develop something like a snapshot of a performer's chord-scale choices. Doing so could perhaps bring some clarity to discussions of jazz style (as in, for example, David Baker's *Giant of Jazz* series) by incorporating the chord-scale transformations developed here. While this is an interesting possibility, we will pursue it no further here.

[79] The only time it is not occurs in Stitt's first chorus ($3A_1$, mm. 67–68), where he plays a sustained D♮ through all four chords.

Original: C-^{7} | F^{7} | B♭^{maj7} | E♭^{maj7} | A-$^{7♭5}$ |

Ammons/Stitt: C-^{7} | F^{7} | B-^{7} E^{7} | B♭-^{7} E♭^{7} | A-$^{7♭5}$ |

Figure 4.23. "Autumn Leaves," mm. 1–5, showing substitutions in mm. 3–4.

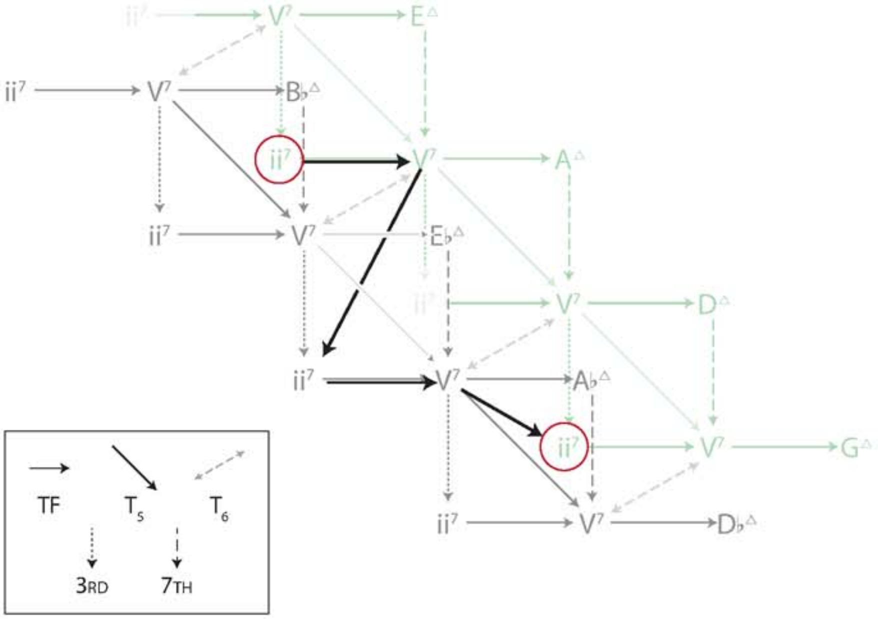

Figure 4.24. The substitutions in mm. 3–4 of "Autumn Leaves" in ii–V space.

home-key dominant D7, on the other hand, enjoys a wider range of scalar options, appearing at various times with the whole–half diminished scale (see mm. 6, 18, 38), the Lydian diminished scale (mm. 14, 26, 78), the diatonic collection (m. 82), and the Lydian ♭7 scale (mm. 102, 110). As first noted in Chapter 1 (p. 22n67), the scale choice for the tonic G minor is somewhat flexible: it is always played with E♮, but sometimes with an upper-neighbor F♮ implying a Dorian scale (mm. 19, 39) and other times with an F♯ implying a melodic minor scale (or ⟨Gm, ♭, Lyd. aug.⟩, as in m. 83).

The analysis of "Autumn Leaves" in Chapter 1 focused on its underlying diatonic nature, but this diatonicism is attenuated somewhat in the Ammons/Stitt recording. Figure 4.23 gives the first five bars of "Autumn Leaves" as analyzed in Chapter 1 (top) and as played on *Boss Tenors* (bottom).

Both major seventh chords in the original diatonic succession F7–B♭maj7–E♭maj7–Am7♭5 are here substituted with chromatically descending ii–V progressions; Figure 4.24 shows this set of substitutions in ii–V space. This set of substitutions is relatively easy to understand: first, the major seventh chords are turned into dominants (B♭7–E♭7); each is preceded by a ii^7 chord (Fm7–B♭7–B♭m7–E♭7); finally, the first ii–V progression is transposed by a tritone. While these chords are close together in ii–V space, they are far apart in chord-scale space, as noted above; Bm7–E7 is played as ⟨3♯, Dia.⟩ in this recording, while B♭m7–E♭7 is played as ⟨4♭, Dia.⟩— a difference of five sharps/seven flats.[80]

The ensemble likely decided to make the substitutions in the solos to prevent the piece from becoming boring: since almost all of the original chords are diatonic in G minor, playing in the 2♭ diatonic collection will work for almost every harmony.[81] The substitutions in mm. 3–4 of the A sections provide some variety in the chord-scale options, leaving the diatonic nature of the tune to manifest in other ways. In both of his choruses, Stitt uses a harmonic generalization from the last four bars of the bridge through the first four of the C section, playing the 2♭ diatonic collection throughout (Figure 4.25 reproduces this passage from his last chorus). Because of the substitutions in the A section, this implicit diatonic cycle (first discussed in connection with Figure 1.8) is the only one remaining in the solo changes; Stitt's choice to use a harmonic generalization highlights this implicit diatonicism.[82] Because the scale stays the same while the chords move through a diatonic cycle, every transformation in this passage is the same: $R\langle t_3, e, 0\rangle$.

4.3.3 Joe Henderson, “Isotope”

Joe Henderson's composition “Isotope” was the subject of Section 3.1.2, and this section will return to Henderson's solo on the tune from *Inner Urge* (1965). Since he is also the composer of the tune,

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[80] This passage also illustrates the property of signature transformations that f_7 is always equivalent to T_{11}; see Hook, “Signature Transformations,” 142–44.

[81] The dominant, D7, is not in the 2♭ collection, since it is not diatonic in G minor. This collection also results in the major scale (not the Lydian) for the B♭maj7 chords, which is borne out by the E♭s in the recording (see mm. 23–24, 55, and 88). The Gm chord itself is usually played with E♮, placing it in the 1♭ collection.

[82] In fact, Stitt's note choices here would also work well if the rhythm section were to play E♭maj7 in the fourth bar, which is suggested in Figure 1.8.

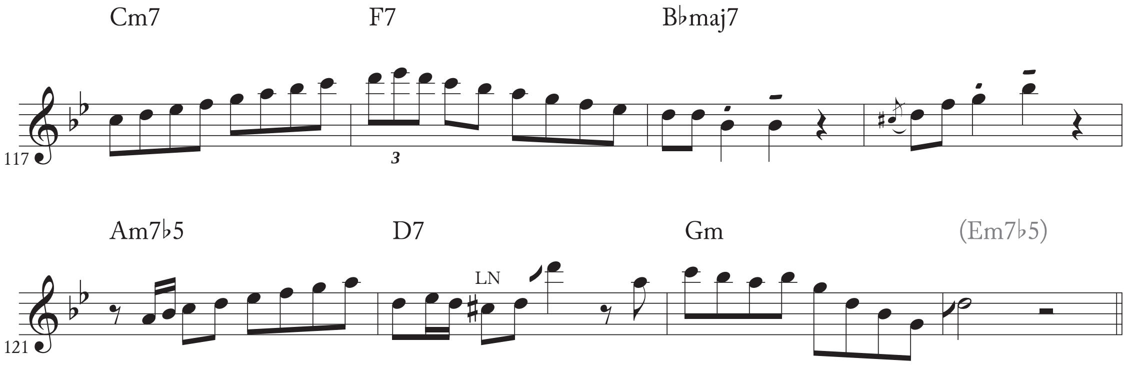

Figure 4.25. Stitt's 2♭ harmonic generalization, highlighting the implicit diatonic cycle (mm. 117–24, 4:11).

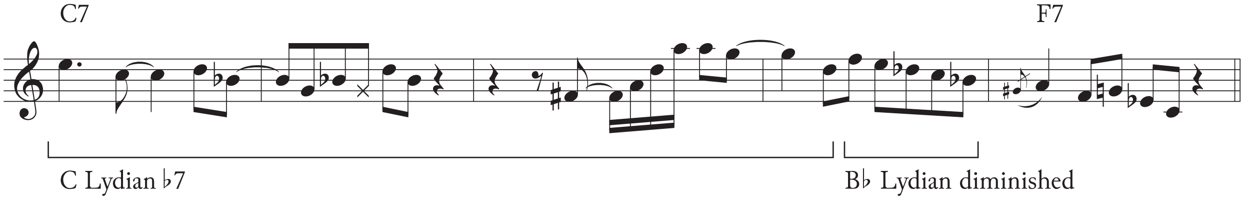

Figure 4.26. A scalar shift highlights a change in function in Joe Henderson's solo on "Isotope" (mm. 13–17, 0:47).

Henderson's fifteen improvised choruses (Appendix B, p. 230) can provide some insight into how he understands its harmony.

As we observed in Section 2.3.2, major-minor seventh chords can function as tonic chords in the blues. This fact presents something of a problem: the C7 at the beginning of "Isotope" acts as tonic at the beginning of its four-bar span, but as a dominant of the following F7 at the end.[83] At some point during this four bars there must be a pivot fifth: a motion from "C7 as tonic" to "C7 as dominant." This is harmonic information that is not readily available in the chord symbols, but can be seen in a soloist's scale choices.

Figure 4.26 gives a representative example from Henderson's second chorus; here, he uses the Lydian ♭7 scale in the first three bars of the chorus before shifting to the Lydian diminished scale

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[83] Other blues tunes negotiate this problem by inserting another chord in the first four bars. Thelonious Monk's "Misterioso," to take a typical example, is a blues in B♭: the first four bars contain the progression B♭7–E♭7–B♭7–B♭7. In a case like this, the first B♭7 functions as tonic, while the last two bars function as dominant of the following IV chord.

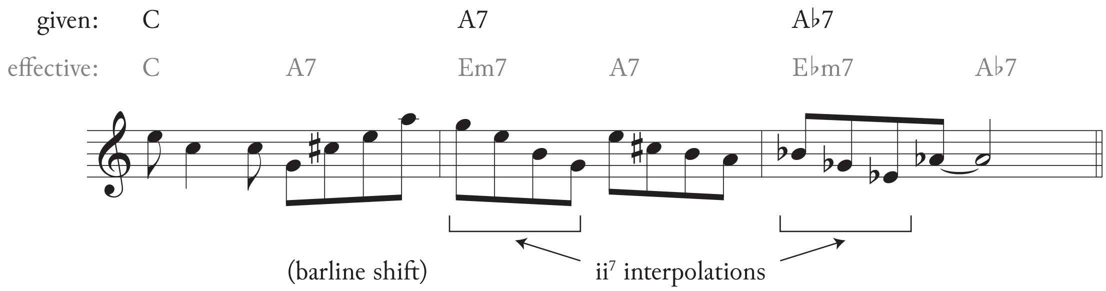

Figure 4.27. Two interpolated ii^7 chords from Henderson's first chorus (mm. 7–9, 0:39).

in the final bar. The addition of ♭9 (D♭) intensifies the motion toward F7, since tonic chords do not usually have this extension. This is a harmonic move that is easily captured using a chord-scale transformation; we can represent the passage as

$$\langle \mathrm{C7}, \sharp, \mathrm{Lyd.}\ \flat 7\rangle \xrightarrow{R\langle \text{pivot 5th}, f_2, -1\rangle} \langle \mathrm{C7}, \flat, \mathrm{Lyd.\ dim.}\rangle.$$

Henderson does not do this in every chorus (he often plays diatonic or blues scales through the entire four bars), but similar effects can be observed in choruses 1, 7, 9, 14, and 15.

In the two analyses above we saw that substitutions can affect chord-scale choices, but not all of them do. One of the most common substitutions is actually an interpolation, in which a performer plays a ii–V progression when only a V^7 chord is given. Because the motion from ii^7 to V^7 does not involve a diatonic shift, these interpolations often do not affect the chord-scale analysis.[84] Two examples from Henderson's first chorus are given in Figure 4.27; in each, the arpeggiation in the first half of the bar could be seen as diatonic (each involving the ninth of the V^7 chord), but they can also be heard as arpeggiations of the ii triad.

The blues offers many opportunities for harmonic generalization, and here I want to focus on two passages in particular.[85] The first is in Henderson's fifth chorus, the opening of which is shown in Figure 4.28. In the third bar of the chorus, Henderson begins playing a lower-neighbor figure

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[84] If a performer chooses non-diatonic scales for the ii^7 and V^7 chords, the analysis would be affected; in the diatonic case, it is not.

[85] Harmonic generalization is part and parcel of the blues; the blues scale derives its name in part from the fact that the tonic blues scale can be (and often is) played over an entire blues chorus. As Mark Levine has it, "playing the blues scale over the I–IV–V chords of a basic blues yields dissonances hardly acceptable in traditional theory. But these dissonances have been present in jazz since its inception" (*The Jazz Theory Book*, 233).

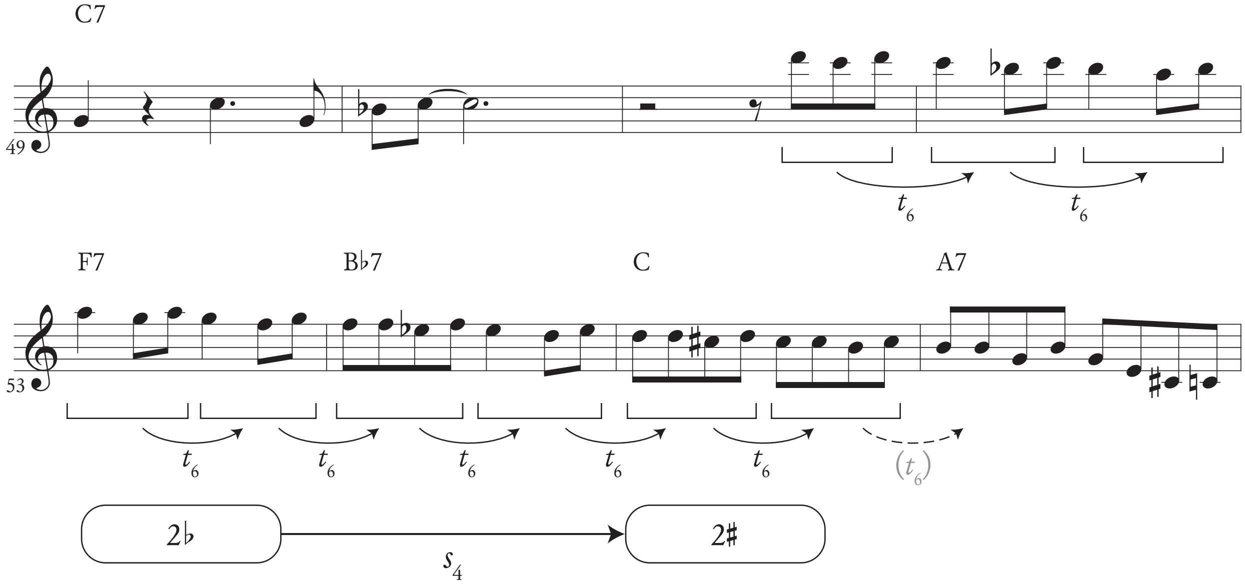

Figure 4.28. Signature transformations in the opening of Henderson's fifth chorus (mm. 49–56, 1:33).

that continues in sequence through the next four bars. Because this passage is so uniform, we can draw on the full power of Hook's signature transformations, including the diatonic transposition operator *t*.[86] Each pattern moves down a diatonic step (t_6); the final arrow is dotted to indicate that the pattern breaks down at this point, and the lower-neighbor is distorted into a descending minor third. In mm. 54–55, the lower-neighbor pattern is constant, while the underlying collection shifts from the 2♭ to the 2♯ collection, an s_4 transformation.[87] Henderson ignores the C chord in m. 55, anticipating the A7 by a full bar, and also deemphasizes the B♭7 (there is no A♭ present).

Henderson plays a similar passage in the middle of chorus 14, shown in Figure 4.29. Like the first, it begins with a diatonic sequence, this time in a stepwise rising motion (a series of t_1s). Here the sequence breaks down much earlier, but the constant eighth notes recall the previous passage; both sound like a single line with a shifting diatonic foundation rather than a series of loosely connected gestures. This passage traverses the same s_4 transformation as the first from the 2♭ to 2♯

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[86] This operator should be read here as Hook's t_k, which operates on "floating diatonic forms," *not* as the t_k operator defined in Section 1.5. See Hook, "Signature Transformations," 139–41.

[87] Incidentally, the diatonic collection of the C7 chord itself is ambiguous. We might hear it as the 1♯ collection (as part of a Lydian ♭7 scale), the 1♭ collection (hearing C7 as a dominant, as the second mode of of B♭ Lydian), or perhaps even the 2♭ collection (as a harmonic generalization of B♭7 in the first six bars of the chorus). Exactly which diatonic collection we choose is not important to the analysis here.

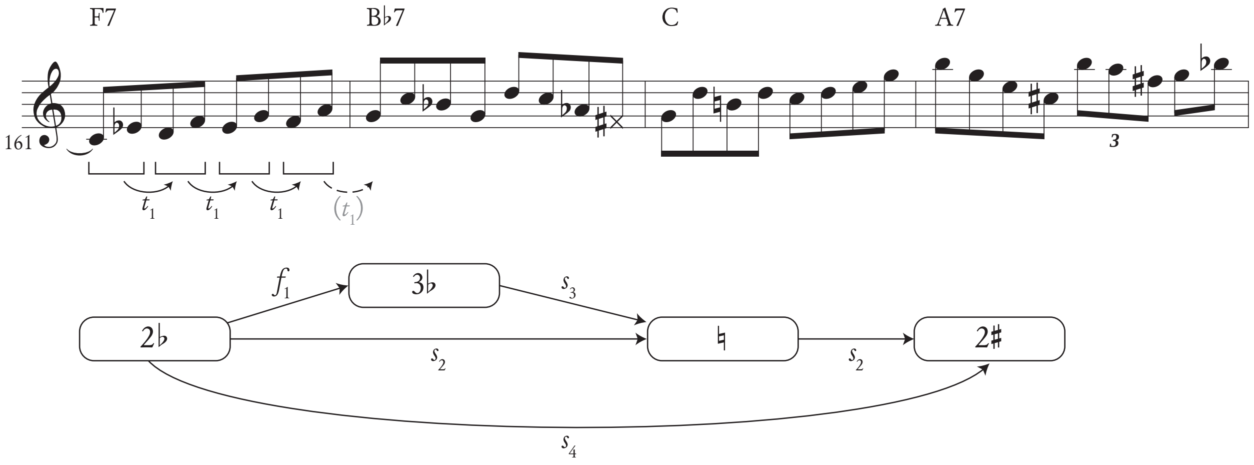
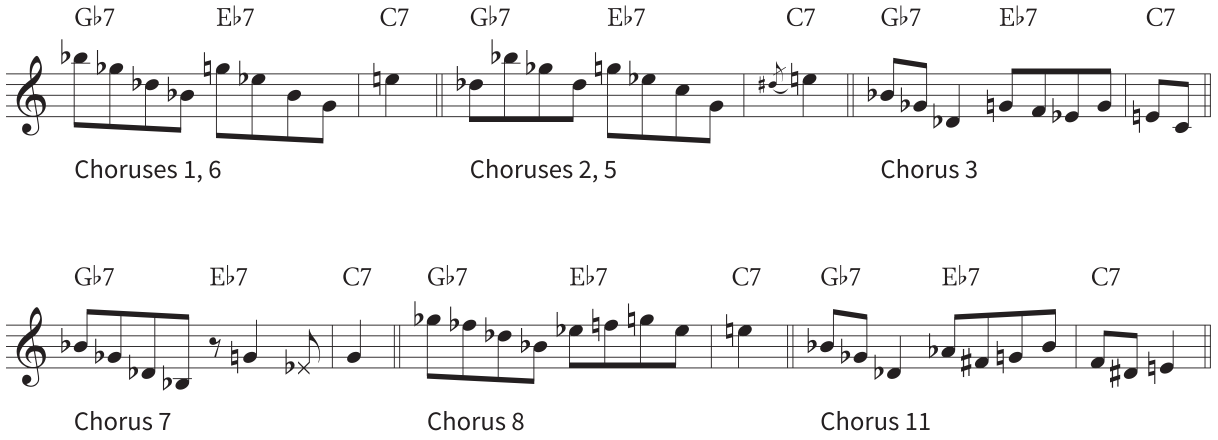


Figure 4.29. A similar passage in chorus 14, with signature transformations (mm. 161–64, 3:52).



Choruses 1, 6

Choruses 2, 5

Chorus 3

Chorus 7

Chorus 8

Chorus 11

Figure 4.30. Henderson's solos on G♭7–E♭7–C7 in several choruses of "Isotope."

diatonic collections, though the route here is more scenic. Henderson does not anticipate the A7 and instead plays the C harmony, splitting the s_4 transformation into two s_2s. The first of these is further subdivided into an f_1/s_3 pair; because the A♭ comes at the very end of m. 162 as part of an enclosure figure, it is not as strong as the larger motion from the 2♭ to the ♮ collection.

In the final pages of this chapter, I want to take a step back to show that there are aspects of Henderson's solo that are not easily expressed with chord-scale transformations. In the analysis of "Isotope" in Chapter 3, we noted that the turnaround, in which dominant sevenths descend by minor thirds, was one of the most interesting parts of the tune. The fact that these chords can be related by smooth voice leading was downplayed there, but Henderson often plays these chords (especially G♭7–E♭7) as arpeggiations that emphasize this voice leading. Figure 4.30 gives several examples of Henderson's solos on the final bar of the chorus, along with their resolutions. In all of these, he emphasizes the G♭–G♮ voice leading from G♭7 to E♭7, and usually brings out the E♭–E♮ over E♭–C7 as well. The importance of these voice leadings disappears in the chord-scale transformations, and would be better shown in, for example, one of Tymoczko's voice-leading spaces.[88]

There are two choruses of Henderson's solo where contrapuntal concerns seem to take precedence over making the changes; Figure 4.31 reproduces chorus 6 in its entirety, but the same principle is at work in chorus 12. The particular scale Henderson plays in the first four bars is clearly not as important as the chromatic line from E to G and back, taking place over the lower G pedal point.[89] This chromatic polyphonic melody continues throughout the chorus, culminating in the arrival at the high C in m. 70. This chorus is one that a Schenkerian voice-leading sketch would describe particularly well (the first four bars unfold the tonic interval E_5–G_5, and so on), but that chord-scale transformations do not. Indeed, it is not at all clear which scales are being used, nor do they even seem particularly important to understanding this chorus.

To the extent that chord-scale transformations describe *harmony*, though, they are quite useful, as these three analytical vignettes have shown. Both of the limitations just discussed involve voice leading, and for jazz musicians, harmony and voice leading do seem to be separate entities.[90] While for Schenker harmony and voice leading are two aspects of the same phenomenon, George Russell instead pairs harmony (or chord) and scale. The two theories seem to be quite similar in their goals; compare Schenker's unification of harmony and voice-leading with Russell's repeated

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[88] The four-voice tesseract in Tymoczko, *A Geometry of Music*, 106 is his way of describing these motions; Cohn's Four-Cube Trio in Figure 3.13 describes the same voice-leading space.

[89] Jerry Coker calls this kind of line CESH, which is short for "contrapuntal elaboration of static harmony" (*Elements of the Jazz Language*, 61–67).

[90] None of the passages discussing CESH or "linear chromaticism" in Coker's book, for example, say anything about harmony, nor do any of the passages that feature discussions of voice-leading (what other authors refer to as "guide tones") mention it as a harmonic phenomenon. In his discussion of the "7–3 resolution", he says that he is "simply concerned with the smooth connection (voice-leading) of two chords, especially with respect to melodic, rather than harmonic, implications" (*Elements of the Jazz Language*, 19).

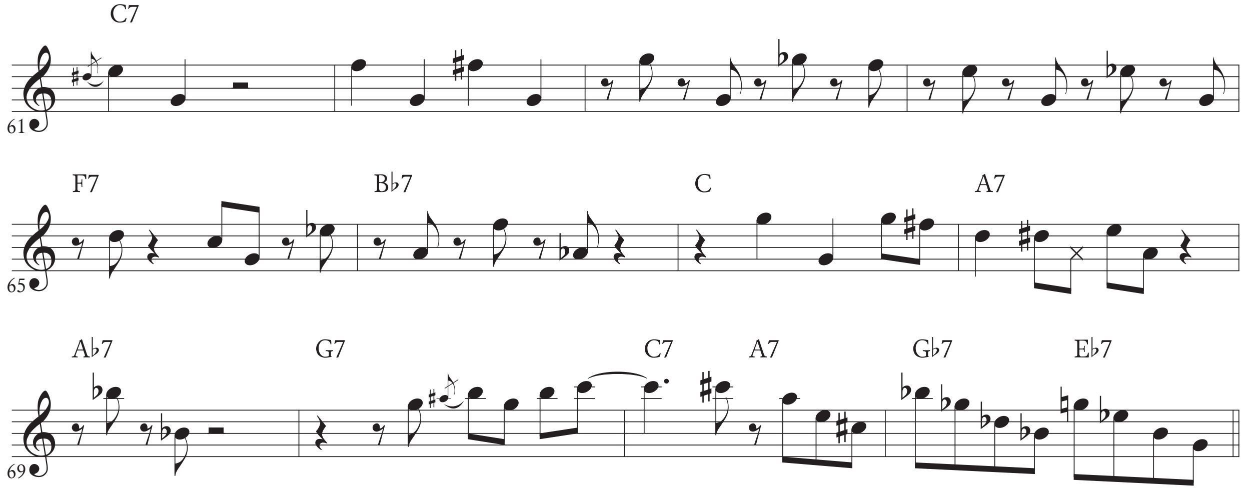

Figure 4.31. Henderson's sixth chorus (mm. 61–72, 1:48), featuring contrapuntal motion.

declarations that the Lydian Chromatic Concept unites both the vertical and horizontal aspects of music. Furthermore, Russell's theory of jazz was no less aspirational than Schenker's in its goal to describe all tonal music; Russell says explicitly that the concept of tonal gravity is "the underlying force of equal tempered music" (*LCC* 223).

Toward the end of the *Concept*, addressing the claim that his theory can describe all music, Russell asks why "such a theoretical work [should] come from the jazz experience" (*LCC* 223). This chapter has asked the related question of why Russell's theory of "the jazz experience" seems not to have been applied in any rigorous theoretical way to jazz itself. If we take seriously the idea that chords and scales are manifestations of a single phenomenon, then we must look at scales if we are to fully understand jazz harmony. Transformational theory provides a means of examining these chord-scales in a systematic way, and the chord-scale transformations developed here have expanded our musical universe beyond the lead sheet and into jazz performance itself.

CHAPTER 5
Rhythm Changes

The discussion of chord-scale transformations in the previous chapter concludes the theoretical portion of this dissertation; this final chapter will synthesize that theoretical framework in a series of three longer analyses. All three of the tunes here—Thelonious Monk's "Rhythm-a-ning," George Coleman's "Lo-Joe", and Sonny Stitt's "The Eternal Triangle"—are instances of a harmonic archetype known as "Rhythm changes," so named for their origin in George Gershwin's "I Got Rhythm."[1] Because tunes that use Rhythm changes all share a common harmonic framework, they are an ideal means to investigate jazz harmony. A complex set of standard substitutions and harmonic patterns have emerged over the many years jazz musicians have been playing Rhythm changes; the three analyses in this chapter will allow us to compare these musicians' manipulation of this basic harmonic framework.

5.1 Rhythm Changes in General

It is hard to overestimate the importance of Rhythm changes on jazz practice; along with the blues, it is one of the most common harmonic types in the bebop era and beyond.[2] David Baker lists more than 150 Rhythm tunes in his *How to Learn Tunes*; some of the most well-known of these are reproduced in Table 5.1.[3] Before moving on to the analyses in the following sections, it will be useful to examine the form itself, along with some of its more common harmonic substitutions.

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[1] "I Got Rhythm" was written in 1930 and first appeared in the musical *Girl Crazy*. Because the phrase "Rhythm changes" has developed a life beyond its initial meaning, it is rendered throughout this chapter without quotes but with a capital "R." The phrase is normally used as a noun, while adjectival uses drop the "changes" (as in "Oleo is a Rhythm tune").

[2] Thomas Owens, *Bebop: The Music and its Players* (New York: Oxford University Press, 1995), 13.

[3] David Baker, *How to Learn Tunes* (New Albany, IN: Jamey Aebersold Jazz, 1997), 42–44. New tunes that are based on the chord changes to other tunes are known as "contrafacts." Part of the reason for the proliferation of contrafacts in general (and the genre of Rhythm tunes in particular) is that jazz musicians could avoid paying royalties to the Gershwins. On contrafacts more generally, see Owens, *Bebop*, 8 and 12–15.

Title	Composer
Anthropology	Charlie Parker/Dizzy Gillespie
Cotton Tail	Duke Ellington
52nd Street Theme	Thelonious Monk
The Flintstones	Hoyt Curtain
Jumpin' at the Woodside	Count Basie
Moose the Mooche	Charlie Parker
Oleo	Sonny Rollins
The Serpent's Tooth	Miles Davis
Tiptoe	Thad Jones
Wail	Bud Powell

Table 5.1. A selection of Rhythm tunes and their composers.

5.1.1 SUBSTITUTION SETS

"I Got Rhythm" is, like many jazz standards, a 32-bar AABA form; the basic progression is shown in Figure 5.1.[4] As Andy Jaffe notes, its changes are "not the least bit astonishing"; the tune is a fairly basic set of turnarounds and dominant cycles.[5] Indeed, this feature is one of the reasons for its popularity: the harmonic framework is something of a blank slate, and allows room for alteration in a way that more specific sets of changes (like Parker's "Blues for Alice," for example) do not. Another thing that is immediately apparent is the quick harmonic rhythm in the A sections, which allows soloists the opportunity to show off as they navigate the rapidly moving changes.[6]

Fundamental to the genre of Rhythm tunes is their "mix-and-match" nature; each part of the form has many different sets of changes, from which the performers may choose freely.[7] Mark Levine explains this issue succinctly:

> When a musician calls a Rhythm tune like "Oleo," there's no discussion of which version of the changes to play. As with the blues, jazz musicians freely mix many versions of Rhythm changes on the spot, as they improvise. Playing Rhythm changes is a little like

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[4] The vast majority of Rhythm tunes are in the key of B♭. Those in Table 5.1 that are not are Hoyt Curtain's theme to the cartoon *The Flintstones* and Bud Powell's "Wail" (both in E♭), along with Thad Jones's "Tiptoe" (A♭) and Thelonious Monk's "52nd Street Theme" (C).

[5] Andy Jaffe, *Jazz Harmony* (Tübingen: Advance Music, 1996), 149.

[6] Adding to this virtuosity is the fact that many recordings of rhythm changes are quite fast. Of the standard bebop recordings of tunes in Table 5.1, Parker's recording of "Moose the Mooche" is 212 bpm, Powell's "Wail" is 270 bpm, while the Parker/Gillespie "Anthropology" burns along at roughly 305 bpm.

[7] The "mix-and-match" metaphor comes from Jaffe, *Jazz Harmony*, 149.

A	B♭	G-^{7}	C-^{7}	F^{7}	B♭	G-^{7}	C-^{7}	F^{7}
	B♭	B♭^{7}	E♭	E♭-	B♭	G-^{7}	C-^{7}	F^{7}
A′	B♭	G-^{7}	C-^{7}	F^{7}	B♭	G-^{7}	C-^{7}	F^{7}
	B♭	B♭^{7}	E♭	E♭-	B♭	F^{7}	B♭	
B	D^{7}				G^{7}			
	C^{7}				F^{7}			
A′	B♭	G-^{7}	C-^{7}	F^{7}	B♭	G-^{7}	C-^{7}	F^{7}
	B♭	B♭^{7}	E♭	E♭-	B♭	F^{7}	B♭	

Figure 5.1. The basic changes to George Gershwin's "I Got Rhythm" (taken from Levine, *The Jazz Theory Book*, 238).

> knowing several tunes and playing them all at once; that's why "Rhythm" tunes are harder to play at first than a tune with only a single set of changes.[8]

Given this background, the analytical discussion below will proceed in segments: the A sections can each be broken into four-bar halves, while the bridge is typically treated as a single eight-bar unit.

The first four bars of the Rhythm A section serve to establish the tonic B♭; Figure 5.2 gives a number of possible harmonizations of this section.[9] Letter *a* gives the original Rhythm changes, while *b* shows what is by far the most common substitution, replacing B♭ with Dm7 in the third bar; this changes the I–vi–ii–V turnaround in the last two bars into a iii–vi–ii-V instead. If the B♭ harmony is voiced with a major seventh and major ninth (B♭–D–F–A–C, a very common voicing), then we can understand the substitution of Dm7 as a simple omission of the root. Letter *c* goes

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[8] Mark Levine, *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 241.

[9] These six harmonizations do not begin to constitute a complete set of substitutions for these four bars. Jamey Aebersold gives 43 harmonizations of the Rhythm A section in *"I Got Rhythm" Changes in All Keys*, Jamey Aebersold Play-A-Long Series, vol. 47 (New Albany, IN: Jamey Aebersold Jazz, 1991), 26.

a) B♭ G-^{7} | C-^{7} F^7 | B♭ G-^{7} | C-^{7} F^7 ‖

b) B♭ G-^{7} | C-^{7} F^7 | D-^{7} G-^{7} | C-^{7} F^7 ‖

c) B♭ G^7 | C-^{7} F^7 | D^7 G^7 | C-^{7} F^7 ‖

d) B♭ B$^{\circ 7}$ | C-^{7} C♯$^{\circ 7}$ | D-^{7} D♭^{7} | C-^{7} F^7 ‖

e) F♯^{7} B^7 | E^7 A^7 | D^7 G^7 | C^7 F^7 ‖

f) B♭ A♭^{7} | G♭^{7} F^7 | B♭^{7} A♭^{7} | G♭^{7} F^7 ‖

Figure 5.2. Several harmonizations of Rhythm changes, mm. 1–4.

further, transforming many of the minor seventh chords into dominant sevenths that serve to lead more strongly into the following harmonies.

Figure 5.3 shows the relevant portion of ii–V space for these first three harmonizations, along with a few annotations. The standard harmonization of figure 5.2a can be seen by following the blue arrows. The substitution of Dm7 in letter *b* is represented in the space by the red arrows: in this reading, first follow the blue arrows until arriving at F7, then follow the red arrows until Gm7 where the blue arrows continue to the tonic B♭. The minor-to-dominant substitutions of letter *c* are not shown in the space, but are easy enough to imagine: both Gm7 and Dm7 are transformed by the 3RD^{-1} operation, and each is replaced by the chord immediately to its north in ii–V space (a substitution which results in the evaded cadence transformation, G7 $\xrightarrow{\text{EC}}$ Cm7, across the bar lines at the end of mm. 1 and 3).

The harmonization in Figure 5.2d is still more complex. The tritone substitution of D♭7 for G7 in m. 3 is by now familiar, but the G7 in m. 2 has been replaced with a passing diminished seventh chord. As we first saw in the analysis of "Have You Met Miss Jones?" in Section 3.2.3, fully-diminished sevenths in jazz can often be understood as V7♭9 chords missing their roots. The

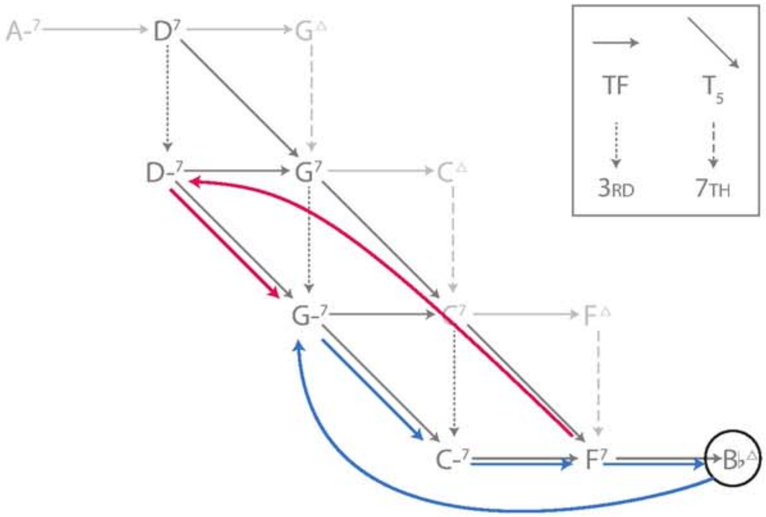

Figure 5.3. The first four bars of Rhythm changes in ii–V space.

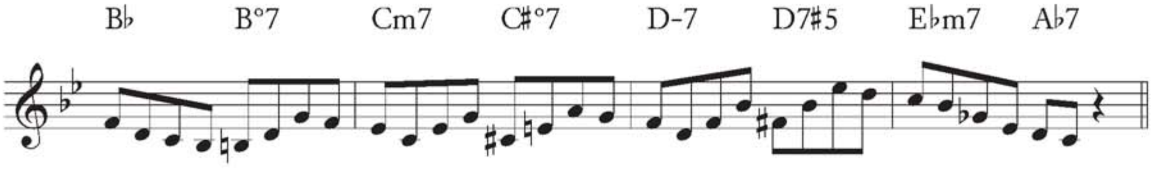

Figure 5.4. "The Serpent's Tooth" (Miles Davis), mm. 1–4.

B°7 here, then, is a logical substitution for G7♭9, and the C♯°7 in the following bar can be understood as the same substitution of an implied A7♭9 chord (the dominant of the following D minor), resulting in a chromatically ascending bass line in the first two bars. Miles Davis's composition "The Serpent's Tooth" (the opening of which is shown in Figure 5.4) uses a variation of this progression. Davis also includes a minor-third substitution in m. 4, substituting E♭m7–A♭7 for the diatonic Cm7–F7.

The last two harmonizations in Figure 5.2 are somewhat different in nature; while any of the substitutions of letters *a*–*d* can be swapped in and out at will (the first two bars of *a* followed by the last two bars of *d*, for example), those in letters *e*–*f* usually appear intact. Letter *e* harmonizes the first four bars with a cycle of dominant seventh chords (a favorite technique of Thelonious Monk, and one we will see in the analysis of "Rhythm-a-ning" below). In contrast to the relatively compact arrangement of letters *a*–*c* in ii–V space, this cycle traverses nearly the entire space before

a) B♭ B♭^{7} | E♭ E♭- | B♭ G-^{7} | C-^{7} F^7 ||

b) F-^{7} B♭^{7} | E♭ A♭^{7} | B♭ G-^{7} | C-^{7} F^7 ||

c) B♭ B♭^{7} | E♭ E°^{7} | D-^{7} G^7 | C-^{7} F^7 ||

d) B♭ B♭^{7}/D | E♭-^{7} A♭^{7} | B♭ D♭^{7} | C^7 B^7 ||

Figure 5.5. Several harmonizations of Rhythm changes, mm. 5–8.

arriving at the tonic B♭.[10] Letter *f* is the harmonization from Jimmy Heath's composition "C.T.A.," and features a repeated lament-bass pattern from B♭ down to F7.

The last four bars of the Rhythm A section contain a shift to the subdominant in the first two bars, followed by a turnaround in the last two; Figure 5.5 gives several common harmonizations of this passage. Once again, letter *a* reproduces the original changes: a seventh is added to the tonic B♭, tipping it towards an E♭ chord that resolves plagally (via minor iv) back to tonic before a vi–ii–V turnaround. This plagal motion in the second bar is often substituted with a backdoor progression, A♭7–B♭, as seen in *b* (which also precedes the B♭7 in the first bar with a ii^7 chord in E♭) and *d* (which elides the E♭ and E♭m harmonies). Letter *c* makes the substitution of Dm7 for B♭ in the third bar and includes E°7 as a substitution for E♭7♭9.[11]

The Rhythm bridge is usually recognizable because of the drastic slowing of the harmonic rhythm; again, Figure 5.6 gives several common harmonizations, and Figure 5.7 shows them in ii–V space. The standard bridge (letter *a*) is a simple cycle of dominants, beginning on the III chord; we will call this the "4-cycle bridge." The most common substitutions here are tritone substitutions of every other chord, as shown in *b* and *c*. The other common option is to insert a ii^7 chord before each of the dominants, as shown in *d*, decomposing each T_5 transformation into

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[10] I have not included another copy of the complete ii–V space here, but one can be found in Figure 2.10 (p. 50).

[11] This E°7 is functionally ambiguous; it could also stand in for an A7♭9 as the dominant (or C7♭9 as part a backdoor progression) to the following Dm7. It is spelled as E°7 to produce a smooth bass line from E♭ in the first half of the bar.

a)	D^7		G^7		C^7		F^7	
b)	D^7		$D\flat^7$		C^7		B^7	
c)	$A\flat^7$		G^7		$G\flat^7$		F^7	
d)	$A{-}^7$	D^7	$D{-}^7$	G^7	$G{-}^7$	C^7	$C{-}^7$	F^7

Figure 5.6. Several harmonizations of the Rhythm bridge, mm. 17–24.

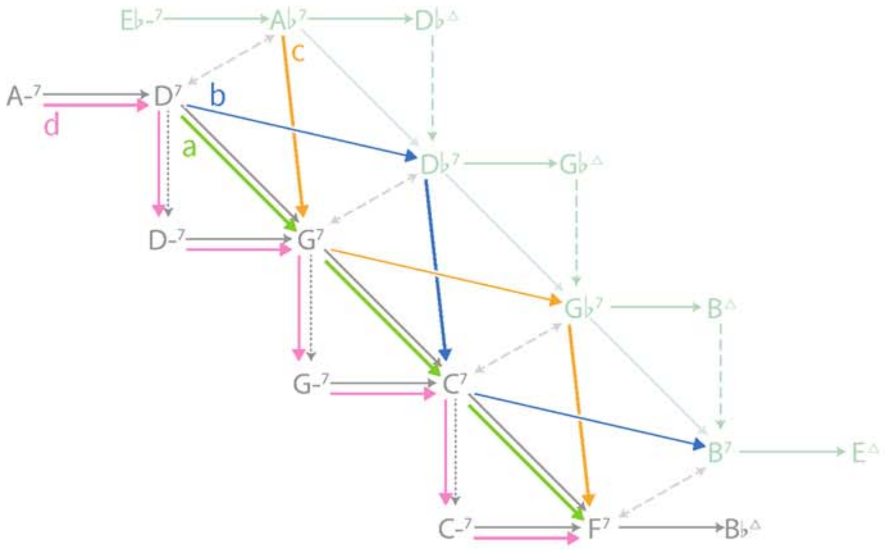
E♭-⁷
A♭⁷
D♭△
c
A-⁷
D⁷
b
d
a
D♭⁷
G♭△
D-⁷
G⁷
G♭⁷
B△
G-⁷
C⁷
B⁷
E△
C-⁷
F⁷
B♭△

Figure 5.7. The four Rhythm bridge harmonizations of Figure 5.6 in ii–V space.

TF • 3RD. (This procedure could of course be combined with the tritone-substituted versions in *b* and *c* as well.) Other less conventional harmonizations are also possible; "The Eternal Triangle" and "Lo-Joe" both use specialized bridges which we will see in later sections.

It should be apparent from this discussion that Rhythm tunes can vary widely in their harmonic particulars. The mix-and-match nature of their construction means that the chords used by an ensemble can change even over the course of a single performance: a rhythm section might prefer one harmonization of the bridge during a saxophone solo and opt for another during a piano solo, for example. The harmonizations given in Figures 5.2, 5.5, and 5.6 have only begun to scratch the surface; because most of the tune consists of turnarounds, any of the countless possible turnarounds could be used instead.[12] It is easy to imagine a Rhythm tune that makes use of the descending minor-third turnaround of Henderson's "Isotope" in the A sections or fast-moving Coltrane changes over the bridge.

Still, though, the many harmonizations of Rhythm changes all share certain attributes: 32-bar AABA form, half-note harmonic rhythm, a move to the subdominant in the sixth bar of the A sections, and so on. These aspects, combined with the popularity of the form, means that Rhythm tunes are usually apparent to performing jazz musicians, even though the changes themselves might be quite removed from Gershwin's original (as in the case of Figure 5.2e above, or in George Coleman's "Lo-Joe" below). To say that a tune is a Rhythm tune is akin to saying a piece is in sonata form: as listeners we can expect certain general things to be true, but the particulars of the instantiation will vary from piece to piece (or performance to performance, or even chorus to chorus).

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[12] Given the modular nature of Rhythm tunes, the genre seems particularly ripe for a schema theory approach, following in the footsteps of Robert Gjerdingen, *Music in the Galant Style* (New York: Oxford University Press, 2007). The two- and four-bar units here (turnarounds, dominant cycles) are not unlike the stock phrases used in the galant style, and jazz pedagogical materials like the previously-cited Aebersold and Jaffe texts could easily serve as analogs to the 18th-century Italian *partimenti* often used by schema theorists.

5.1.2 HARMONIC SUBSTITUTION VS. CHORD-SCALE ELABORATION

Before moving on to the three analyses proper, it will be helpful to return to an issue first mentioned in the last chapter in connection with Rahsaan Roland Kirk's solo on "Blues for Alice." In many cases, it is not clear whether a particular improvised passage should be heard as a harmonic substitution or as an outgoing chord-scale choice over a more basic harmony. In the case of non-Rhythm tunes, we can usually rely on the head to provide the authoritative changes for the tune, and it is likely that we choose to hear that particular set of changes throughout the performance. Rhythm changes, though, bring this problem to the fore, since we cannot depend on a single set of canonical changes.

By way of a short illustration, consider again the melodic passage that opens Miles Davis's "Serpent's Tooth" (first shown with Davis's original progression in Figure 5.4). If this were an improvised passage, it seems likely that the first choice of harmonies would *not* be those used by Davis, given the clear outlines of both G7 and A7 chords in the second halves of mm. 1–2. It is also possible to hear this passage as a series of outgoing scale choices over a standard diatonic progression, hearing the C♯–E–G fragment as part of a diminished scale over F7. Three possible hearings of these first two bars are shown in Figure 5.8, which gives locations in chord-scale space for each harmony. They are shown here in ingoing-to-outgoing order: *a* uses only diatonic scales, *b* uses the same collections but hears the Lydian diminished scales over the diminished seventh chords, while *c* emphasizes more widely shifting diatonic collections and scale choices.

While this prismatic approach to analysis may have seemed excessive for the relatively insignificant passages in the last chapter where it was used, it will take a central role in our study of Rhythm changes. Because the harmonic structure of the tunes is so fluid, it is impossible to claim with any certainty that a particular set of changes constitutes some Platonic TUNE, in the same way that we might be able to for "Autumn Leaves" or "All the Things You Are." To fix a set of definitive changes for a particular passage is to misrepresent the fundamental nature of Rhythm tunes in jazz practice; the changes are often ill-defined even among the players themselves (as the above quotation from Levine attests). Engaging with a single Rhythm tune, then, constitutes an

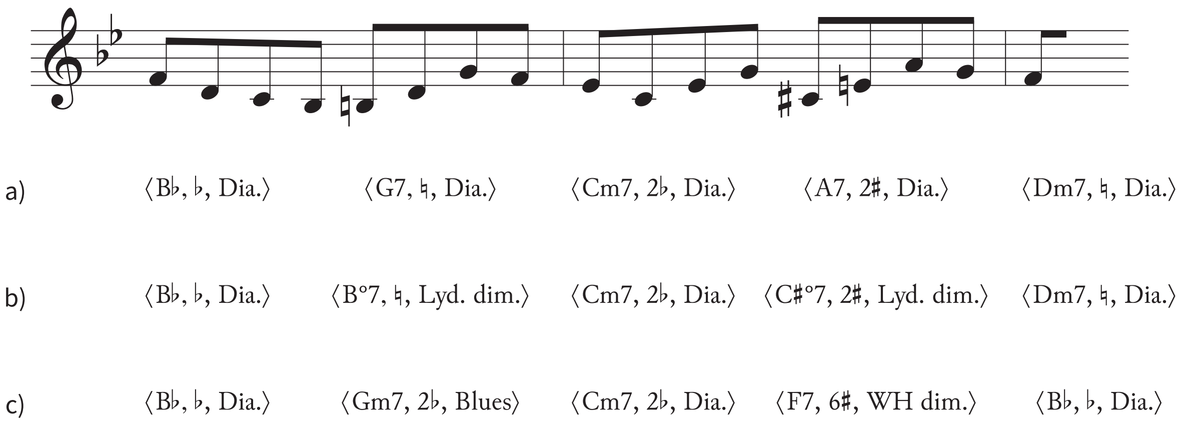
a) ⟨B♭, ♭, Dia.⟩ ⟨G7, ♮, Dia.⟩ ⟨Cm7, 2♭, Dia.⟩ ⟨A7, 2♯, Dia.⟩ ⟨Dm7, ♮, Dia.⟩
b) ⟨B♭, ♭, Dia.⟩ ⟨B°7, ♮, Lyd. dim.⟩ ⟨Cm7, 2♭, Dia.⟩ ⟨C♯°7, 2♯, Lyd. dim.⟩ ⟨Dm7, ♮, Dia.⟩
c) ⟨B♭, ♭, Dia.⟩ ⟨Gm7, 2♭, Blues⟩ ⟨Cm7, 2♭, Dia.⟩ ⟨F7, 6♯, WH dim.⟩ ⟨B♭, ♭, Dia.⟩

Figure 5.8. Three possible hearings of the opening of "The Serpent's Tooth."

engagement with an entire genre of tunes, with all their attendant history.[13] Transformational theory, with its ability to refract a passage into many possible interpretations, offers us a way into this rich network of harmonic possibilities inherent to the genre.

5.2 Thelonious Monk, "Rhythm-a-ning"

5.2.1 HEAD

Thelonious Monk's "Rhythm-a-ning" is a basic Rhythm tune, and as such will be an illustrative first example. The head of the tune is shown in Figure 5.9 as it appears in the *Thelonious Monk Fake Book*.[14] The source recording for this lead sheet is from Monk's album *Criss-Cross* (1963); we will analyze a different performance below, but the differences in the head are insignificant. What

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[13] A lack of understanding about the recorded history of jazz is often seen as something of a social mistake, especially in the case of Rhythm changes. Mark Levine tells a story of playing a Rhythm tune with Sonny Stitt, when Stitt began to play the cycle progression of Figure 5.2e over the A sections. Levine recounts: "after a couple of choruses, glares from Sonny, and a growing sense of feeling smaller and smaller, I finally 'strolled,' or stopped playing. After the set, I asked him what were the changes he was playing, and he growled 'just listen, man.'" This story is accompanied by a footnote that (based on the word of saxophonist Don Byas) attributes the cycle progression to pianist Art Tatum. Levine, *The Jazz Theory Book*, 242.

Phil Ford notes that this kind of secret knowledge is fundamental to understanding jazz as a part of an emerging hip culture in the 1940s–50s, which coincides with the time that Rhythm tunes began to proliferate. As he puts it, "'knowing the score' is what practically defines the hipster: *hip*, in its original meaning, means to be aware" ("Somewhere/Nowhere: Hipness as an Aesthetic," *The Musical Quarterly* 86, no. 1 [April 2002]: 54). We might instead read "knowing the changes," since to really know Rhythm changes involves knowing about the wide variety of harmonic possibilities that defines the genre.

[14] Don Sickler, ed., *Thelonious Monk Fake Book*, Steve Cardenas, transcriber (Milwaukee: Hal Leonard, 2002).

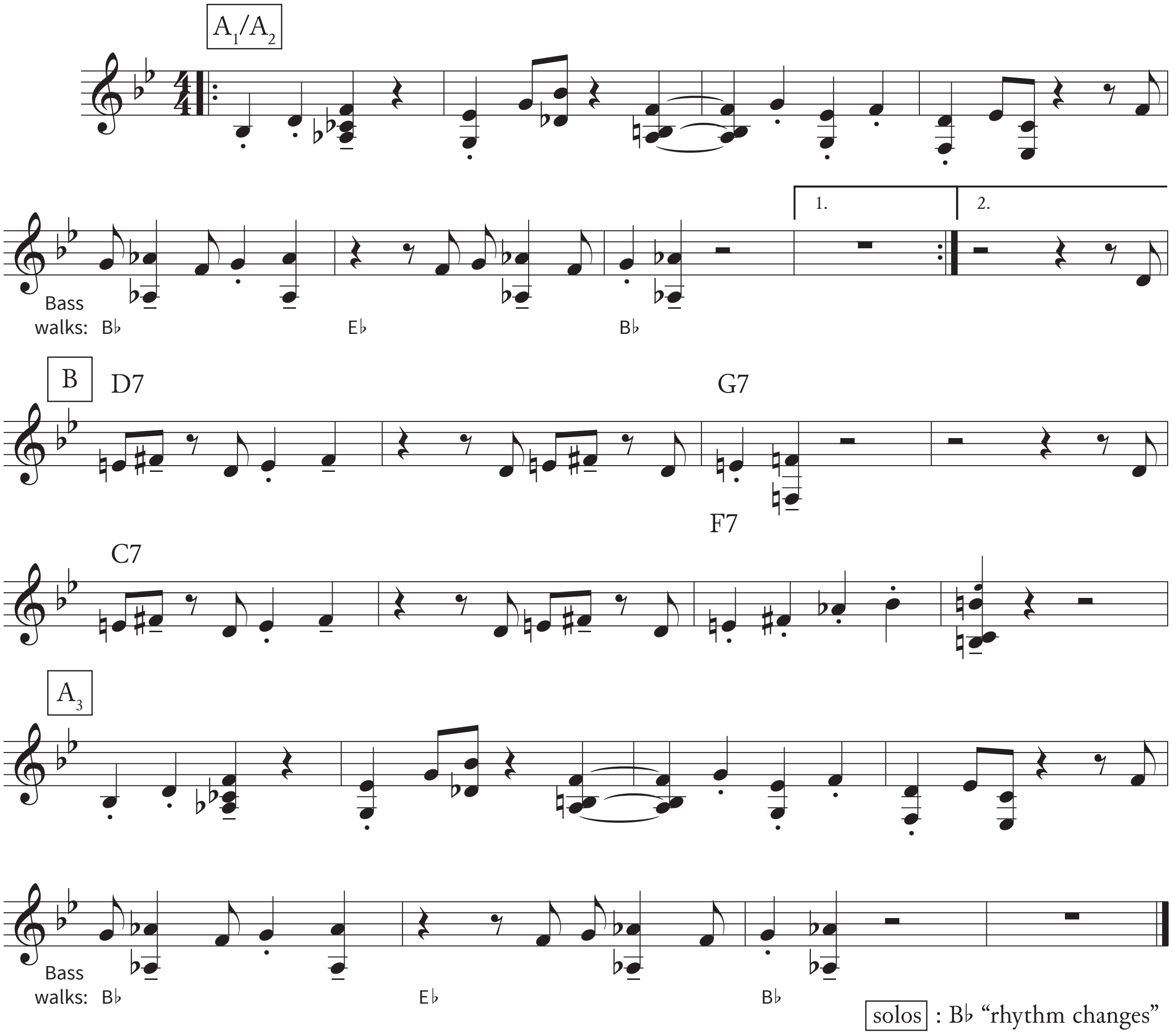
A1/A2
1.
2.
Bass walks: B♭
E♭
B♭
B
D7
G7
F7
C7
A3
Bass walks: B♭
E♭
B♭
solos : B♭ "rhythm changes"

Figure 5.9. Thelonious Monk, "Rhythm-a-ning," head.

is noteworthy about this lead sheet is that there are no changes given in the A sections; there is only an indication that the solos are to be played over Rhythm changes.[15] This speaks not only to the ubiquity of the form, but also to its fluidity, since a single definitive version is not given.

Nevertheless, there are a few aspects of the head that might have an impact on a soloist's harmonic choices. The first is the arpeggiation of an E♭ major triad in the second bar of the A sections. None of the common sets of changes in Figure 5.2 use E♭ in the second bar, but this plagal motion is essential to the tune.[16] The other important feature of the tune is the whole-tone ascent at the end of the bridge. Monk is well-known for his propensity towards the whole-tone scale, and we will see this manifest below in his solo on the tune.

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[15] The second volume of the old *Real Book* does give changes during the A sections, but they are somewhat inaccurate; in any case, it also includes the indication to "solo over Rhythm changes."

Instead of the *Criss-Cross* recording, we will instead focus our analytical attention on a live recording made in 1958, on the album *Thelonious in Action*.[17] This recording is attractive for a number of reasons. First, tenor saxophonist Johnny Griffin takes eleven full choruses on the tune, allowing the opportunity to analyze a somewhat longer selection of music than we did in the previous chapter. Second, after Griffin's second chorus, Monk does not play at all, leaving only the bass and drums to accompany the tenor saxophone.[18] This, combined with the ambiguity of Rhythm changes, provides something of a blank harmonic slate, leaving Griffin's improvised lines to do the bulk of the harmonic work.

5.2.2 Johnny Griffin's Harmonic Strategies

When approached with the fast-moving harmonies in the Rhythm A sections, Johnny Griffin's preferred strategy seems to be to ignore them: he frequently uses harmonic generalizations in the A sections. Often these generalizations are diatonic, using the 2♭ collection; Figure 5.10 gives a representative example from chorus $8A_2$. While we could perhaps imply a diatonic set of chord changes like those in Figure 5.2a–b, the rising arpeggios harmonizing the top line F_5–F_6 seem to take precedence over any particular harmonization. Similar rising diatonic patterns can be found in mm. 1–4 (chorus $1A_1$) and mm. 169–72 ($6A_2$).

Other times, Griffin plays passages that are nearly diatonic, but altered somewhat to fit an underlying harmony. Figure 5.11 gives an example from chorus $9A_1$ (chorus $11A_1$ is similar). In

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16. The harmonization Cm7–C♯°7 fits the melody, but does not appear in Monk's recordings, where the bassist consistently arrives on E♭ on the downbeat of the second bar.

17. Robert Hodson provides a similar analysis of the *Criss-Cross* recording (though not from a transformational perspective) in *Interaction, Improvisation, and Interplay in Jazz* (New York: Routledge, 2007), 66–74. His analysis focuses more strongly on the interactive elements of the performance than the harmonic ones.

18. Again, this is a common occurrence for Monk. During particularly good solos, he would rise from the piano and dance around the stage (an aspect of his performance on display throughout Charlotte Zwerin's documentary of Monk, *Thelonious Monk: Straight No Chaser* [Warner Bros., 1988], VHS). In the live recording here, he can occasionally be heard shouting words of encouragement to Griffin.

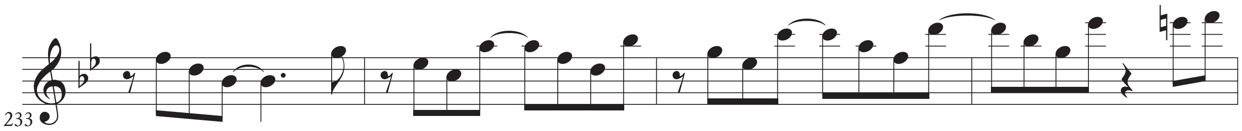

Figure 5.10. Diatonic harmonic generalization in Johnny Griffin's solo on "Rhythm-a-ning" (mm. 233–36, 4:01).

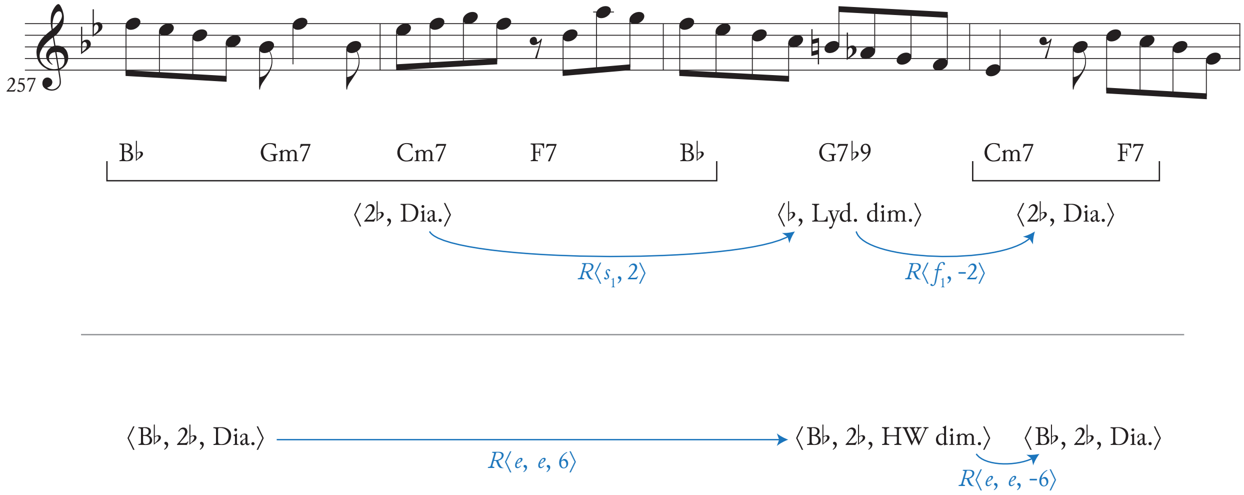

Figure 5.11. Altered diatonic generalization in Griffin's solo (mm. 257–60, 4:22), with two possible transformation networks.

this passage, the line is mostly diatonic, with the exception of the B♮ and A♭ in m. 259, implying a G7♭9 harmony. This passage, unlike the diatonic ascent in Figure 5.10, fits better with a diatonic chord progression, as shown in the upper transformation network. While it is certainly possible to hear a 2♭ diatonic swath throughout these four bars (represented in the lower network), hearing the half-note harmonic rhythm brings out the contrast between G minor in the first bar and the altered G dominant seventh in the third.

The most common harmonic generalization Griffin uses is the B♭ blues scale, which often appears in the last A section of a chorus. The clearest example of this is also the first, at the end of his third chorus; this passage is reproduced in Figure 5.12, and similar clear statements of the blues scale can be found in $4A_3$, $8A_3$, $10A_3$, and $11A_3$. Because Griffin generalizes the A sections so frequently, the two-bar harmonic rhythm of the bridge often sounds like an acceleration of

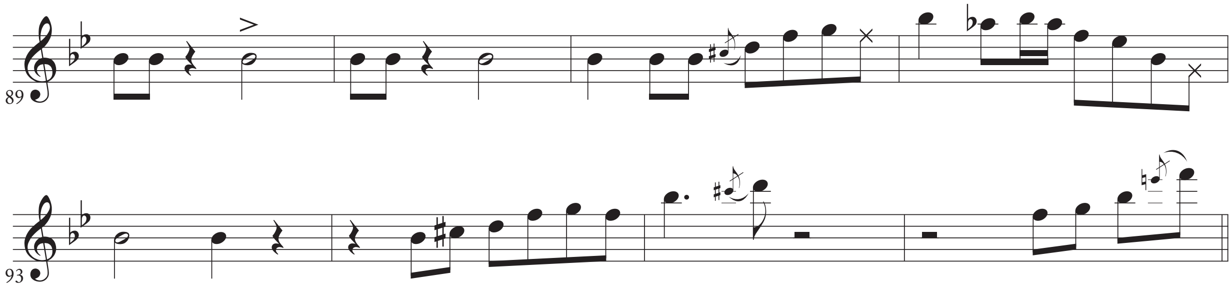

Figure 5.12. Griffin's blues generalization in a final A section (mm. 89–96, 1:55).

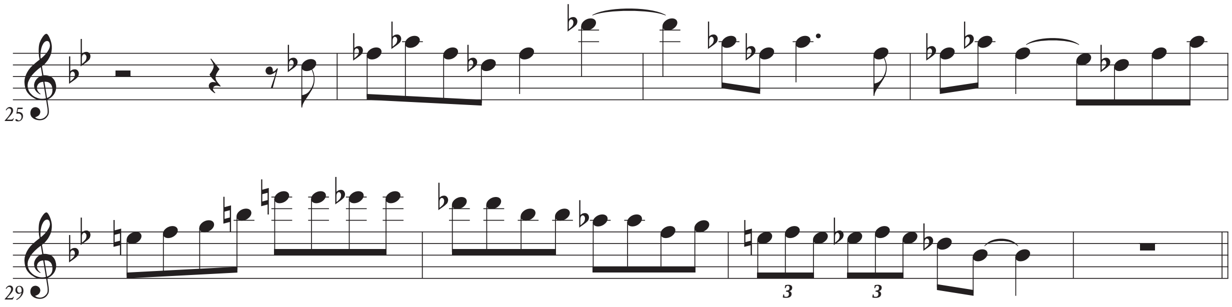

Figure 5.13. A D♭m triad as a B♭ blues subset in Griffin's solo (mm. 25–32, 0:58).

harmonic activity rather than its usual role as a relaxation of the half-note harmonic rhythm of the A sections. This blues generalization in the last A section of a chorus, then, helps to increase the contrast to the dominant cycle of the bridge.

The blues scale also provides an explanation for Griffin's seemingly unusual implication of D♭m at the end of the first chorus, shown in Figure 5.13. It is not immediately apparent how to understand the passage in mm. 25–28 (see Figure 5.14): Griffin could be superimposing D♭m over a B♭ diatonic progression, implying B♭m7♭5, or using a B♭ half–whole diminished scale generalization. Given his inclination for the blues scale in the last A section of the tune, though, my own hearing leans towards this D♭m triad as a subset of the B♭ blues scale. Griffin also emphasizes the pitches D♭ and A♭ at the end of his second and tenth choruses; the former implies D♭ major, while the latter leans more clearly toward B♭.

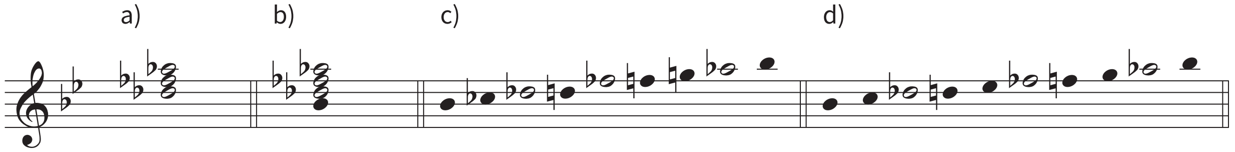

Figure 5.14. Four possible harmonic contexts for a D♭m triad: a pure triad; the upper tones of a B♭m7♭5; a diminished scale subset; and a blues scale subset.

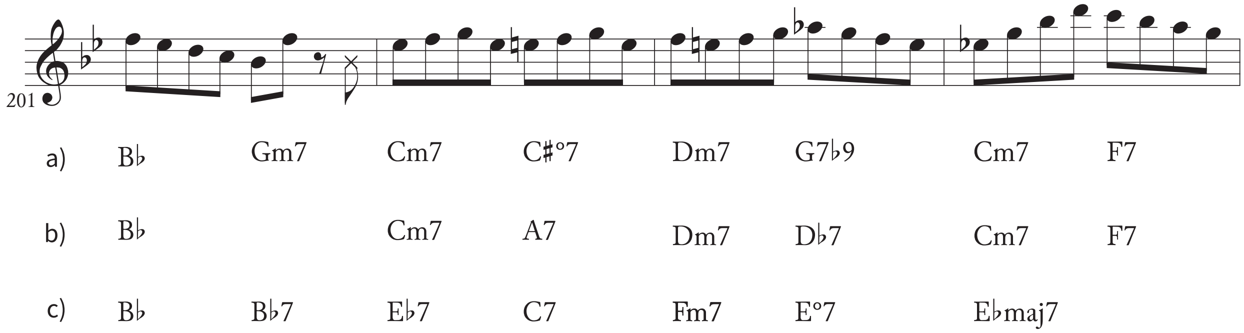
201
a) B♭ Gm7 Cm7 C♯°7 Dm7 G7♭9 Cm7 F7
b) B♭ Cm7 A7 Dm7 D♭7 Cm7 F7
c) B♭ B♭7 E♭7 C7 Fm7 E°7 E♭maj7

Figure 5.15. Half-note harmonic rhythm in an A section of Griffin's solo (mm. 201–4, 3:33).

Griffin does not always generalize the A sections; he sometimes plays the half-note harmonic rhythm of the tune itself. The clearest example of this occurs in chorus 7A₂, which is reproduced in Figure 5.15. While it seems clear that Griffin hears the half-note harmonic changes here, the melodic patterns he plays are harmonically ambiguous, owing to their limited range. While the progression shown in *a* is most likely—it combines the passing diminished seventh in the second bar with the applied dominant of C in the third—we might hear the harmonization at *b* instead, with A7 in the place of C♯°7 and a tritone substitution for G7♭9. Still other, less conventional, hearings are possible; letter *c* shows a hearing that moves to E♭, by first moving to E♭7 in the second bar (a nod towards the head's tilt towards the subdominant in the same formal location), and then via an applied dominant to a modified ii–V–I in E♭.

Admittedly, this last hearing may be difficult to discern, not least because it is so far from the typical first four bars of Rhythm changes. Another important reason, and one we have yet to consider, is the role of the other band members in shaping harmony. Indeed, the principal role of the rhythm section is to provide the harmonic framework for the soloists. Since Monk does not

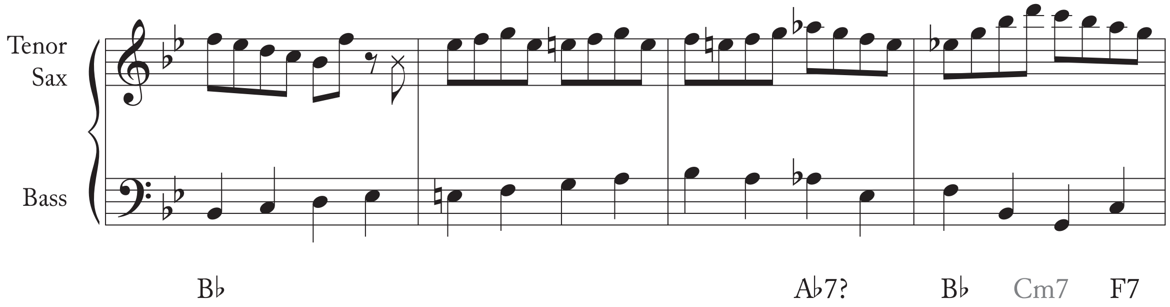

Figure 5.16. Griffin's improvised solo line, along with Ahmed Abdul-Malik's bass line (mm. 201–4, 3:33). Bass sounds as written.

play during Griffin's solo, we might look to bassist Ahmed Abdul-Malik's line during these four bars, which is shown in Figure 5.16.[19] Abdul-Malik does not seem to use the half-note harmonic rhythm here, and instead plays a generalization in the first two bars, walking up the B♭ major scale (or Lydian, depending on whether the E♭ or E♮ is heard as the chromatic pitch). The strong tonic–dominant motion from A♭ to E♭ in the third bar seems to imply some A♭ harmony, perhaps as a backdoor substitution to the downbeat B♭ in the fourth bar. He does gesture towards the home-key ii–V in the last bar: we might hear the B♭–G on beats 2–3 as a weak arpeggiation of Cm7, and the final C as a representative of F7 (which resolves to B♭ in the next bar). While this bass line certainly provides insight into Abdul-Malik's conception of the harmony of these four bars, it does not necessarily tell us anything more about *Griffin*'s harmonic understanding; it is entirely possible (and common, as here) that all band members do not share exactly the same harmonic framework, especially in a Rhythm tune.[20]

As we have noted before, our job as listeners and analysts is not necessarily to decide on a set of definitive changes. This ambiguity is a critical part of understanding exactly what jazz harmony *is*, and carries with it important epistemological questions—questions, incidentally, which relate to my own suspicion of the Schenkerian analysis of jazz first sketched in Section 1.2. If we take

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[19] A detailed account of exactly how walking bass lines project harmony is beyond the scope of this project; for a good overview, see Todd Coolman, *The Bottom Line: The Ultimate Bass Line Book* (New Albany, IN: Jamey Aebersold Jazz, 1990).

[20] There is much more to be said about the role of interaction in negotiating harmony in Rhythm tunes; we will return to this idea in more detail in the analysis of "The Eternal Triangle" in Section 5.4.

Abdul-Malik's bass line as *the* harmony, are we then to understand some of Griffin's note choices as incorrect? Or vice versa, if Griffin's solo line represents the true version of the harmony, why does Abdul-Malik choose to ignore it? In "Rhythm-a-ning," do the changes for the head (determined by whom?) hold through all of the solos, or is the harmonic framework considered anew in every chorus? In order to make a Schenkerian voice-leading sketch of the passage in Figure 5.15, we would be forced to contend with these issues, since determining what pitches are consonant (or more structural) is dependent on being able to identify the underlying harmony unambiguously.[21] My own contention is that the realities of jazz performance necessitate a more fluid theoretical conception of harmony; the prismatic transformational approach allows us to make these kinds of distinctions by presenting multiple transformation networks (representing multiple harmonic hearings) of a single passage.

Compared to his strategies for the A sections, Griffin's bridges are much less varied. In most choruses, he uses the standard 4-cycle bridge; Figure 5.17 gives an example from his final chorus. In this passage, Griffin repeats the rising arpeggio, altering it in each two-bar phrase to fit with the descending-fifths harmonic pattern.[22] He often provides additional harmonic interest by changing the scale to lead more strongly to the following harmony (not unlike the technique Joe Henderson used in the first four bars of his solo on "Isotope," examined in Section 4.3.3). Figure 5.18 gives a passage from Griffin's second chorus; here, both the D7 and G7 gain a ♭9 in their last two beats, while the F7 gets a ♯5 (or ♭13) in its final bar, which acts as a common-tone connection with the B♭ blues scale that follows in chorus 2A3. The other common alteration Griffin makes is the tritone substitution, as shown in Figure 5.19.

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[21] This difficulty is perhaps one of the reason that Steve Larson's book on the subject (*Analyzing Jazz: A Schenkerian Approach* [Hillsdale, NY: Pendragon Press, 2009]) focuses primarily on solo piano recordings of " 'Round Midnight," a piece in which the harmonies are well-defined—unlike Rhythm changes—and there are no other band members to muddy the waters. (Larson does include a live recording from Bill Evans and a partial transcription of a Bud Powell recording, each of which uses a piano trio, plus an "ensemble" recording from Evans's *Conversations with Myself*, a multi-track recording in which Evans acts as all three members of the ensemble.) Garrett Michaelsen critiques Larson on this same point, and also suggests that Larson overemphasizes the importance of harmony in general ("Analyzing Musical Interaction in Jazz Improvisations of the 1960s" [PhD diss., Indiana University, 2013], 9–10).

[22] The main interest in this particular passage is metric: Griffin superimposes a three-beat pattern over the quadruple meter starting in m. 341.

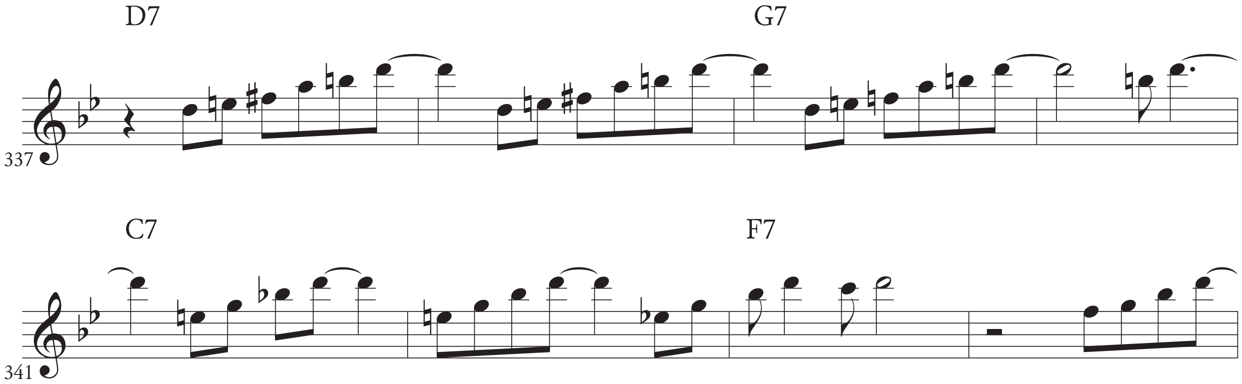
D7
G7
337
C7
F7
341

Figure 5.17. A 4-cycle bridge from Griffin's last chorus (mm. 337–44, 5:30).

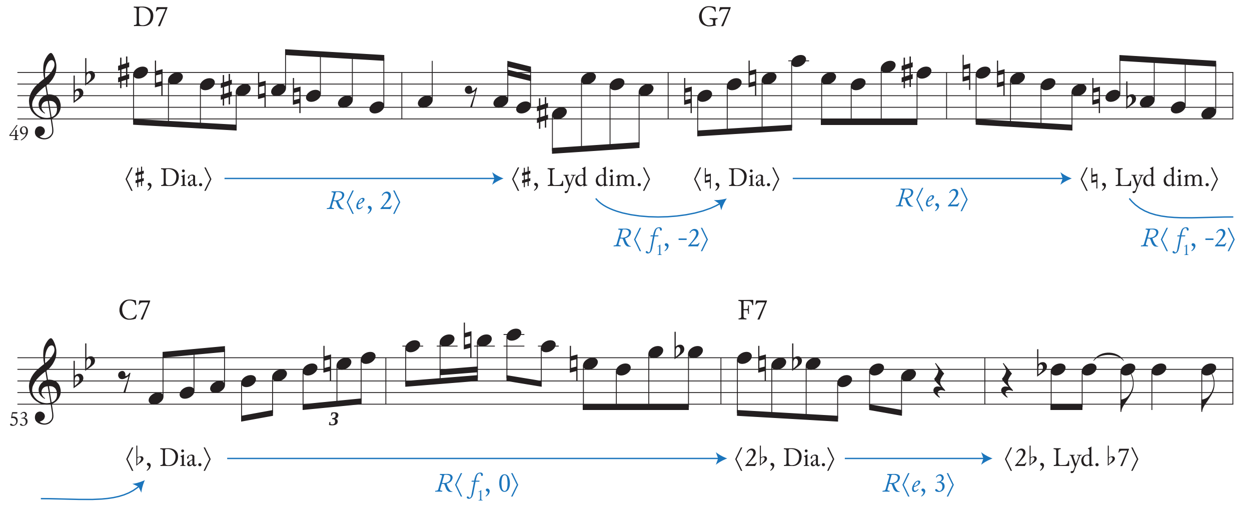
D7
G7
49
⟨♯, Dia.⟩
R⟨e, 2⟩
⟨♯, Lyd dim.⟩
R⟨f₁, -2⟩
⟨♮, Dia.⟩
R⟨e, 2⟩
⟨♮, Lyd dim.⟩
R⟨f₁, -2⟩
C7
F7
53
3
⟨♭, Dia.⟩
R⟨f₁, 0⟩
⟨2♭, Dia.⟩
R⟨e, 3⟩
⟨2♭, Lyd. ♭7⟩

Figure 5.18. A 4-cycle bridge, with chord-scale elaborations that lead more strongly toward the following harmony (mm. 49–56, 1:19).

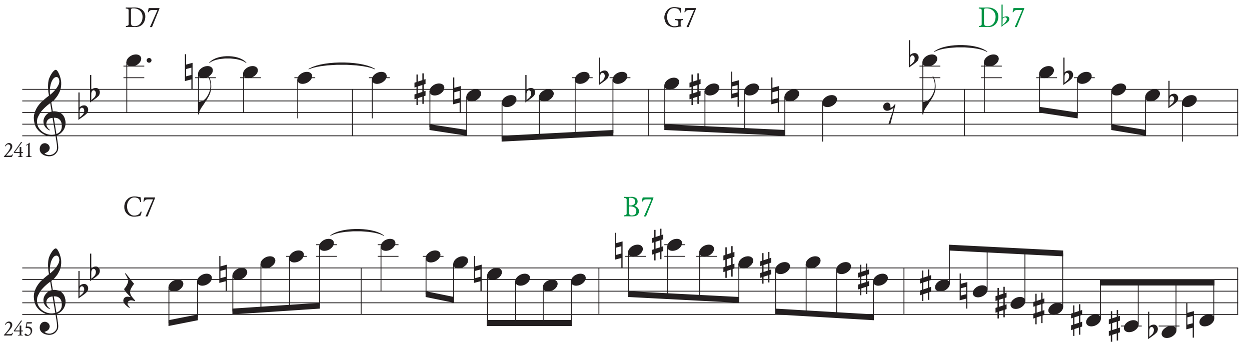
D7
G7
D♭7
241
C7
B7
245

Figure 5.19. The bridge from Griffin's eighth chorus, with tritone substitutions shown in green (mm. 241–48, 4:08).

5.2.3 Monk's Solo Harmony

While Griffin's tenor saxophone solo displays a number of interesting harmonic formations, Thelonious Monk's own solo on "Rhythm-a-ning" exhibits a few more, and is worth a brief visit here. Monk only plays three choruses on the tune, and his first is characteristically sparse. Throughout all three A sections of this first chorus (chorus 12 in the transcription), he plays only pitches from the B♭ pentatonic collection—the same collection he uses to comp behind Griffin's first three choruses.[23] The recurring rhythmic motive is altered slightly so that it fits the harmonies of the standard 4-cycle bridge in mm. 369–76, before returning to the B♭ pentatonic collection for the final eight bars of the chorus.

Beginning in chorus 13, Monk consistently plays an 8-chord dominant cycle in the A sections (the harmonization first seen in Figure 5.2e). Because this harmonization is so distinct from the ordinary Rhythm A section, Monk simply arpeggiates each chord to avoid blurring the overall progression. Playing a winding bebop line through the dominant cycle might risk the coherence of the substitution, especially if the bass player did not pick up on this harmonization and played a B♭ diatonic bass line.[24] Figure 5.20 reproduces chorus $13A_1$, showing the dominant cycle in the first four bars, followed by a B♭ blues harmonic generalization in the next four.

These dominant-cycle A sections are always paired with bridges that use the whole-tone scales. The head of "Rhythm-a-ning" uses the whole-tone scale in the bridge (clearly over the F7, and implied over C7 as well), and its use in Monk's solo helps to provide coherence to the performance as a whole. The bridge from chorus 14 is shown in Figure 5.21; though the whole-tone collection shifts between adjacent dominant seventh chords, the passage is harmonically consistent. This uniformity is easy to see in the chord-scale analysis: every change of harmony is represented by the transformation $R\langle T_5, f_1, 0\rangle$.

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[23] "Comping" is what jazz musicians call the act of ac**comp**anying (or **comp**lementing) a soloist; see Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 223–34.

[24] *Thelonious in Action* is a live recording made at the end of an eight-week run at the Five Spot Cafe in New York (Robin D. G. Kelley, *Thelonious Monk: The Life and Times of an American Original* [New York: Free Press, 2009], 242–43). Ahmed Abdul-Malik was used to playing with Monk by the time of the recording, and catches the dominant-cycle A section almost immediately; Monk's strong left-hand entrance on F♯ at m. 385 removes any doubt as to the progression that will follow.

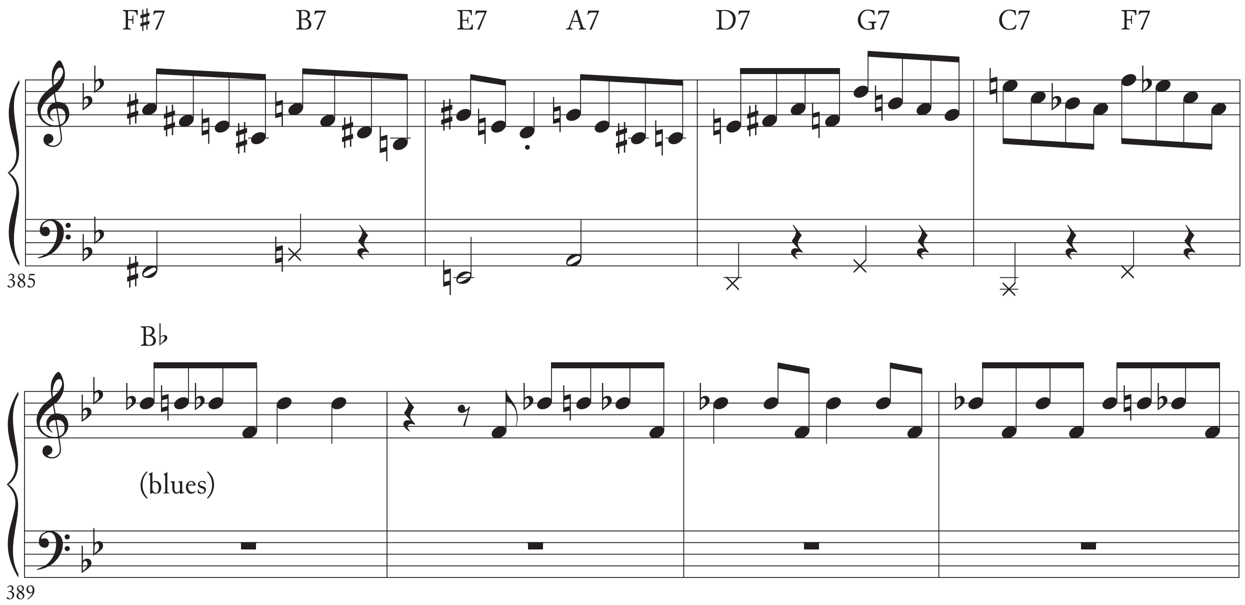
F♯7
B7
E7
A7
D7
G7
C7
F7
385
B♭
(blues)
389

Figure 5.20. Monk’s dominant-cycle A section and blues generalization from chorus 13A_1 (mm. 385–92, 6:12).

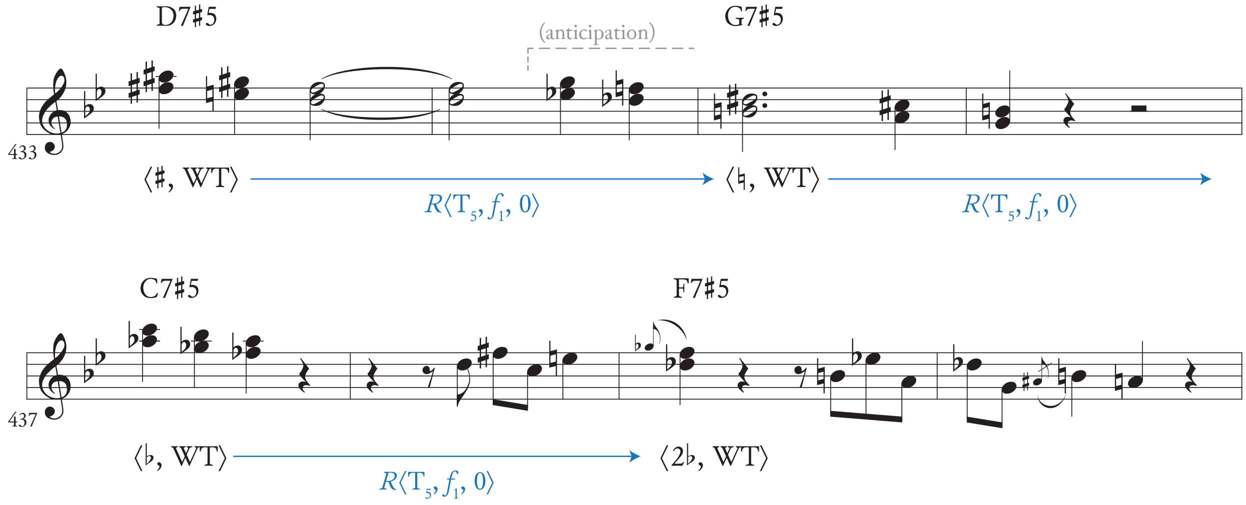
D7♯5
(anticipation)
G7♯5
433
⟨♯, WT⟩
R⟨T5, f1, 0⟩
⟨♮, WT⟩
R⟨T5, f1, 0⟩
C7♯5
F7♯5
437
⟨♭, WT⟩
R⟨T5, f1, 0⟩
⟨2♭, WT⟩

Figure 5.21. Monk’s bridge from chorus 14, using the whole-tone scale (mm. 433–40, 6:53).

This brief analysis of "Rhythm-a-ning" has illustrated that no single set of changes can adequately describe this tune. While in a more standard tune like "Autumn Leaves," individual chords might change slightly (adding an extension or using a tritone substitution), the basic progression remains intact; rarely do we encounter a situation like that of Monk's 8-cycle A section, where an entire set is replaced with another. This mix-and-match approach to harmony is an essential element of Rhythm tunes. Though the dozens of Rhythm contrafacts are all based on a single chord progression, the wide range of harmonic approaches means that no two Rhythm tunes sound exactly alike.

5.3 George Coleman, "Lo-Joe"

To this point in this study, the analyses have focused on harmony as reflected in a lead sheet, or how particular performances confirm (or contradict) these given lead-sheet harmonies. This approach naturally requires a lead sheet to exist in the first place, which is not always the case. Even in cases where one does exist, there are several reasons a jazz musician might want to create a lead sheet anew: it may be inaccurate (often the case with fake books); it may not reflect a particular recording the musician wants to emulate (John Coltrane's recording of "Body and Soul," for example, does not use the standard changes); or the musician may simply want to practice ear training.

In these cases, the transformational approach to harmony can be used "in reverse," so to speak; rather than analyzing how a soloist elaborates on a given set of changes, we can take the raw material of a recording and deduce a likely set of changes. This section will do just that, using George Coleman's composition "Lo-Joe," recorded on the album *Amsterdam After Dark* (1979).[25] "Lo-Joe" was recorded somewhat later than the other tunes analyzed here, and as such is somewhat more harmonically adventurous. It is recognizably a Rhythm tune, though with a highly altered bridge, and in the key of D♭ rather than the usual B♭.

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[25] I am grateful to Prof. Tom Walsh for bringing this tune to my attention.

Before beginning with the analysis, a few disclaimers are in order. While a transcription of the head (complete with piano and bass parts) can be found in Appendix B on p. 236, I want to emphasize the fact that a full transcription is in general *not* necessary to create a lead sheet, and is provided here only as an expedient to writing about the process. A skilled jazz musician would likely transcribe only the melody, and determine the harmonies simply by ear, without necessarily writing anything down. Next, there is the question of whether or not Coleman and his bandmates ever played from a lead sheet at all; might we be manufacturing a somehow "false" lead sheet rather than "reconstructing" one? This question is not important for our purposes here, since lead sheets are such a common way of conveying jazz tunes. Even if Coleman did not give his bandmates a lead sheet, he must have had *some* means of communicating the harmonic progression of the tune, and a lead sheet is the canonical way to notate this kind of progression in jazz.

The opening of melody of "Lo-Joe," shown in Figure 5.22, appears to be straightforward, outlining mostly major triads and major seventh chords. The resulting succession of harmonies, though, does not seem to reflect any of the usual Rhythm openings, or indeed any ordinary jazz progression at all. When combined with the ensemble, though, it becomes clear that the melody consists primarily of upper extensions to harmonies.[26] Figure 5.23 gives the same passage from the second A section along with the piano and bass parts.[27] On the downbeats of the first and third bars, bassist Sam Jones plays a D♭, and pianist Hilton Ruiz plays an identical voicing. Combined with the knowledge of standard Rhythm A sections, we can be relatively confident that the harmony here is the tonic D♭maj7; Coleman's opening figure in ascending fourths is then understood as outlining the 13th, 9th, and 5th of this chord.[28]

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[26] This focus on melodic upper extensions becomes more common after the bebop era; Robert Hodson discusses this (in connection with how players create individual melodic profiles) in *Interaction, Improvisation, and Interplay*, 42–46.

[27] The second A section is given here only because there is more activity in the piano, providing more harmonic information. Because there are three nearly identical A sections in Rhythm tunes, they can generally be exchanged freely. In this analysis, only one A section is usually given, but the reader is encouraged to compare the corresponding locations in the other two A sections.

[28] One of the hallmarks of later (post-1960) jazz is the increase of the use of perfect fourths, especially in piano voicings; see Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 357–60 for an overview of this practice.

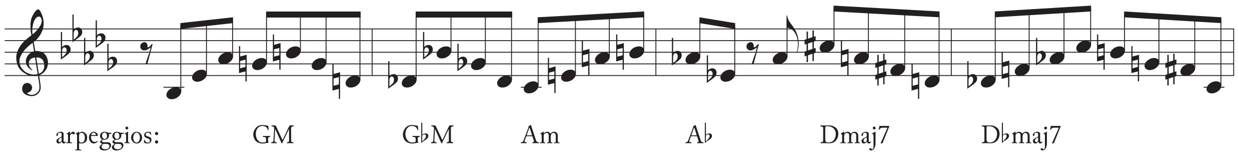
arpeggios:
GM
G♭M
Am
A♭
Dmaj7
D♭maj7

Figure 5.22. George Coleman, “Lo-Joe,” melody, mm. 1–4.

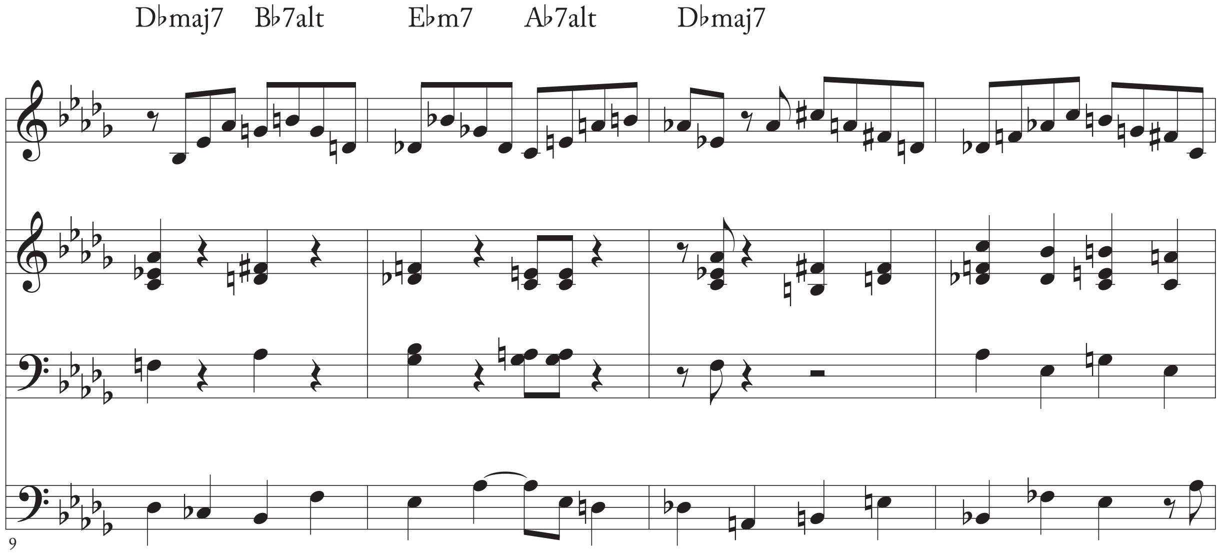
D♭maj7
B♭7alt
E♭m7
A♭7alt
D♭maj7
9

Figure 5.23. The first four bars of the A section of “Lo-Joe,” with ensemble (mm. 9–12).

The third-bar tonic is preceded, as usual, by a ii–V progression, but the melody over the A♭7 contains the pitches E♮, A♮, and B♮. These pitches can be understood as the ♯5 (or ♭13), ♭9, and ♯9, of the chord, which are all representative extensions of the altered chord (hereafter, 7alt).[29] Depending on our analytical priorities, we might analyze this harmony as A♭7alt (in which case the extended tones are first-class chord members) or as a diatonic A♭7 in which the melody notes were part of an outgoing scale choice: ⟨A♭7, 4♯, Lyd. dim.⟩.[30] In either case, understanding the A♭7 helps to understand the harmony in the previous bar, which is a B♭7, again with ♯5 in Ruiz's voicing and ♭9 in the melody (and, in section A_3, with ♯9 in the voicing). The first two bars of the A section, then, contain a standard I–VI–ii–V progression, with both dominant sevenths played as 7alt chords.

The next two bars are perhaps the most difficult in the entire piece; Figure 5.24 provides these two bars from all three A sections of the head. While the rhythm section pitches are mostly consistent, the harmony is not so clear. The starting and ending points are stable, and correspond with ordinary Rhythm changes, with D♭maj7 in bar 3 of the A section moving to D♭7 in bar 5. Beat three following the D♭maj7 seems to be Bm7 (arpeggiated in the melody), but at this point Ruiz seems to double the harmonic rhythm, playing four chords in the last bar of this passage. Jones's bass line is also very consistent here, but it is unclear whether this acceleration of the harmonic rhythm is real or only a surface elaboration.[31]

Figure 5.25 gives several interpretations of this passage. All three of the interpretations begin with D♭maj7 and end with D♭7, with Bm7 on beat 3 of the first bar. Letter *a* conforms most strongly with the melody, but the progression is unusual: there is a chromatic slipping effect from Bm7 to B♭m7 that prefigures the chromatic motion to the tonic via a tritone-substituted

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[29] Recall that the "altered chord" has a specific meaning in jazz, as a dominant that includes both ♯9 and ♭9 (along with other possible extensions). To avoid confusion, and because they are prevalent throughout "Lo-Joe," in this analysis the altered chord will be called "7alt."

[30] The unusual signature designation here is an artifact of Russell's categorizations: normally dominant chords have the parent Lydian tonic a whole-step below their roots (the parent Lydian tonic of G7 is F), but for 7alt chords the Lydian tonic is a half-step above (the parent of G7alt is A♭). Here, the chord is effectively A♭7alt (given the E♮, A♮, and B♮ in the melody), which Russell would analyze as part of the A Lydian diminished scale (A–B–C♮–D♯–E–F♯–G♯, or enharmonically starting on A♭: A♭–A♮–B♮–C–D♯–E♮–G♭).

[31] During the solos, there there are consistently only two chords in this bar, but then again, "Lo-Joe" could be a tune in which there are separate sets of head and solo changes.

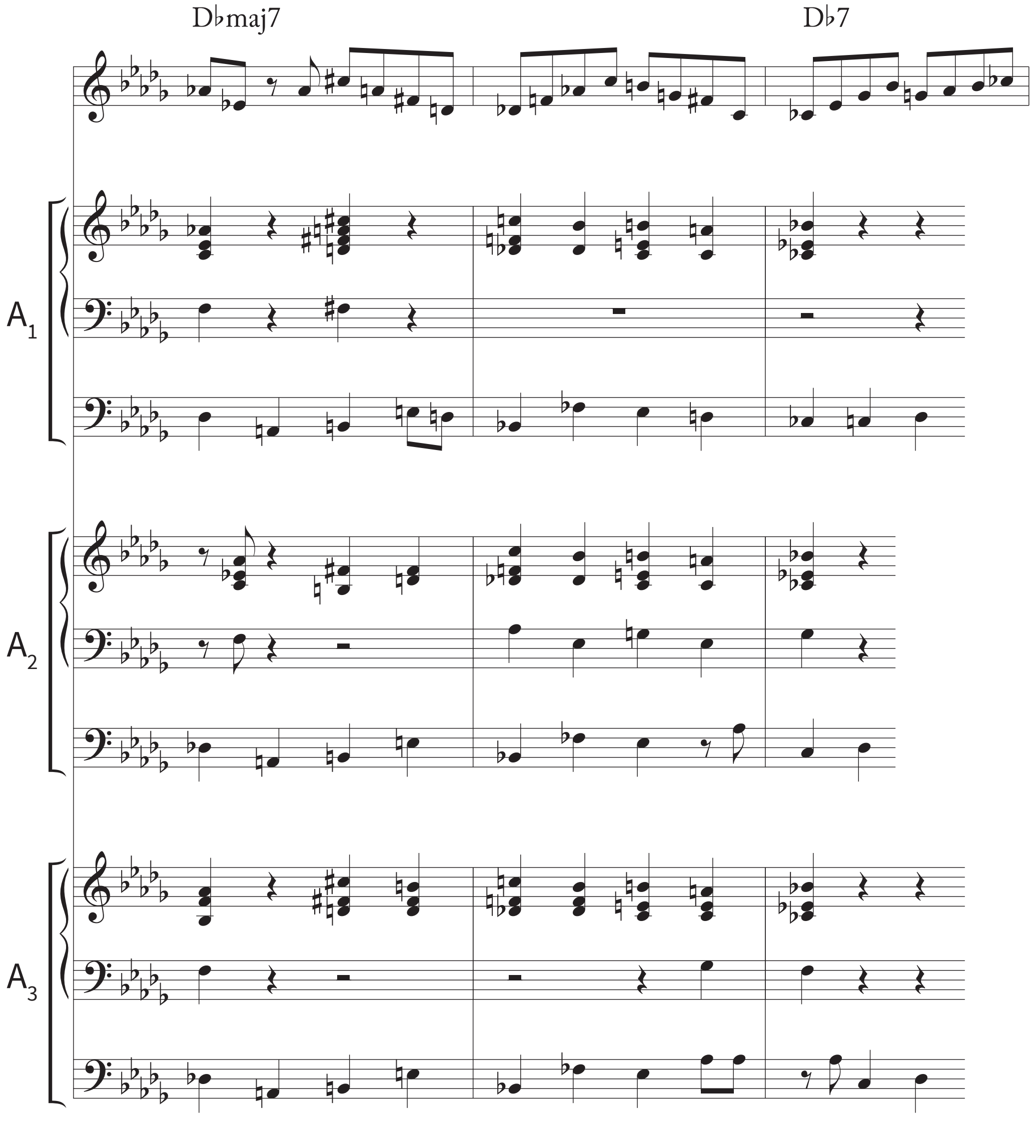
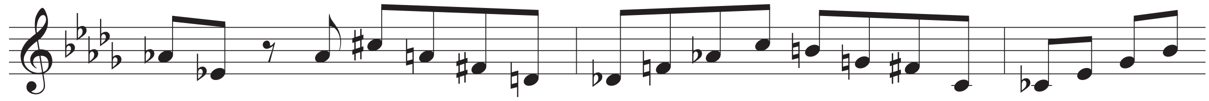


Figure 5.24. The third and fourth bars of all three A sections of "Lo-Joe." The melody is shown on the top staff, and each grouping of three staves below contains piano and bass parts. Each group ends when Jones's bass reaches the tonic D♭.



a)	D♭maj7		Bm7		B♭m7		D7		D♭7
b)	D♭maj7	A7	Bm7	E7	B♭m7	E7	E♭7	A♭7	D♭7
c)	D♭maj7		Bm7		A7alt		A♭7alt		D♭7

Figure 5.25. Possible interpretations of the preceding passage.

dominant, D7–D♭maj7. Letter *b* is a hearing with doubled harmonic rhythm that follows the bass line. The progression here makes more harmonic sense, featuring mostly ii–V progressions, though the melodic support for some of the chords—the A7 and both E7s—is weak at best. Letter *c* focuses on the piano line, returning to two chords per bar. In this hearing the top notes of the voicings in the second bar are heard in both cases as ♯9 moving to ♭9: a logical hearing, but one that is not strongly supported by the melody or bass line. None of these hearings seem to fit the music perfectly, but each does fit some aspect of it. As we have seen before, it is of little analytical use to decide on a single "true" analysis, though it does have consequences for our imaginary lead sheet author (who must put a set of changes with this melody). This may be a passage in which a lead sheet is not sufficient; no single fixed interpretation can adequately capture the essence of this harmonic motion. Only in their interaction (and in the recording itself) does a full picture of the harmony emerge.

The second half of the A section is much simpler, though the first A section ends differently than the other two. (This is typical of Rhythm tunes, and would probably be notated on a lead sheet as first and second endings, as in "Rhythm-a-ning" in Figure 5.9.) The fifth bar of the A section contains a D♭7 chord, which moves to the subdominant G♭maj7 in the following measure. The second half of m. 6 moves to some kind of C♭ chord, though Ruiz's piano voicings in the head are unhelpful in determining its quality (in the solos it is usually played as a dominant seventh).

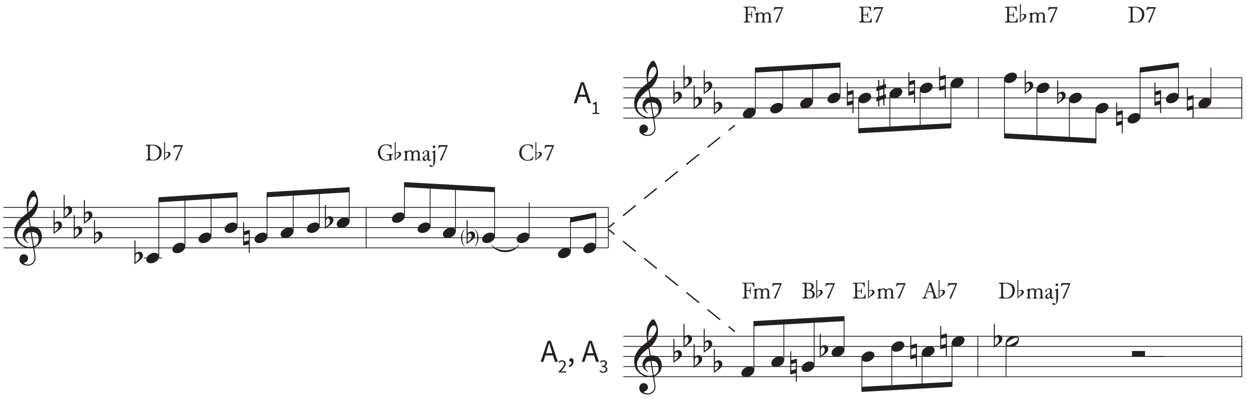


Figure 5.26. A section endings in "Lo-Joe."

The first A section then moves to a tritone-substituted turnaround, while the other two double the harmonic rhythm to arrive on tonic in the eighth bar (see Figure 5.26).

The bridge of "Lo-Joe" is its most distinctive feature, and is given with chord changes in Figure 5.27. This bridge is clearly inspired by the last half of the bridge of "Eternal Triangle" (discussed in the next section), and its sequential nature is helpful to our imagined lead-sheet author: once a single bar is determined, it can simply be transposed to all of the others. Here, each bar contains a single ii–V progression, made explicit in the bass and with basic three- and four-note voicings in the piano.[32]

This bridge is phenomenologically rich: the ii–V progressions themselves are clear, but the connections between them admit of multiple possibilities (as a listener, or for an improvising musician). Figure 5.28 gives several possible transformation networks for the first half of bridge of "Lo-Joe". The analysis at *a* is a "horizontal" one: do a ii–V, move up a tritone and do another, move down a half-step and do it again, and so on. Letter *b* emphasizes the descending fifth motion: play a ii–V and its tritone transposition, then move down a fifth and repeat. This hearing respects the descending fifths present in the standard Rhythm bridge, and also reflects the organization of ii–V space, shown (in letter *c*). Letter *d* highlights the tritone relationships between bars, and also encourages hearing the 3RD transformation connection between F7–Fm7

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[32] This progression is given (in the key of B♭) in Aebersold, *"I Got Rhythm" Changes in All Keys*, 27.

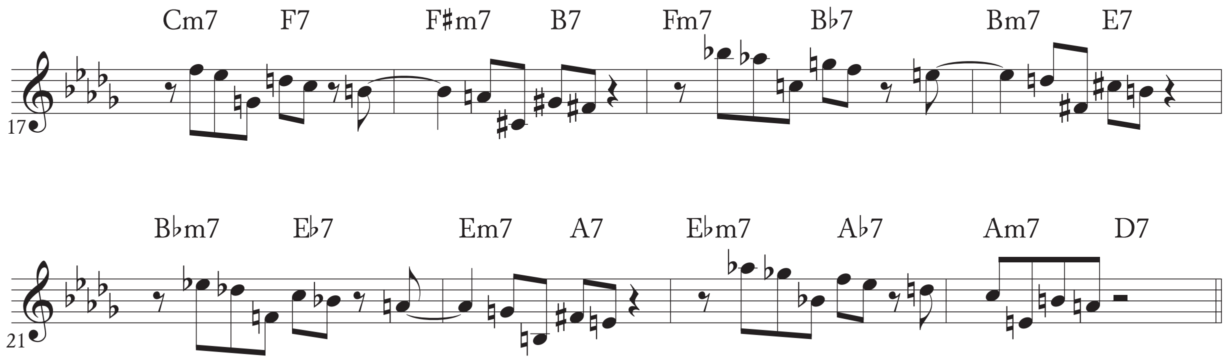
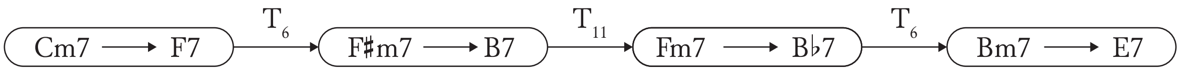

Cm7
F7
F♯m7
B7
Fm7
B♭7
Bm7
E7
17
B♭m7
E♭7
Em7
A7
E♭m7
A♭7
Am7
D7
21

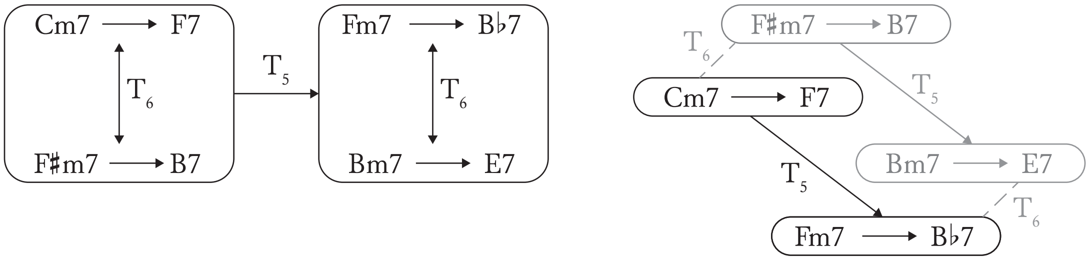
Figure 5.27. “Lo-Joe,” bridge (mm. 17–24).

a) a “horizontal” reading


Cm7 → F7
T6
F♯m7 → B7
T11
Fm7 → B♭7
T6
Bm7 → E7

b) emphasizing T_5

c) cf. ii–V space


Cm7 → F7
T6
F♯m7 → B7
T5
Fm7 → B♭7
T6
Bm7 → E7
F♯m7 → B7
T6
Cm7 → F7
T5
Bm7 → E7
T5
T6
Fm7 → B♭7

d) emphasizing T_6

e) emphasizing T_{11}

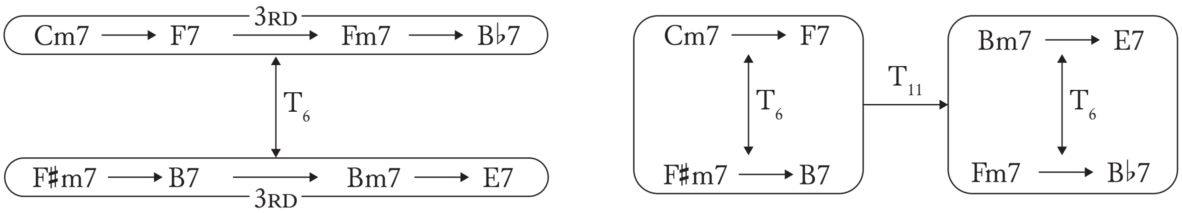

Cm7 → F7
3RD
Fm7 → B♭7
T6
F♯m7 → B7
Bm7 → E7
3RD
Cm7 → F7
T6
F♯m7 → B7
T11
Bm7 → E7
T6
Fm7 → B♭7

Figure 5.28. Several possible transformation networks of the bridge (first four bars). Unlabeled arrows indicate the TF transformation.

and B7–Bm7 chords. Hearing the bridge this way allows hearing as if the music is bouncing back and forth between two normal Rhythm bridges, one in D♭, the other in G. Network *e* emphasizes the half-step relationships, and encourages a connection between the first and last pairs of chords and the central two.

It may seem as though transformations themselves have not played an important role in reconstructing a lead sheet for "Lo-Joe." This reconstruction, though, has taken place against the background of the musical spaces developed in the earlier chapters of this study (all of which have transformations as their logical basis). These musical spaces provide a mostly-unseen structuring principle to the analytical work in this section. Because the spaces were developed to demonstrate functional jazz harmony, to recognize that "Lo-Joe" uses functional harmony is to recognize that it likely reflects an orderly representation in (say) ii–V space. This, combined with the knowledge of Rhythm changes in general, means that we could easily reject the triadic analysis in Figure 5.22 as a nonsensical jazz progression. This notion of syntax is one that is often implicit in the construction of the spaces, but comes to the fore when used for the kinds of harmonic determination done here.[33]

5.4 Sonny Stitt, "The Eternal Triangle"

5.4.1 HARMONIC PECULIARITIES

The final Rhythm tune of this chapter was the inspiration for the bridge of "Lo-Joe": Sonny Stitt's "The Eternal Triangle." The canonical recording appears on Dizzy Gillespie's album *Sonny Side Up* (1957), featuring Stitt along with Sonny Rollins, both on tenor saxophone. The album is widely regarded as one of the best "jam session" albums in jazz, and "Eternal Triangle" is often singled out as the standout performance of the record.[34] This two-tenor format will allow us the opportunity to explore more deeply the role of interaction between players in shaping harmony.

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[33] It is also a concept that is absent from other prevalent theories of jazz harmony: it is easy to imagine an analysis in which the arpeggiated triads and seventh chords of the melody of "Lo-Joe" form coherent voice-leading strands to and from structural tonic chord members, for example.

[34] See, for example, Stephen Cook's review on *AllMusic*, where he notes that on "'The Eternal Triangle,' in particular, Stitt and Rollins impress in their roles as tenor titans . . . an embarrassment of solo riches comes tumbling

First, though, a brief analysis of the tune itself is in order. The head of "Eternal Triangle" is shown in Figure 5.29; the A sections are standard Rhythm changes, featuring fast-moving bebop melodic lines. The B section, though, is unique to this tune, and features ii–V progressions descending by half-step. We might imagine this bridge as being derived from the standard Rhythm bridge, as shown in Figure 5.30. In the first step, the typical III–VI–II–V is compressed into the second half of the bridge. To preserve the correct length, Stitt extends the fifths cycle backward by two chords to E7, maintaining the original harmonic rhythm of one chord every two bars, increasing the harmonic work done by the bridge (and consequently, the area of ii–V space it traverses). In the next step of the derivation, each dominant seventh is replaced by a ii–V progression; finally, every other ii–V progression is replaced with its tritone substitute, resulting in the chromatic descent of the bridge itself.

Because the A sections of "Eternal Triangle" use typical harmonies, both Rollins's and Stitt's solos display many of the same solo approaches we saw in Griffin's solo on "Rhythm-a-ning" above. Both players use harmonic generalizations of various types: diatonic (choruses $2A_3$, $5A_3$, $8A_2$, and $11A_1$, for example); blues ($4A_1$, $9A_2$, $11A_3$); and other scales (the half-whole diminished scale in $9A_1$ and $12A_2$). Stitt in particular emphasizes the half-note harmonic rhythm of the A sections, often playing bebop lines that change accidentals frequently to highlight the harmonic shifts (see choruses $6A_3$ and $10A_2$, for example). Many other common harmonic devices can also be found, including tritone substitutions (mm. 133) and CESH (mm. 33–35, 193, and 395–96), in which a chromatic melodic line embellishes a single unchanging harmony (see p. 147n89).

One harmonic aspect of Rollins's solo does deserve special mention, as it has not yet appeared in the analytical examples. Figure 5.31 shows the fifth and sixth bars of an A section, where we would normally expect the harmonies B♭7–E♭7. Here, Rollins clearly arpeggiates a Bm7 instead; this is a feature which is often called "side-slipping" or "side-stepping."[35] The overall harmony of this bar is B♭7 (sometimes with its preceding ii^7), but here Rollins plays a harmony a half-step

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out of both these men's horns" (Stephen Cook, Review of Dizzy Gillespie, *Sonny Side Up*, *AllMusic.com*, accessed July 13, 2015, http://www.allmusic.com/album/sonny-side-up-mw0000188698).

[35] Jerry Coker defines side-slipping as "deliberately playing 'out-of-the-key' for the sake of creating tension" (*Elements of the Jazz Language for the Developing Improvisor* [Miami: Belwin, 1991], 83).

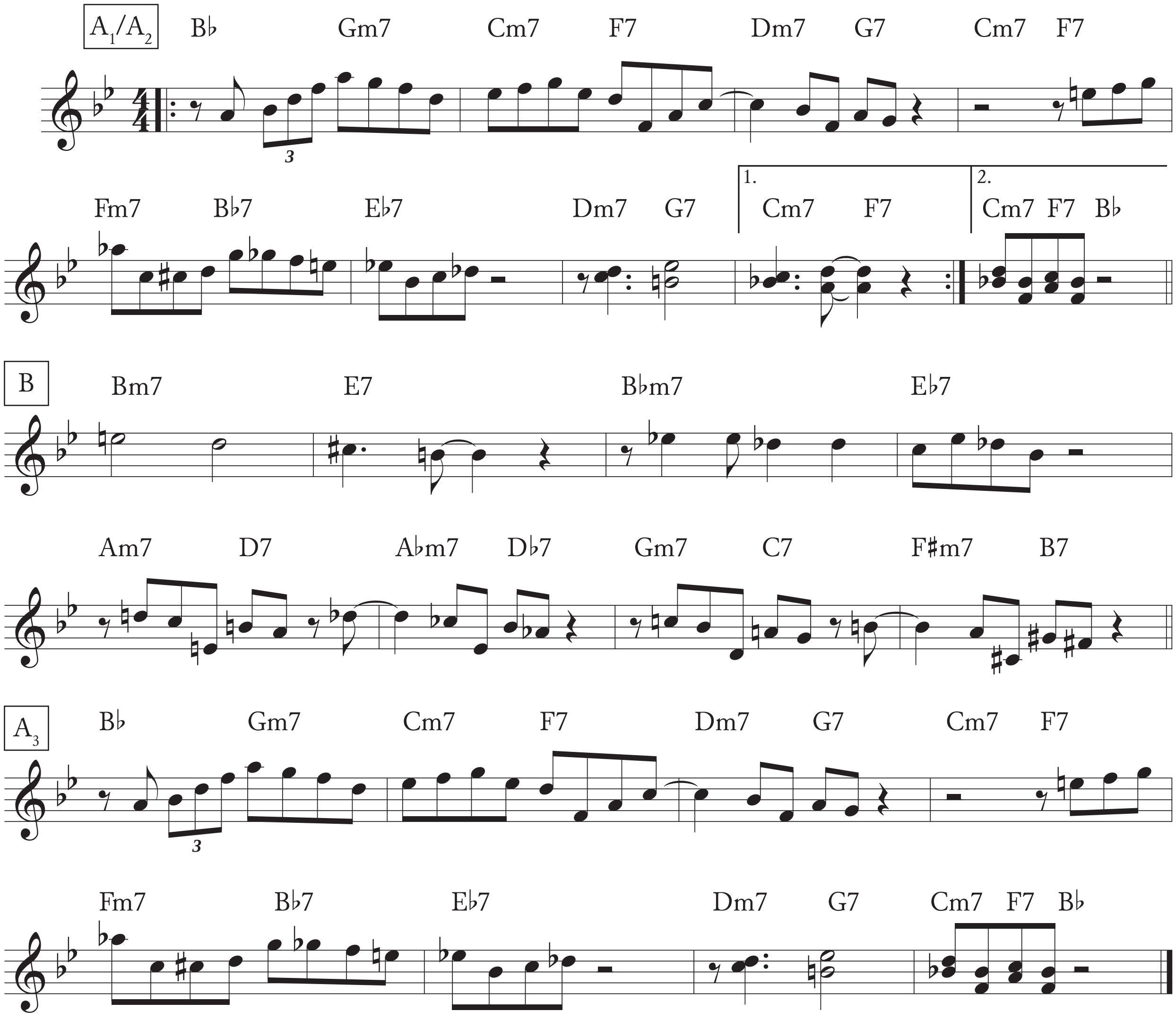
A$_1$/A$_2$
B♭
Gm7
Cm7
F7
Dm7
G7
Cm7
F7
3
Fm7
B♭7
E♭7
Dm7
G7
1.
Cm7
F7
2.
Cm7
F7
B♭
B
Bm7
E7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
Gm7
C7
F♯m7
B7
A$_3$
B♭
Gm7
Cm7
F7
Dm7
G7
Cm7
F7
3
Fm7
B♭7
E♭7
Dm7
G7
Cm7
F7
B♭

Figure 5.29. Sonny Stitt, “The Eternal Triangle,” head.

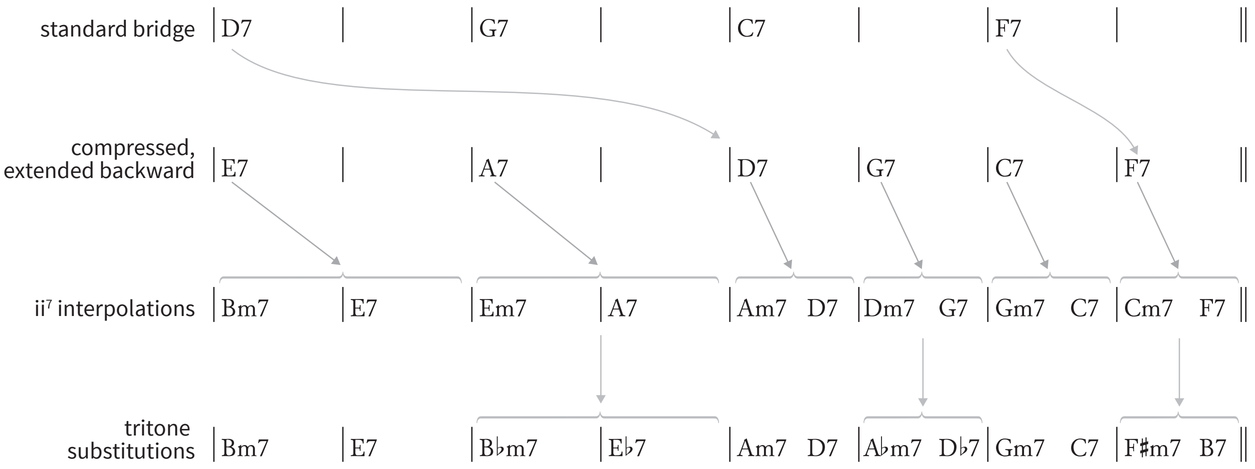
standard bridge
D7
G7
C7
F7
compressed, extended backward
E7
A7
D7
G7
C7
F7
ii7 interpolations
Bm7
E7
Em7
A7
Am7
D7
Dm7
G7
Gm7
C7
Cm7
F7
tritone substitutions
Bm7
E7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
Gm7
C7
F♯m7
B7

Figure 5.30. Derivation of the bridge of "Eternal Triangle" from a standard Rhythm bridge.

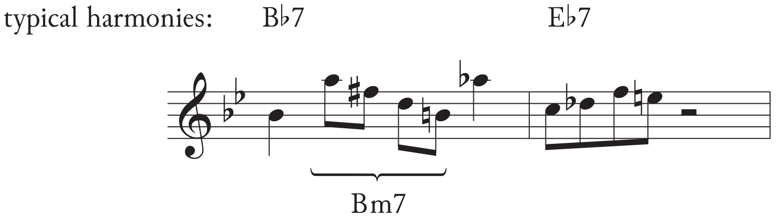
typical harmonies:
B♭7
E♭7
Bm7

Figure 5.31. Side-slipping in Sonny Rollins's solo (mm. 45–46, 1:15).

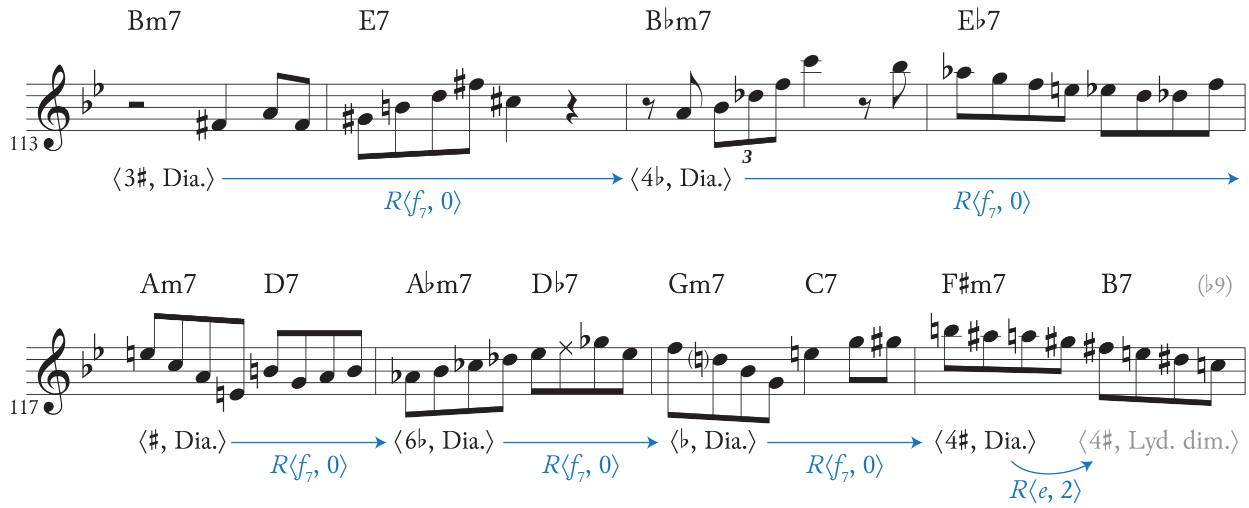

Figure 5.32. A non-sequential bridge from Sonny Rollins's solo (mm. 113–20, 2:12).

away. The motion from B♭7 to Bm7 is distant in ii–V space, though is closely related to the SLIDE_7 transformation (B♭7 $\xrightarrow{7\text{TH}^{-1} \bullet \text{SLIDE}_7}$ Bm7).[36] It is also a convincing way of playing "outside," which is what jazz musicians call improvised lines that do not seem to connect with the underlying harmony.[37] Outside playing becomes an important feature of more modern jazz improvisations, but it also features prominently in this recording when Rollins and Stitt begin trading (a section to which we will return below).

Given that the bridge of "Eternal Triangle" is its most interesting feature, it will be worthwhile to examine the solo strategies of Rollins and Stitt separately before moving on to discuss how the two interact with each other. These strategies can be broken down into two basic types: sequential and non-sequential solo approaches. The non-sequential solo approach to this bridge is less common, and a single example should suffice. Figure 5.32 gives the bridge from Rollins's fourth chorus; because the harmonies are so fast-moving, he uses mostly diatonic scales. Though the melodic line is not sequential, Rollins's shifting diatonic palette works in conjunction

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[36] Dmitri Tymoczko notes that side-slipping usually demonstrates efficient voice-leading; he also emphasizes its role in creating polychordal or polytonal effects; see *A Geometry of Music*, 374–78.

[37] I suspect that the "outside" terminology is related to Russell's "outgoing" scale choices, but I have not been able to corroborate this suspicion.

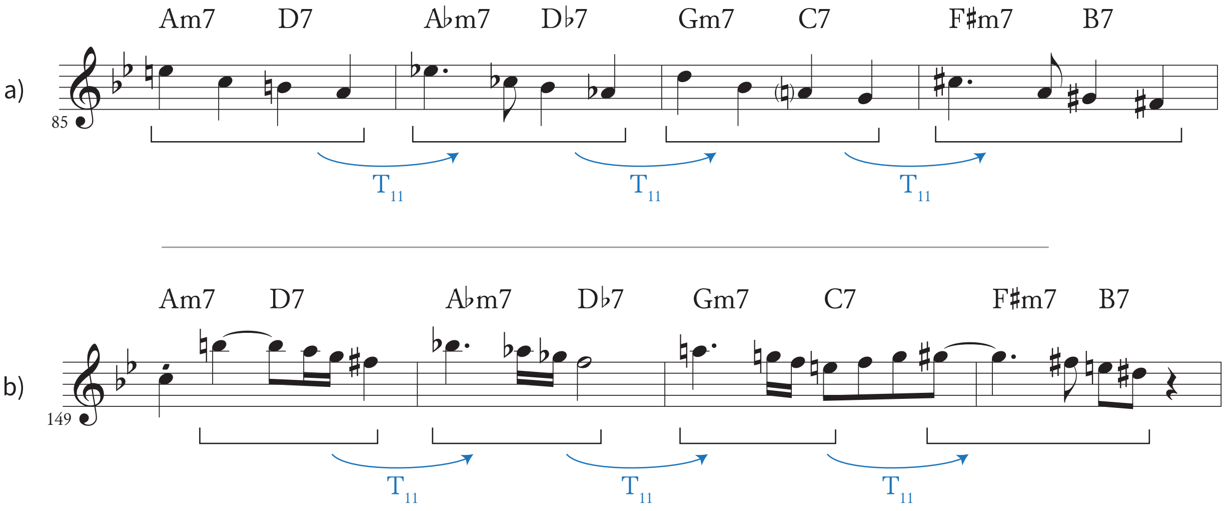

Figure 5.33. Two sequential bridges from Rollins's solo (mm. 85–88, 1:48 and mm. 149–52, 2:42).

with the chromatically descending ii–V progressions; the chord-scale analysis highlights that the f_7 signature transformation is equivalent to T_{11}.[38]

Because the bridge is made up of these chromatically descending ii–Vs, the most obvious approach for an improviser is to play sequentially, repeating a single pattern over each ii–V progression. This approach is seen most often in the second half of the bridge, where the harmonic rhythm doubles; Figure 5.33 gives two examples of this strategy from Sonny Rollins's solo. In *a*, from the third chorus, Rollins plays the same quarter-note pattern in all four bars, highlighting the T_{11}s of the progression itself.[39] Letter *b*, from the fifth chorus, is more complicated, but the basic idea is the same. Here, Rollins begins on the ninth of the minor seventh chord and descends to the third of the dominant seventh (diatonically, in the key of the dominant). The rhythm here is more varied, and linking material is inserted in the third bar, but the sequential pattern is still clear.

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[38] That $f_7 = T_{11}$ is one of the properties of signature transformations; see Julian Hook, "Signature Transformations," in *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha M. Hyde, and Charles J. Smith (Rochester: University of Rochester Press, 2008), 142–44.

[39] This repeating pattern is one of what are sometimes called "digital patterns." Though these patterns are more common at faster tempos (and in eighth notes), this is a standard 5–3–2–1 pattern, where the numbers indicate scale degrees of the minor seventh chord. Jerry Coker gives the passage at Figure 5.33a as an example of digital patterns in *Elements of the Jazz Language*, 11.

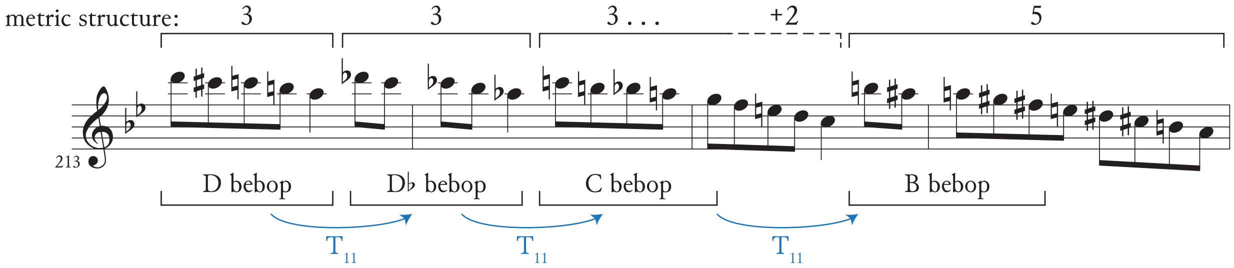

Figure 5.34. A sequential bridge with metric shift, from Sonny Stitt's solo (mm. 213–16, 3:34).

Stitt seems to prefer sequential bridges more than Rollins does, but Stitt often introduces a metric shift as well; Figure 5.34 gives a representative example from chorus 7 (similar passages can be found in choruses 9, 10, and 12). Here, Stitt plays a descending bebop scale over every dominant seventh chord. The first two of these are three-beat patterns, creating a metrical grouping dissonance (G3/4).[40] When Stitt repeats the pattern on the C bebop scale, it should end on the downbeat of the third bar. As if realizing he has arrived too early—after all, the C7 chord does not *begin* until the third bar—Stitt extends the scale another two beats, resulting in a five-beat scale that descends an entire octave. With five beats remaining in the bridge after the end of this pattern, he repeats it using the B bebop scale; a final A♮ in the last beat acts as an enclosure to the tonic B♭ that begins the next A section. While the harmonic rhythm of "Eternal Triangle" normally doubles in the last half of the bridge, Stitt's frequent metric shifts accelerate it even further, giving his solos even more momentum into the final A section of each chorus.

Though the strictly sequential patterns are mostly restricted to the last half of the bridge, the first half can also support sequential patterns; Figure 5.35 shows an example from the eighth chorus. Because the harmonic rhythm is slower at the beginning of the bridge, the sequences are usually somewhat looser. Here, Stitt plays a decorated Bm7 arpeggio, followed by the same pattern a bar later over B♭m7. (While in the passage in Figure 5.34 Stitt ignored the ii^7 chords, here he seems to ignore the E7 instead.) The pattern breaks in the middle of the third bar, where Stitt moves toward a standard diatonic pattern for the final E♭7 chord.

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[40] On metrical dissonance in jazz, see especially Stefan Love, "Subliminal Dissonance or 'Consonance': Two Views of Jazz Meter," *Music Theory Spectrum* 35, no. 1 (Spring 2013): 52–54 and throughout.

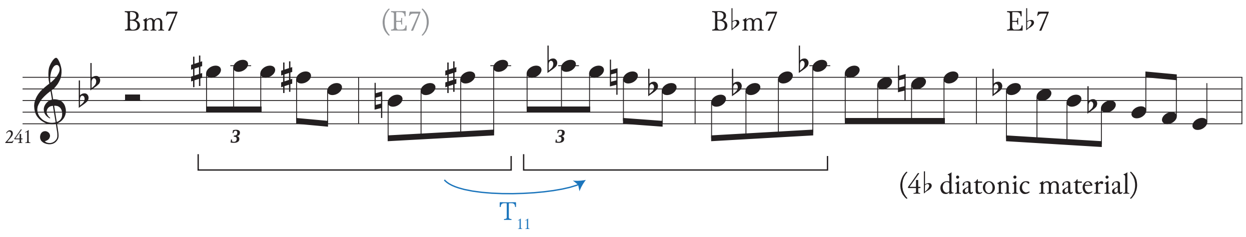

Figure 5.35. A loosely sequential bridge from Stitt's solo (mm. 241–44, 3:57).

5.4.2 INTERACTIONAL ELEMENTS

With a few exceptions (including "Lo-Joe" in the previous section), the analyses so far have been interested primarily in a soloist's improvised line, paying only passing attention to the fact that these improvisations occur within a framework of group improvisation. Sonny Rollins's solo on "Eternal Triangle," for example, does not take place in isolation; he is supported by the rhythm section, and in later choruses he and Stitt "trade," alternating 4- or 8-bar segments of improvised melodies. Understanding this interaction is crucial to understanding the performance as a whole. This section will focus on these moments of interaction in "Eternal Triangle," acknowledging the role that interaction plays in harmony, and vice versa.

One of the first models of interaction in jazz is found in Robert Hodson's *Interaction, Improvisation, and Interplay in Jazz*. He uses a semiotic model borrowed from Jean-Jacques Nattiez, in which a work of music is both a product (a score/sound) and a set of processes: both the poietic process of composition and the esthesic process of a listener.[41] Hodson adapts this model for jazz performance, since a jazz musician simultaneously creates the sound and listens to the other band members' sounds. The two separate components of poiesis and esthesis in Nattiez's model form something of a feedback loop for jazz musicians, since their musical utterances are often shaped by those of their fellow musicians (while at the same time, potentially influencing those musicians themselves).[42]

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[41] Hodson, *Interaction, Improvisation, and Interplay*, 14–15.
[42] Ibid., 15–16.

Garrett Michaelsen, in his dissertation on interaction in jazz, critiques Hodson on this model. He argues that while Hodson's focus on the ensemble as a whole (via the esthesic process) is valuable, Hodson does not go far enough. In his analyses, Hodson places himself in an improviser's perspective, and as Michaelsen notes, "it is . . . unclear how this vantage point enables musical analysis because it leaves no room for the outside observer's non-poietic perspective."[43] Instead, Michaelsen offers a listener-based approach, in which a particular auditory stream (a bass line, for example) might be heard as influencing another (like an improvised saxophone line).[44] As with harmony, there is often not a single "correct" analysis of a given interaction, and so Michaelsen's listener-based approach fits nicely with the prismatic approach to harmony taken here.

Michaelsen discusses harmony only in passing (usually in connection with a particular musical example), while Hodson dedicates an entire chapter to the role of interaction in harmony.[45] Many of the questions that concern Hodson are the same as those we have confronted during the course of this study:

> How can one reconcile the disparity between different versions, both written down and performed, of the "same" harmonic progression? Does it even need to be reconciled? Some scholars criticize the effort to reconcile these variants as an attempt to force a Western ideology of coherence—and a modernist ontology of the piece—onto a music to which it doesn't really apply. But, if this kind of [harmonic] coherence is not a part of jazz, then why do jazz musicians talk about a soloist "making the changes," or an improvised line as either "making sense" or not? There must be *some* criteria for musical coherence.[46]

Hodson answers these questions by borrowing a linguistic metaphor from Noam Chomsky: he argues that jazz musicians play the "deep structure" of a tune, which might be realized in any number of ways—and can be revealed by analyzing musical interaction. Rhythm tunes, he argues, can be generated from the deep structure of "I Got Rhythm": its A sections consist of prolongations of B♭, while the bridge begins off tonic and contains a motion back towards it. This appeal to linguistics does provide some way of understanding the myriad of Rhythm harmonizations, but it leaves something to be desired: it is not as though *any* set of chord changes can appear in the Rhythm A section, provided it starts and ends with B♭. Again, the transformational approach to harmony developed here allows us a means to specify the ways in which this deep structure is modified, and an interactional analysis of "Eternal Triangle" seems a natural way of exploring these modifications.

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[43] Michaelsen, "Analyzing Musical Interaction," 24–25.

[44] Ibid., 36–38 and throughout. Michaelsen's work draws on Albert Bregman's schema-based stream segregation, while acknowledging that auditory streams in jazz are more flexible than Bregman's own; see Albert S. Bregman, *Auditory Scene Analysis* (Cambridge, MA: MIT Press, 1990).

[45] The closest Michaelsen gets to an outright discussion of harmony comes in his discussion of interaction with "referents": he notes that "chord changes are inherently open-ended expressions of harmony that permit a wide variety of possible chord voicings," and that "different referents will motivate projections of varying specificity" ("Analyzing Musical Interaction," 90–91).

[46] Hodson, *Interaction, Improvisation, and Interplay*, 53.

As a first step in that direction, consider Figure 5.36, from the end of chorus 11. In the final A section of the chorus, Stitt plays a very strong blues generalization. Though this is a common choice (especially in A_3 sections), this particular occurrence is marked by the strong emphasis on the blues in the rhythm section parts as well.[47] After the fast-moving harmonies of the bridge (the end of which is given in the transcription), the group's convergence on eight bars of blues has a striking effect.

The instigating factor for this blues generalization might well have been Tommy Bryant's decision to play a tonic pedal in m. 345. In a Rhythm tune like "Eternal Triangle," the constant half-note harmonic rhythm can become tedious, and a pedal point is one of the most effective ways a bass player can counteract this tendency. This is the first time in the 5½ elapsed minutes of the recording Bryant uses such a pedal, and Stitt and Bryant's brother Ray on piano are very likely to have noticed.

Stitt responds to this tonic pedal by playing an emphatic blues lick with a prominent A♭.[48] This pitch, $\flat\hat{7}$, is unlikely to occur over any of the common harmonies of the first bar of the Rhythm A section (except perhaps as a ♭9 over G7), and its repetition as a long note in the second bar cements its status as a member of the B♭ blues scale. Pianist Ray Bryant, hearing the bass pedal

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[47] Charlie Persip's drums are not included in the transcription, since they do not contribute to the harmony.

[48] Throughout these interactional analyses, I follow Michaelsen's own use of language, which relocates a listener's perception into the player. Thus, "Stitt responds to this tonic pedal" should be understood as "I, the listener/analyst, hear Stitt's musical utterance as a response to the tonic pedal of the bass." This language, though potentially confusing, is in general clearer and easier to read. See Michaelsen, "Analyzing Musical Interaction," 46–47.

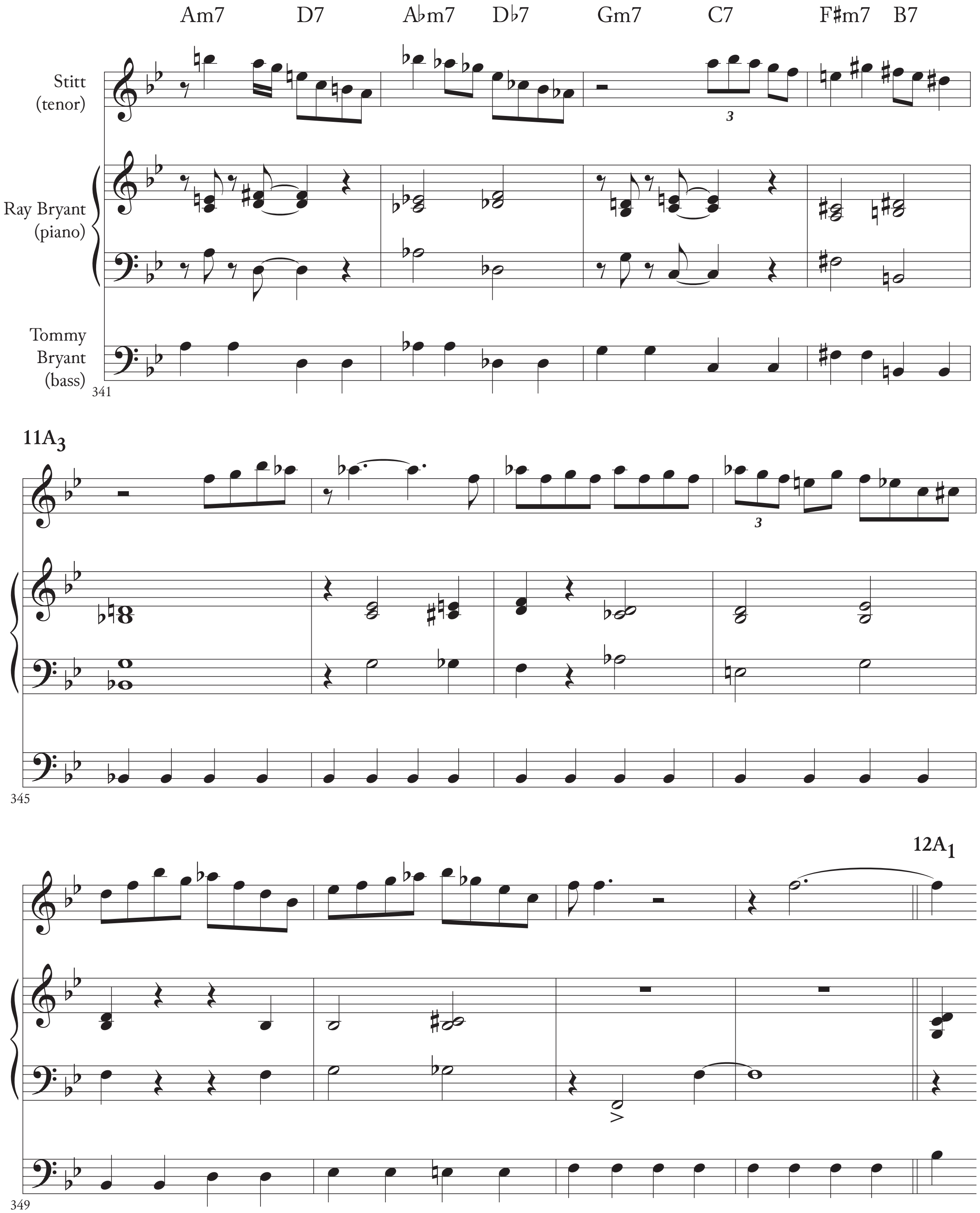
Am7
D7
A♭m7
D♭7
Gm7
C7
F♯m7
B7
Stitt (tenor)
Ray Bryant (piano)
Tommy Bryant (bass)
3
341
11A3
3
345
12A1
349

Figure 5.36. The end of chorus 11, including rhythm section (mm. 341–53, 5:19; bass sounds as written).

along with Stitt's blues lick, then launches into a series of blues voicings.[49] Combined with the tonic pedal, Bryant's diminished chords and neighboring $^{6}_{4}$ chord of m. 348 give the section a strong blues feel that both reinforces and is reinforced by Stitt's solo line.

This group interaction is what Michaelsen would call strongly "convergent," in that all three of the members involved play music that supports the others.[50] The convergence continues in the last four bars of the section, as Tommy Bryant moves away from the tonic pedal to outline the blues-inflected progression B♭–B♭7/D–E♭–E°7–F7. In m. 351, he arrives on a dominant pedal at the same moment that Stitt also concludes his line on F, all reinforced by Ray Bryant's strong left-hand F in the piano.[51] This convergence on the dominant (combined with Charlie Persip's drum fill) provides a strong push into the following chorus, where the three fall back into their usual, less convergent, roles.

The most clearly interactional moments of "Eternal Triangle" come between the two saxophonists themselves after the trading begins in chorus 14. Stitt and Rollins "trade fours" for three choruses, then "trade eights" for three more to end the saxophone soloing. The fours supply many good examples of harmonic interaction; chorus $15A_2$ (shown in Figure 5.37) gives a representative example. Here, Rollins plays a diatonic figure in the first two bars, then side-slips in the next two to play a figure that outlines B major rather than B♭. Stitt's response is at once both convergent and divergent: he enters on the same pitch that Rollins did and plays a very similar figure in his first bar (marked with an *x* in Figure 5.37), but his starting pitch is F♮, dissonant with the B major triad Rollins is playing at the same time. The last A section of the same chorus (shown in Figure 5.38) illustrates a more convergent interaction. Here, Rollins begins with a motive outlining the pitches G–G♭–F–D. Stitt seamlessly picks up this line on the downbeat of

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[49] At the same time, Bryant's piano voicings project some continuity with the end of the bridge, since he continues moving his right hand in parallel thirds.

[50] In Michaelsen's terminology, the three auditory streams "project similar futures" ("Analyzing Musical Interaction," 60). The particulars of auditory streams and implications need not concern us too much here, since it is relatively clear that all three members project the blues.

[51] Incidentally, the arrival on F in the piano is perhaps unusually contrapuntal: it seems as though Bryant's voicing in m. 350 is dictated by the lower-voice motion G–G♭–F, while the upper voice C♯ demands resolution to D, fulfilled on the downbeat of the next chorus. These contrapuntal motions all combine with the bass E♮ to produce a German seventh chord in the second half of m. 350—a fairly typical predominant in classical music, but extremely rare in jazz.

m. 477, continuing the motive for another four bars, at which point Rollins reclaims it for a bar before moving on to new material in the beginning of the next chorus.

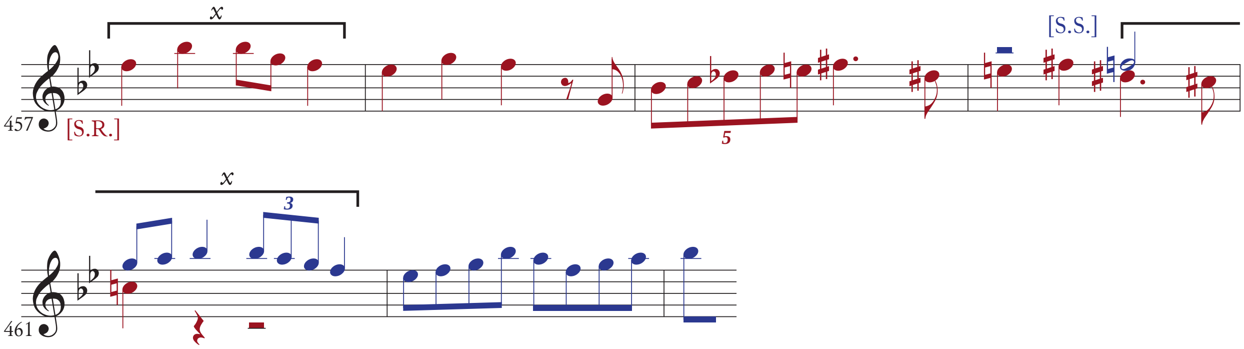

Figure 5.37. A harmonic interaction between Rollins (red notes) and Stitt (blue notes) from chorus 15 (mm. 457–63, 6:52).

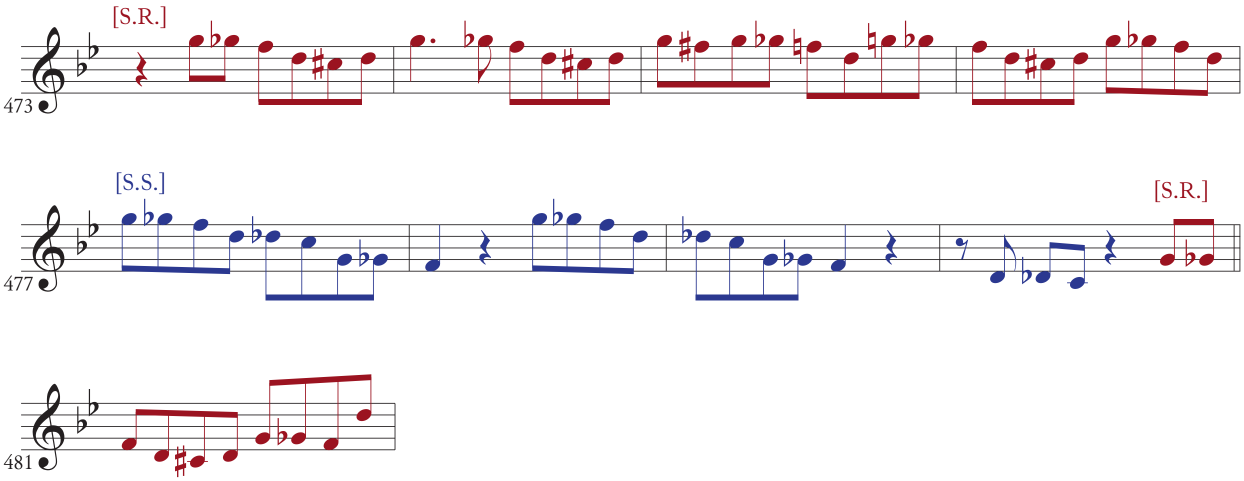

Figure 5.38. A convergent interaction from later in chorus 15 (mm. 473–81, 7:05).

5.4.3 Extended Analysis: Trading Eights, Harmony, and Interaction

For the final analysis in this dissertation, I want to take a more detailed look at the way harmony functions over a longer period of time, using the final three choruses of saxophone soloing on "Eternal Triangle." These are the choruses where Stitt and Rollins trade eights; since each soloist gets an entire section of a chorus, he has more harmonic leeway than in the rapid four-bar segments of the preceding three choruses. Since these are the final three choruses of their solos, this portion is also where the "saxophone dueling" comes to a head. In it, the suggestions of outside playing initially suggested in Rollins's very first chorus (recall Figure 5.31) reach a final realization, before winding down again in the final chorus.

The eight-bar trading begins at the beginning of chorus 17, after a non-musical interaction: someone (probably Stitt) can be heard on the recording asking "keep going?" Stitt's first eight bars are a typical diatonic A section, with only incidental chromaticism. Rollins's response in chorus $17A_2$ is more outside, and is shown in Figure 5.39. The most blatant chromaticism here is in the second bar, where we would normally expect Cm7–F7 or Cm7–C♯°7. As is usually the case with outside playing, it is not clear how we should interpret Rollins's note choices here, though the chord-scale triples below the staff give a few suggestions. After this second bar he returns to more inside playing, though with a few more outgoing scale choices than usual: he plays both ♯5 and ♭9 over the F7 in m. 524.

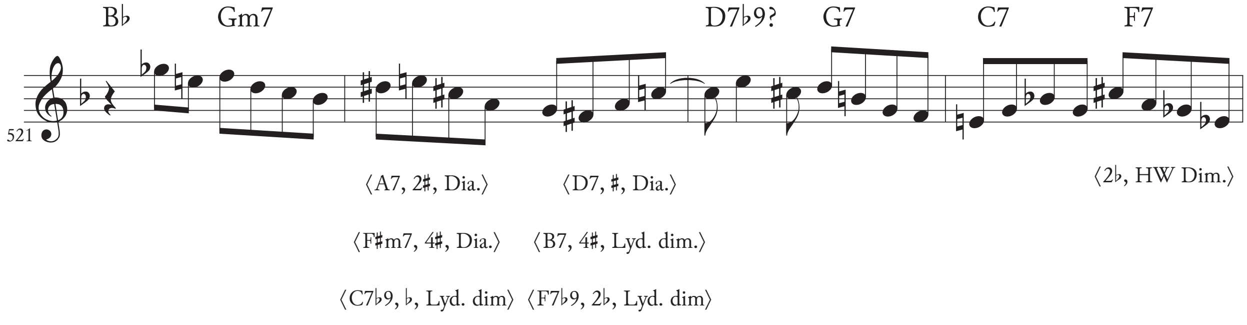

Figure 5.39. Rollins's outside playing in the second A section of chorus 17 (mm. 521–24, 7:44).

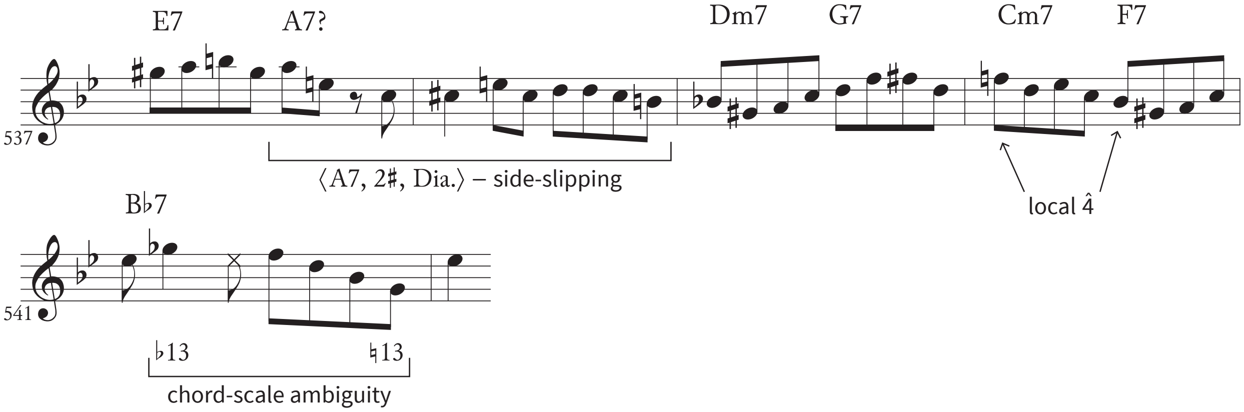

Figure 5.40. Rollins's increased chromaticism in the final A section of chorus 17 (mm. 537–42, 7:56).

Stitt's ensuing bridge is again typical, using only the expected diatonic collections. That he chooses not to play outside on the bridge illustrates a basic principle of jazz harmony, in that progressions that are less commonplace are generally played more inside. The bridge of "Eternal Triangle" is its most distinctive feature, so soloists typically play improvisations that highlight these harmonies, whereas the A sections of the tune are more typical and admit of greater elaboration. Playing far outside the changes on the bridge risks obscuring the harmonic progression that is an essential feature of this particular Rhythm tune (and perhaps, in an amateur performance, giving the impression that outside playing will be mistaken for not *knowing* the changes).[52]

In the final A section of the chorus, Rollins increases the chromaticism even further, as shown in Figure 5.40. His first bar seems to imply a motion from E7 to A; while we might understand the E harmony as a tritone substitution for the tonic B♭, the A chord in the second half of the first bar does not make sense as an ordinary substitution in any of the usual harmonizations of the A section. Given that the A7 harmony seems to continue into the second bar, we might instead hear this outside playing as a downward side-slipping, substituting an A diatonic collection for the tonic B♭. Side-slipping is one of Rollins's preferred methods of playing outside: we observed it in his first chorus, and he repeats the technique in chorus 14A$_3$ (mm. 441–42). Rollins returns to more typical harmonies after these first two bars, though he still emphasizes dissonant tones: m. 540, for example, features strongly accented dissonances, with the local $\hat{4}$ (Levine's "avoid note") appearing on beats 1 and 3. All of this chromaticism combines to form an improvised line even more outside than Rollins's first eight bars of trading, as though his own sense of harmony is being slowly detached from that of both his own rhythm section and from the A section of the tune itself.

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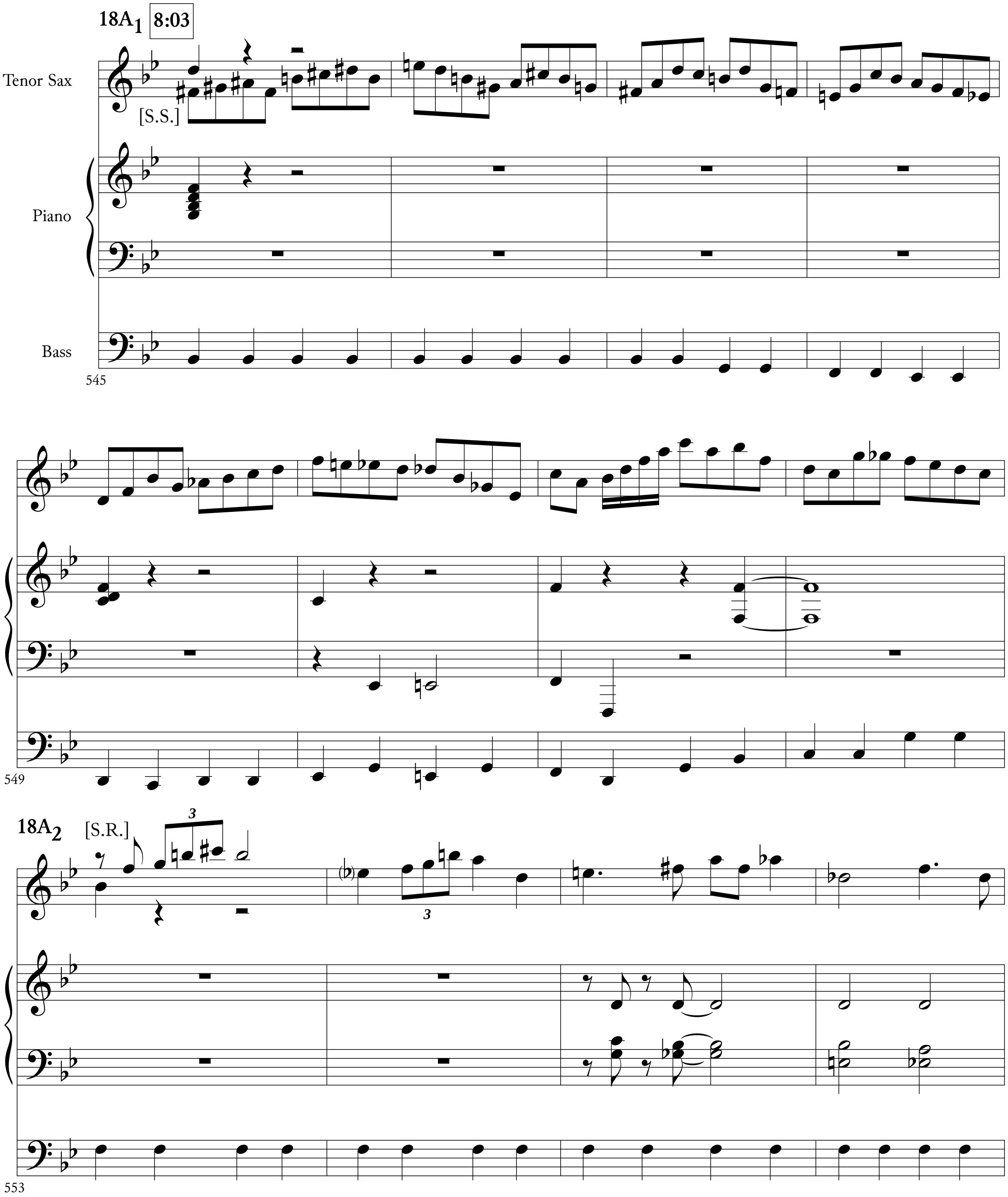
[52] A similar phenomenon occurs in the final turnaround of Joe Henderson's "Isotope," discussed in Section 4.3.3. Because the turnaround is so distinct, Henderson simply arpeggiates the harmonies in most of his improvised choruses to make them as clear as possible.

In the beginning of the next chorus, Stitt seems to take the outside-playing bait, launching into a dominant-cycle A section very much like Monk's solo on "Rhythm-a-ning." This harmonic move seems to take the rhythm section by surprise, and draws our attention to the interaction not only between the two soloists, but between them and the rhythm section as well (a transcription including the piano and bass parts is given in Figure 5.41). Faced with this unexpected dominant cycle, pianist Ray Bryant's solution is simply to stop playing, while bassist Tommy Bryant instead plays a B♭ pedal, as if to stress the tonic in the midst of Stitt's cycle. Since the 8-chord dominant cycle lasts only four bars, all three band members all return to a typical A-section harmonic structure in m. 549.

Perhaps anticipating another outside response from Rollins, Ray Bryant does not immediately begin playing in the next A section, and Tommy Bryant opts this time for a dominant pedal. In the face of harmonic uncertainty, this approach from the rhythm section makes sense: a dominant pedal in the bass will work to build tension no matter what Rollins decides to play, and Ray Bryant's wait-and-see approach prevents any harmonic clashes. As it turns out, Rollins does play a non-diatonic line, and again it is not entirely clear what harmonic framework he has in mind. His line in the first four bars of chorus $18A_2$ is a loose, descending, motivic repetition that recalls the "C.T.A." harmonization of Figure 5.2f (with its whole-step descent). While the motive and direction of the line are relatively clear, we might also hear this line, in conjunction with Ray Bryant's comping in the last two bars, as a series of outgoing scale choices on a more standard progression, as shown in Figure 5.42. Like Stitt, Rollins returns to a more diatonic approach in the last four bars of the section, setting up the way for Stitt's bridge.



18A1
8:03
Tenor Sax
[S.S.]
Piano
Bass
545
549
18A2
[S.R.]
3
(♭)
3
553

Figure 5.41. Transcription of chorus 18, including rhythm section (mm. 545–76, 8:03).

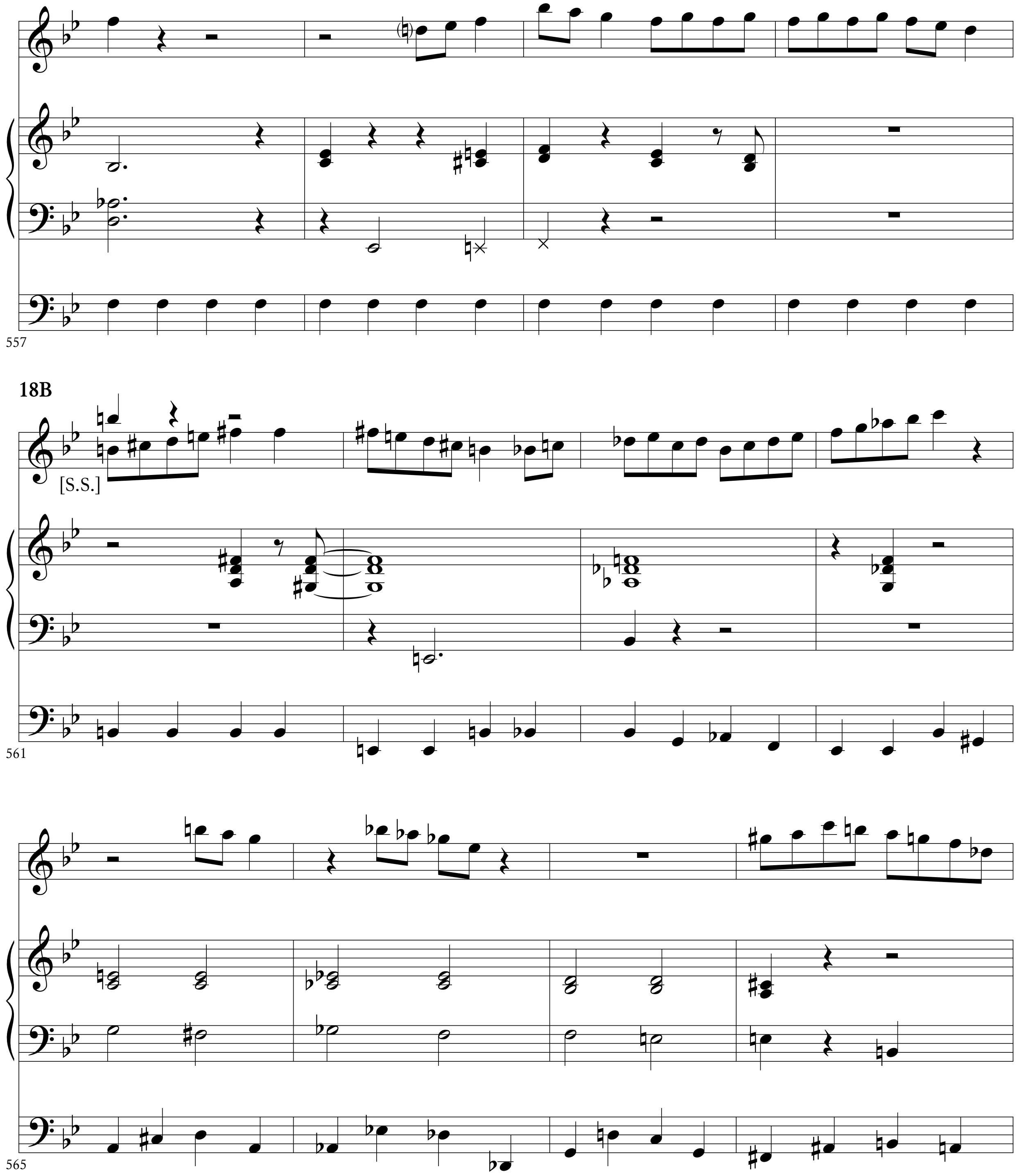

557
18B
[S.S.]
561
565

Figure 5.41 (continued). Complete transcription of chorus 18.

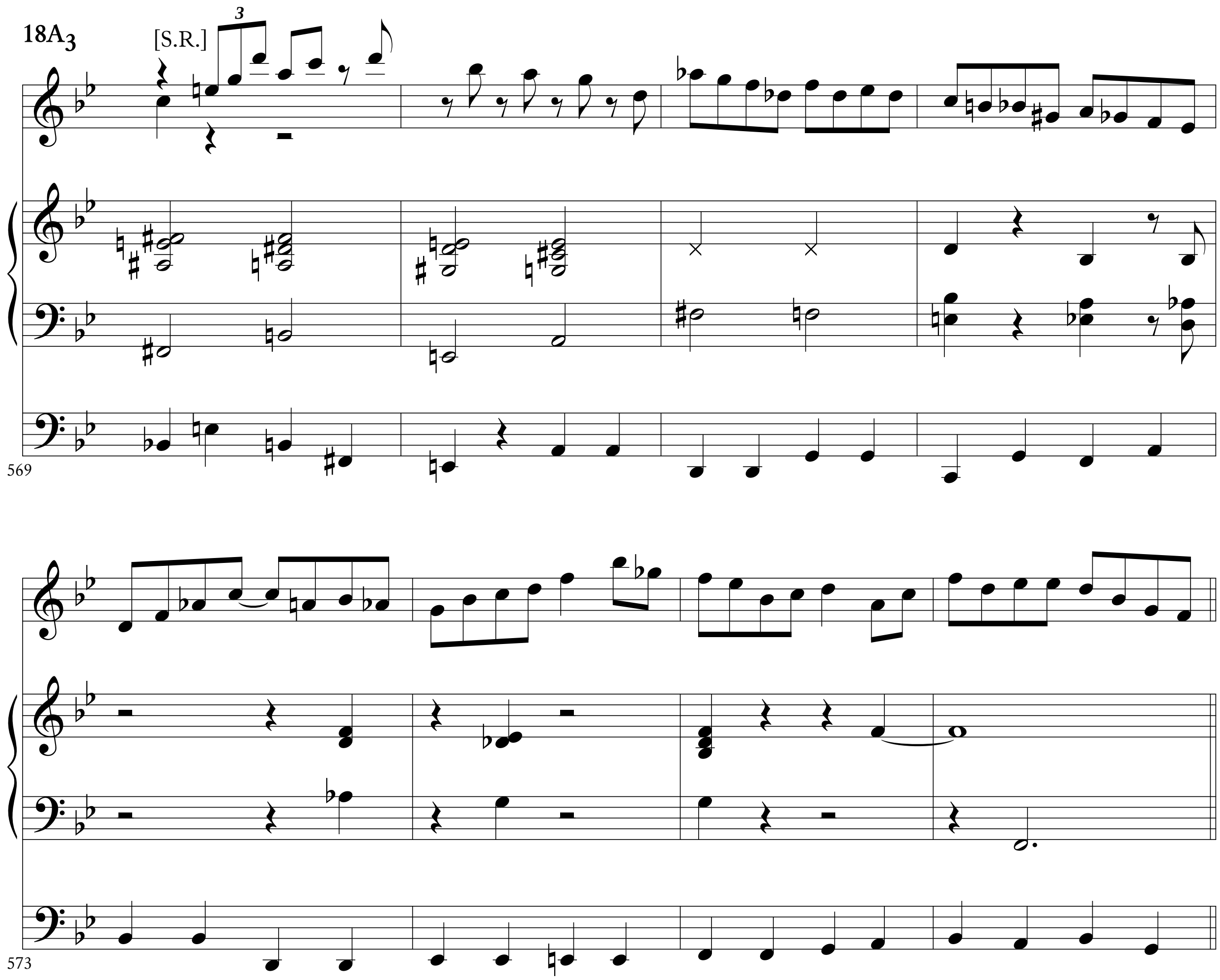

18A3
[S.R.]
3
569
573

Figure 5.41 (continued). Complete transcription of chorus 18.

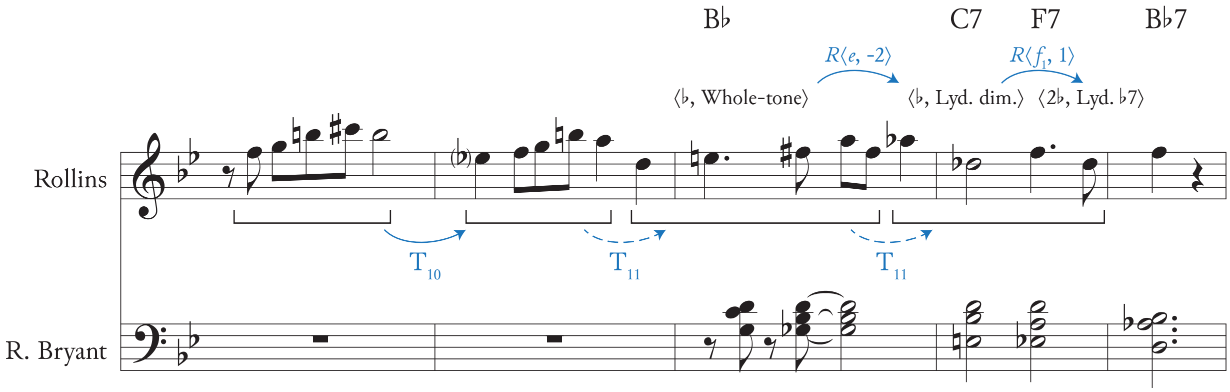

B♭
C7
F7
B♭7
R⟨e, -2⟩
R⟨f1, 1⟩
⟨♭, Whole-tone⟩
⟨♭, Lyd. dim.⟩
⟨2♭, Lyd. ♭7⟩
Rollins
(♭)
T10
T11
T11
R. Bryant

Figure 5.42. One possible interpretation of Rollins's descending line (mm. 553–56, 8:09).

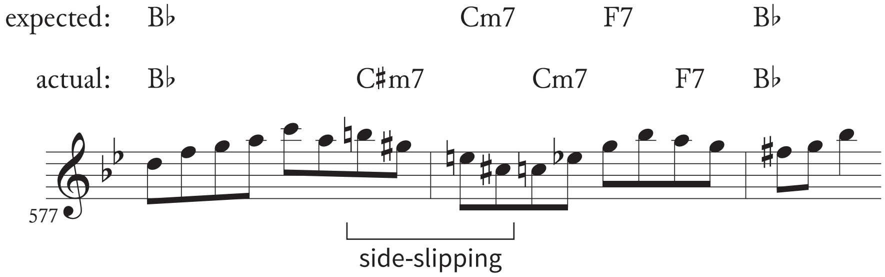


Figure 5.43. Stitt's side-slipping at the beginning of chorus 19 (mm. 577–79, 8:29).

The bridge of chorus 18 proceeds normally in all three instruments: Stitt's scale choices are almost completely diatonic, Ray Bryant uses standard 3-note voicings, and Tommy Bryant plays a bass line emphasizing the T_{11} root motion. While the bridge of "Eternal Triangle" usually functions as the locus of harmonic activity in a chorus, the bridge here has an almost calming effect after the harmonic disruption in the first half of the chorus. The tension of all of the outside playing seems to disappear as all of the band members (and we as listeners) relax into the bridge, relatively confident that it will progress as expected.

This harmonic tranquility does not last long, though, as Ray Bryant instigates another dominant cycle at the beginning of chorus $18A_3$. He enters emphatically on an F♯7 chord, and includes the bass note (typically omitted by pianists) in his left hand, as if to demonstrate that he understood Stitt's cycle in $18A_1$ and is willing to support it for this A section. His brother Tommy catches the cycle almost immediately: after playing a B♭ on the downbeat of m. 569, he makes his way to a B♮ by beat 3, and continues with the cycle all the way through the first four bars. This time, though, Rollins does not follow along, playing the B♭ Lydian scale for the first two bars. Ray Bryant's enthusiasm for the dominant cycle fades quickly, hearing that Rollins does not follow along; by the third bar the piano voicings are nearly inaudible, returning only in the final bar (where C7–F7 is a characteristic choice regardless of the particular harmonization used).

After a more common diatonic ending in the last four bars of chorus 18, Stitt begins the next chorus with a side-slipping gesture, arpeggiating a C♯m7 chord before the expected Cm7 in the

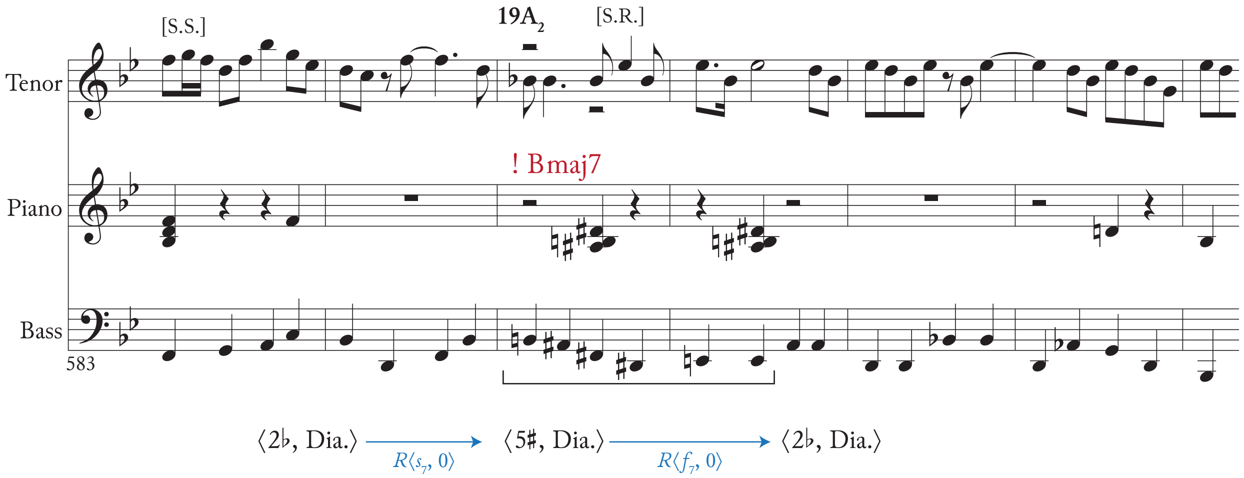


Figure 5.44. Ensemble arrival on Bmaj7 at chorus 19A$_2$ (mm. 583–88, 8:33).

second bar of the chorus (see Figure 5.43). After this brief moment, though, Stitt plays more or less diatonically until the end of this A section.

At the beginning of chorus 19A$_2$, the side-slipping occurring sporadically throughout the saxophone solos reaches its culmination in a remarkable moment of ensemble convergence. Stitt ends his eight bars with a tonic triadic descent, ending on B♭. At the same moment, both Ray and Tommy Bryant land on a Bmaj7 chord, turning Stitt's tonic B♭ into a chordal major seventh (see the transcription in Figure 5.44).[53] This side-slipped B major lasts only two bars, and by m. 587 the rhythm section moves back to B♭. For his part, Rollins plays a repeating motive over all eight bars of this section consisting of the pitches E♭, D, and B♭. Over the Bmaj7, this lick emphasizes the major third and seventh of the chord, but when the rhythm section slips back to B♭, Rollins is left emphasizing the dissonant $\hat{4}$. This approach is certainly not as outside as Stitt's dominant cycle of chorus 18A$_1$, but the dissonance still lends a sense of "outsidedness" to the section as a whole. In the final bar of his A section, Rollins resolves the E♭ to D, after which the bridge and final A section proceed almost completely diatonically.

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[53] Given the convergence of all three members of the ensemble on this Bmaj7 chord, it seems likely that this harmonic move was planned in advance. Though it is impossible to say for certain, the fact that it occurs on the last chorus of trading provides some support; it is easy to imagine a situation in which the band decides (before recording) that a move to B major in the second A section will mark the last chorus of saxophone solos.

Heard as a unit, the final three choruses form a progression from inside to outside playing and back again. Stitt's diatonic playing in chorus 17 is challenged by Rollins's suggestions of outside playing, weakly at first in $17A_2$ but more strongly in $17A_3$. Stitt takes the suggestion in the opening of chorus 18, playing a dominant cycle that takes the rhythm section by surprise. Rollins responds not with a dominant cycle, but with a motivic response that seems almost completely disconnected from typical Rhythm harmony. The rhythm section tries to anticipate a dominant cycle in the chorus's final A section, only to be disregarded by Rollins. Amid the harmonic confusion, Stitt begins chorus 19 with some mild side-slipping before returning to more inside playing. This side-slipping is answered strongly by the rhythm section, which begins the following A section a half-step off on Bmaj7. After a mildly dissonant eight bars from Rollins, the bridge and final A section return inside to conclude the saxophone trading and make space for Dizzy Gillespie's trumpet solo that follows. The harmony in these choruses is far more nuanced than the chord changes of the lead sheet might suggest; it is a dynamic, ever-changing attribute, revealed not only in the interaction among the musicians themselves, but also between the musicians and us as listeners.

5.5 Concluding Remarks

As the end of this study approaches, it may be useful to return to the fundamental question: what *is* jazz harmony, really? Or, as Hodson states the problem, "what exactly are you analyzing when you analyze jazz harmony?"[54] Does the harmony exist in the chord symbols on a lead sheet? In the voicings of a pianist or walking line of a bassist? Does harmony exist in a tune itself, independent of any particular performance of it? If a performer's harmony conflicts with the chord symbol, which one is "correct"?

These are fundamental questions, and questions that do not often arise in the study of notated music. The answer, I think, is that harmony in jazz is all of these things, and more. A single harmony can be captured by a chord symbol, but this chord symbol is only part of the story. A transcription of a piano voicing for that symbol does not represent a final solution either, since in jazz a repeating harmonic framework forms the basis of a performance, and pianists do not usually play identical voicings for a single chord throughout. We may be tempted to turn towards the soloist as the arbiter of harmonic identity, but as we have seen, even a seemingly clear progression in a solo instrument can be obscured by the choices of a rhythm section.

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[54] Hodson, *Interaction, Improvisation, and Interplay*, 52.

It is my hope that the transformational approach developed throughout this dissertation is better equipped to deal with this chimerical nature of jazz harmony. Chapter 1 began by treating harmony in something of a "clean-room" fashion, grouping chord symbols into diatonic sets without worrying too much about what the chord symbols actually represented. Chapters 2 and 3 took steps towards clarifying this nature, representing chord symbols in their most basic form of root, third, and seventh. This abstraction allowed us to explore connections between harmonies that do not share a diatonic collection, using variations on a basic musical space of ii–V–I progressions. In Chapter 4, our conception of harmony expanded from three-note chords into many-note scales, as we drew on the work of George Russell to create a chord-scale space. This work permitted a change of focus from chord symbols and abstract tunes to jazz performance itself, investigating the way in which harmony can function for improvising performers. This final chapter has continued this work by looking through the lens of Rhythm changes, itself a harmonic archetype, variously instantiated in countless Rhythm contrafacts, each of which is elaborated in individual performances.

In this last chapter, the transformations themselves receded into the background somewhat, used only as a tool for discussing harmony in service of other analytical points. This is intentional, and as Julian Hook notes in his review of Lewin's *GMIT*, "transformation theory is a large and varied toolbox; there are only some minimal instructions for using the tools, and no designs at all for what one can build with it."[55] For all the focus on harmony, it is often the case that harmony is not the most interesting aspect of a particular passage. Just as we tell our students that a Roman numeral analysis does not mark the end of the analytical process, neither does a completed chord-scale labeling mean that we can check off a passage as "analyzed" and move on to the next chorus. This dissertation has developed a flexible set of tools for analyzing harmony—tools that can be used as necessary whenever a need to discuss harmony arises.

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[55] Julian Hook, "David Lewin and the Complexity of the Beautiful," *Intégral* 21 (2007): 166.

Given that harmony touches nearly every aspect of jazz performance and analysis, the foundational work here might be applied or extended in any number of ways. It is easy to imagine other transformations that might appear more commonly in other (especially later) jazz repertoires, where either the ordered-triple or chord-scale approaches might be fruitfully applied. Dmitri Tymoczko has argued that jazz is a "modernist synthesis," and that jazz musicians "act as custodians of a tradition of advanced tonal thinking."[56] Though I tend to disagree with this historical view of jazz, it may well be the case that the techniques developed here might lend insight into the musics Tycmozko identifies on either historical end of common-practice jazz: the tonality of the impressionists on one side and that of the minimalists on the other.

Even within the common-practice jazz era of this dissertation, there is room for expansion. The solo analyses done here only begin to scratch the surface, and a detailed investigation of chord-scale choices among different performers might well lead to some meaningful distinction between, for example, a Johnny Griffin solo and a Joe Henderson solo. Certainly every soloist has their own style, and the way in which they interpret harmony is often an essential component of that style. It is also easy to imagine that saxophonists in general have a different kind of harmonic language than do trumpeters, trombonists, pianists, or banjo players; chord-scale transformations could help to tease out these distinctions.

The analyses here have focused exclusively on small-group jazz, since there are fewer moving parts to manage, but harmony is of course present in (nearly) all jazz performance. One of my own interests lies in the implications of harmony for composers and arrangers of big-band music. When an arranger like Thad Jones or Jim McNeely sits down to arrange a jazz standard, they typically do so with a knowledge of harmony earned from experience as a player. The fact that large ensemble music is typically composed and written down means that the author maintains a tighter control

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56. Tymoczko, *A Geometry of Music*, 389.

over the harmony than in an improvised setting; this allows the opportunity for a detailed shaping of harmony over the course of a tune. These standards are often reharmonized in interesting ways, and the transformational approach is ideally suited to discussing these arrangements in dialogue with their original sources.

One of the aims of this dissertation has been to take seriously the manner in which jazz musicians themselves discuss harmony. Since these musicians do not often speak technically about harmony, I have used pedagogical materials as a way of getting at this "insider's view." This focus has often been implicit: there has been no extended literature review of pedagogical texts, nor was there a need to use these texts to the exclusion of other, more academic, treatments of jazz harmony. When a choice arose, however, I usually opted to cite Jerry Coker, Mark Levine, or Jamey Aebersold rather than Henry Martin, Dmitri Tymoczko, or Steve Larson. This choice was made not to disparage the important work of other theorists working on jazz harmony, but rather as a means to acknowledge the real work on harmony in the jazz community (in addition to the developments of the theory community).

At the outset of this study, I suggested that a transformational approach to jazz harmony might constitute a set of analytical values different from the Eurological values of Schenkerian analysis (as often applied to jazz). The prismatic style of transformational analysis, borrowed from Steven Rings, has helped to enable this alternate value system. Nearly all of the analyses here have avoided taking a single synthetic view of a passage, opting instead for a perspective in which multiple, sometimes conflicting, analyses can be considered individually in turn. This multifaceted approach is intended to reflect the nature of jazz harmony itself, and its application can enable us to narrow the gap between bring jazz theory and jazz practice.

Appendix A
List of Recordings

This appendix provides a list of all of the musical examples used in this dissertation, along with recordings where they can be found. For each tune, I have tried to provide at least two reference recordings; they have been chosen because they present clear statements of the standard changes analyzed in the text. Recordings are listed here only by the artist and album title; full details can be found in the discography that follows. In cases where a tune has a recording that is considered definitive, it is marked with a star. (This list reflects my own proclivity towards pianists and saxophonists, and is by no means exhaustive.)

Chapter 1

"Alice in Wonderland" – Sammy Fain/Bob Hilliard

- Chick Corea, *Trilogy* (2013)
- Bill Evans, *Sunday at the Village Vanguard* (1961)
- Oscar Peterson, *The Way I Really Play* (1968)

"Autumn Leaves" – Joseph Kosma/Jacques Prévert

- Gene Ammons and Sonny Stitt, *Boss Tenors* (1961)
- Bill Evans, *Portrait in Jazz* (1960)
- Wynton Kelly, *Someday My Prince Will Come* (1961)

"How My Heart Sings" – Earl Zindars

- Bill Evans, *How My Heart Sings* (1962)
- Tony Williams, *Young at Heart* (1998)

"All the Things You Are" – Jerome Kern/Oscar Hammerstein

- Chet Baker, *The Chet Baker Quartet* (1953)
- The Quintet, *Jazz at Massey Hall* (1953)
- Sonny Rollins and Coleman Hawkins, *Sonny Meets Hawk!* (1963)

Jazz discography in the 21st century can be frustrating, given the many different formats and reissues of original sources. I have tried to be as clear as possible in the discography, listing both original release information and more readily available reissues, when available. Three online sources have been invaluable in compiling this information: *The Jazz Discography Project* (http://www.jazzdisco.org; Nobuaki Togashi, Kohji "Shaolin" Matsubayashi, and Masayuki Hatta, maintainers); *Discogs* (http://www.discogs.com; created by Kevin Lewandowski); and *AllMusic* (http://www.allmusic.com; All Media Network).

Chapter 2

“Blues for Alice” – Charlie Parker

- [Rahsaan] Roland Kirk, *We Free Kings* (1961)
- ⋆ Charlie Parker, *The Magnificent Charlie Parker* (1955)
- Red Rodney, *One for Bird* (1989)

“Ceora” – Lee Morgan

- Benny Carter, *Elegy in Blue* (1994)
- Joey DeFrancesco, *Live: The Authorized Bootleg* (2007)
- ⋆ Lee Morgan, *Cornbread* (1965)

“Solar” – Miles Davis

- ⋆ Miles Davis, *Walkin’* (1957)
- Lee Konitz and Hal Galper, *Windows* (1977)
- Pat Metheny, *Question and Answer* (1990)

Chapter 3

“Giant Steps” – John Coltrane

- ⋆ John Coltrane, *Giant Steps* (1959)
- Tommy Flanagan, *Giant Steps: In Memory of John Coltrane* (1982)
- McCoy Tyner, *Remembering John* (1991)

“Have You Met Miss Jones” – Richard Rodgers/Lorenz Hart

- Chet Baker, *Smokin’ with the Chet Baker Quartet* (1965)
- Joe Pass, *Virtuoso* (1974)
- Oscar Peterson, *We Get Requests* (1964)

“Isotope” – Joe Henderson

- Joe Henderson, *Big Band* (1996)
- ⋆ Joe Henderson, *Inner Urge* (1965)
- Phil Woods and Gordon Beck, *Live at Wigmore Hall, London* (1996)

“Lady Bird” – Tadd Dameron

- Art Blakey and the Jazz Messengers, *At the Cafe Bohemia, Vol. 1* (1956)
- Dexter Gordon, *More Power!* (1969)
- ⋆ Fats Navarro, *The Fabulous Fats Navarro, Vol. 2* (1956)

Chapter 5

"Anthropology" – Charlie Parker and Dizzy Gillespie

- Dizzy Gillespie, *Dizzy Gillespie* (recorded 1946, released 1966)
- Barry Harris, *Newer than New* (1961)
- ⋆ Charlie Parker, *Summit Meeting at Birdland* (recorded 1951, released 1977)

"C.T.A." – Jimmy Heath

- ⋆ Miles Davis, *Miles Davis, Vol. 2* (1953)
- Lee Morgan, *Candy* (1957)
- Art Taylor, *Taylor's Wailers* (1957)

"Moose the Mooche" – Charlie Parker

- Barry Harris, *At the Jazz Workshop* (1960)
- Hank Jones, *'Bop Redux* (1977)
- ⋆ Charlie Parker, *Charlie Parker, Vol. 1* (1946)

"Serpent's Tooth" – Miles Davis

- ⋆ Miles Davis, *Collector's Items* (1956)
- Jeff Hamilton Trio, *Symbiosis* (2009)

"Wail" – Bud Powell

- ⋆ Bud Powell, *The Amazing Bud Powell, Vol. 1* (1955)
- George Shearing, *I Hear a Rhapsody: Live at the Blue Note* (1992)

Appendix B
Transcriptions

Transcription is a notoriously difficult problem in jazz. As Steve Larson (among others) has noted, any transcription is also, in some sense, an analysis.[1] In the transcriptions that follow, I have (in general) notated only the solo line, focusing primarily on the pitches and rhythms. This means that many of the so-called "secondary parameters" are absent from the notation: dynamics, intonation, phrasing, issues of timing, etc. To that end, the transcriptions should be seen as companions to the source recordings, not replacements for them.

In the transcriptions of the Rhythm tunes, no chord symbols are given. One of the principal arguments of Chapter 5 is that the chord symbols are somewhat fluid through the course of a performance; I have simply omitted the chord symbols rather than assigning them based on the solo line. In the transcriptions, the notes remain more-or-less uninterpreted (though the notation of accidentals is a question of interpretation); examples in the text refer to a given passage's harmonic context.

There are a few other minor things to note:

- Formal designations follow Larson's convention: $2A_3$ refers to the third A section of the second chorus, 4B refers to the B section of the fourth chorus, and so on. Timestamps from the reference recording are given at the beginning of each chorus.
- Throughout, the tenor saxophone is notated at concert pitch but sounds down an octave, and soprano saxophone is notated at pitch.
- "X" noteheads indicate either ghosted notes (which are much lower in volume than surrounding notes) or alternate fingerings for the same pitch (a timbral effect often used in the high register by saxophonists).

List of Transcriptions

"Autumn Leaves" – Gene Ammons and Sonny Stitt 206
"Blues for Alice" – Rahsaan Roland Kirk 210
"The Eternal Triangle" – Sonny Stitt and Sonny Rollins 213
"Isotope" – Joe Henderson 230
"Lo-Joe" (head only) – George Coleman 236
"Rhythm-a-ning" – Johnny Griffin and Thelonious Monk 239

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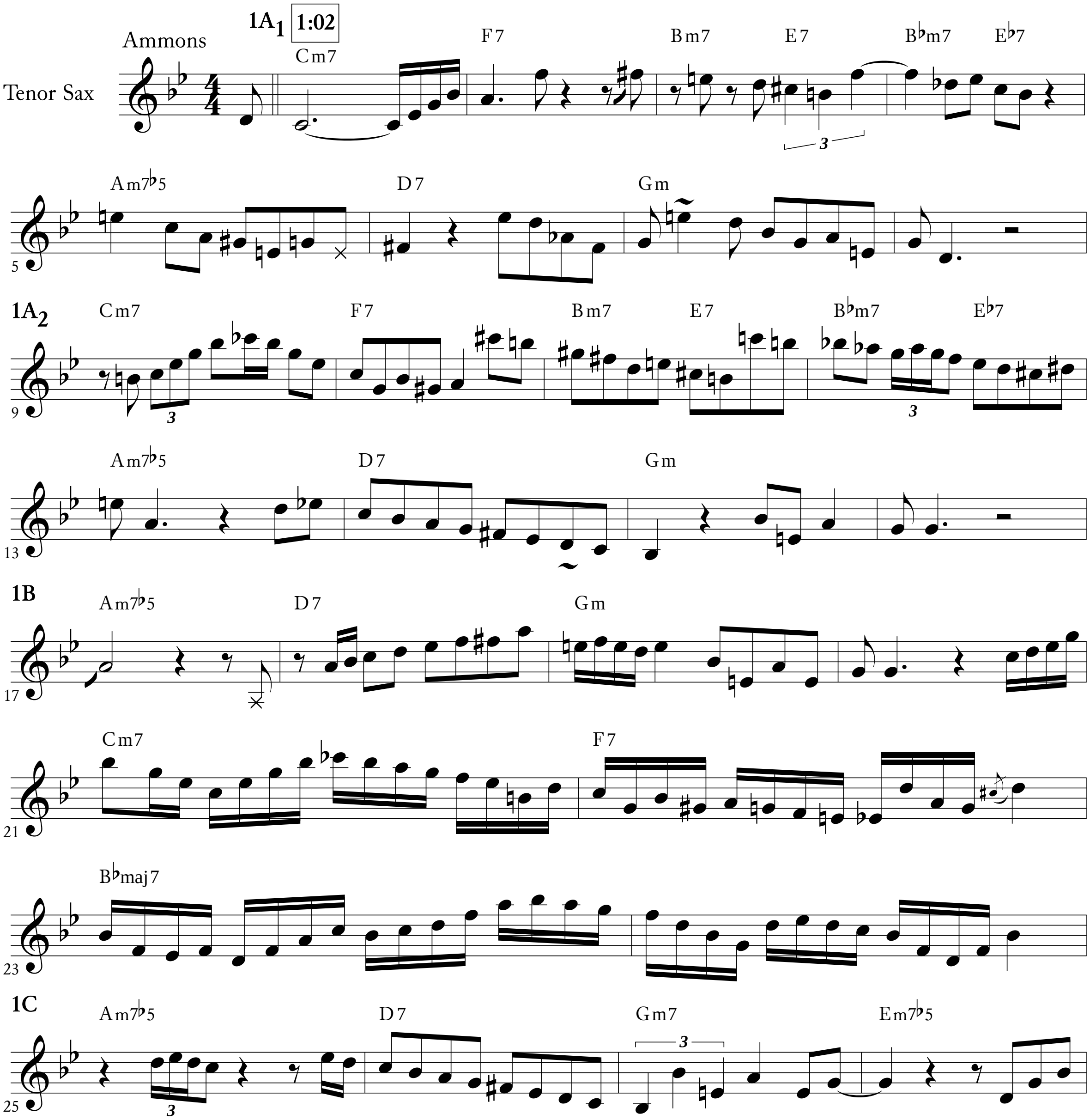
[1] Steve Larson, *Analyzing Jazz: A Schenkerian Approach* (Hillsdale, NY: Pendragon Press, 2009), 2.

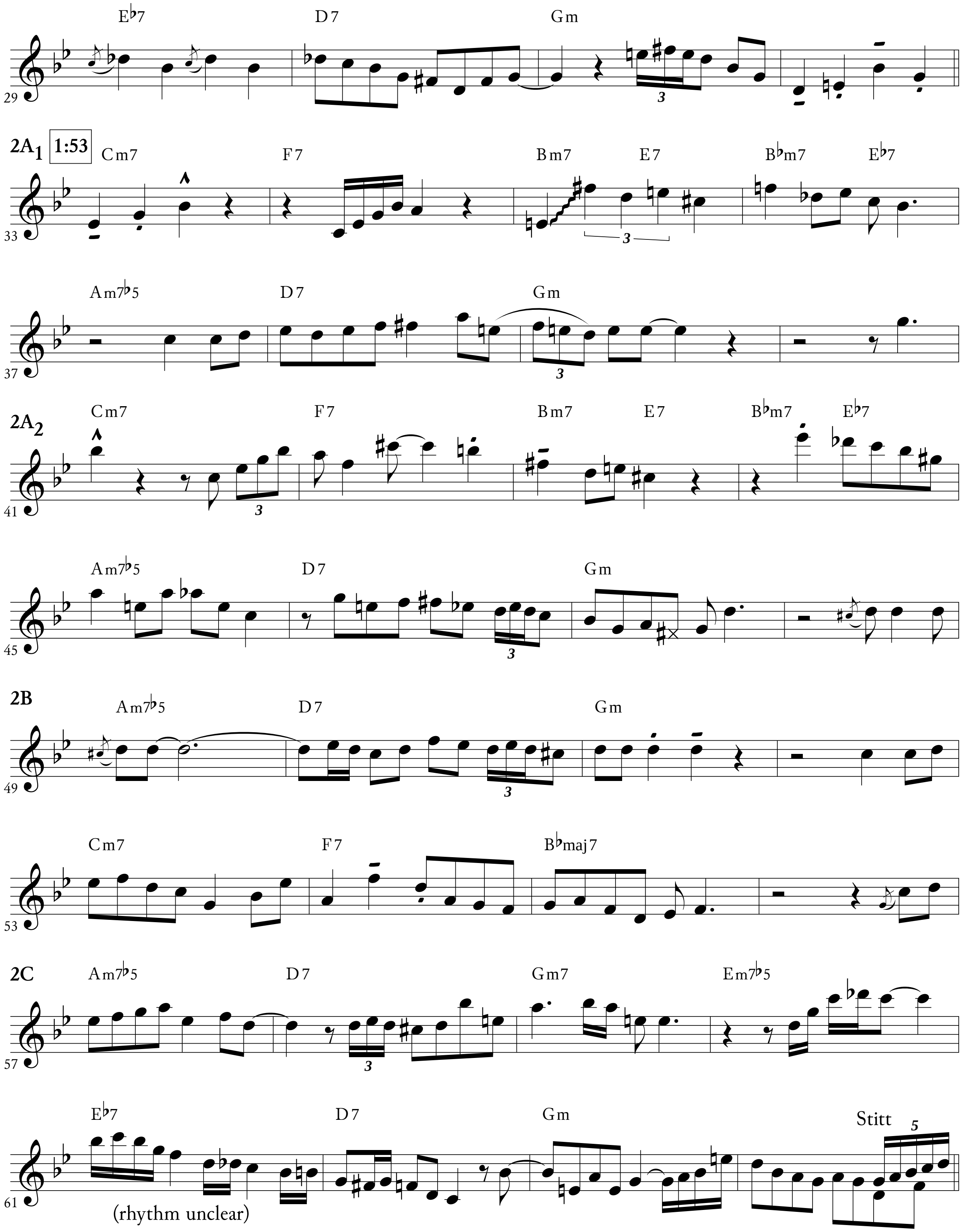
Autumn Leaves

As played on *Boss Tenors* (1961)
Gene Ammons/Sonny Stitt, tenor saxophones

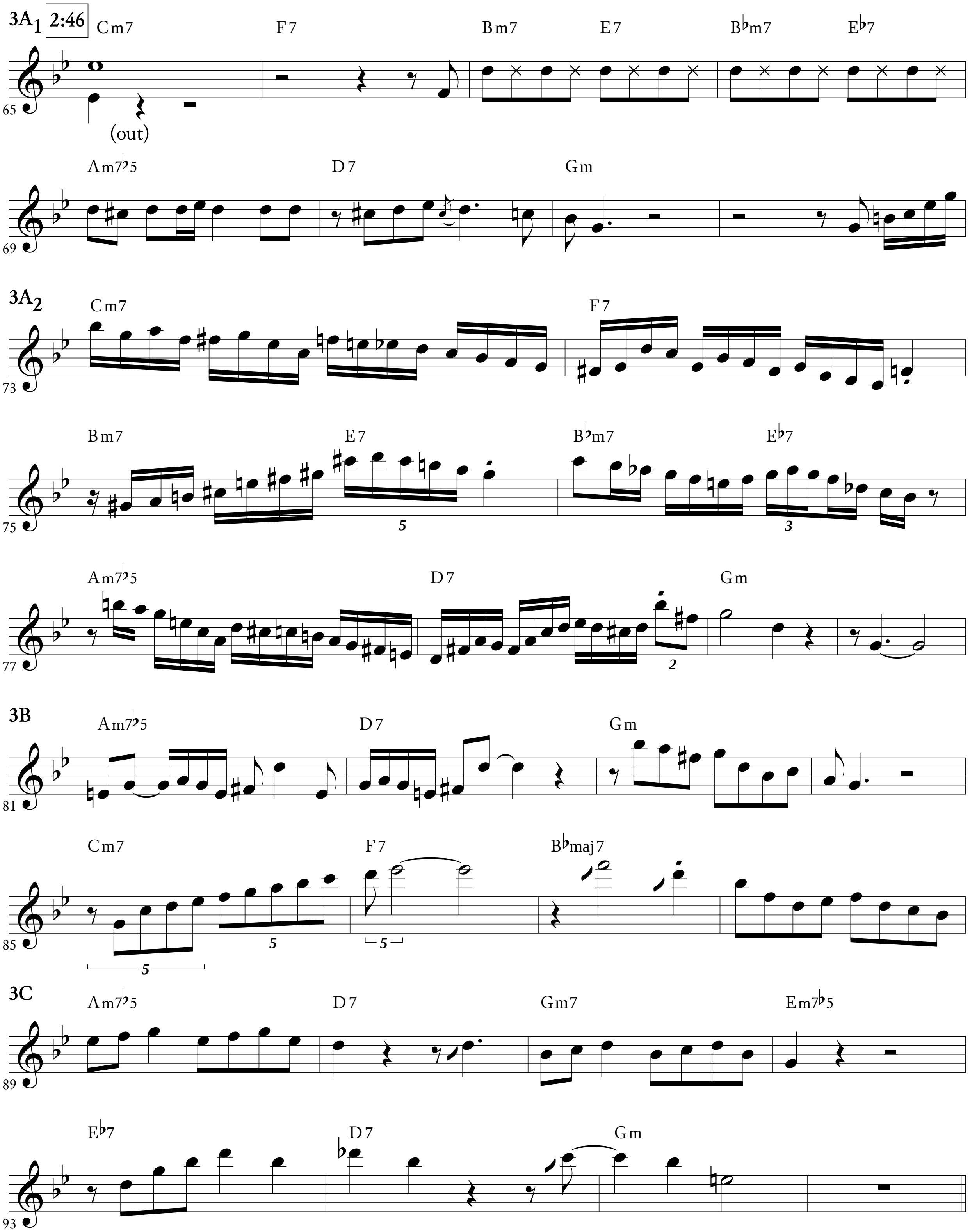
Joseph Kosma/Jacques Prévert
trans. Michael McClimon

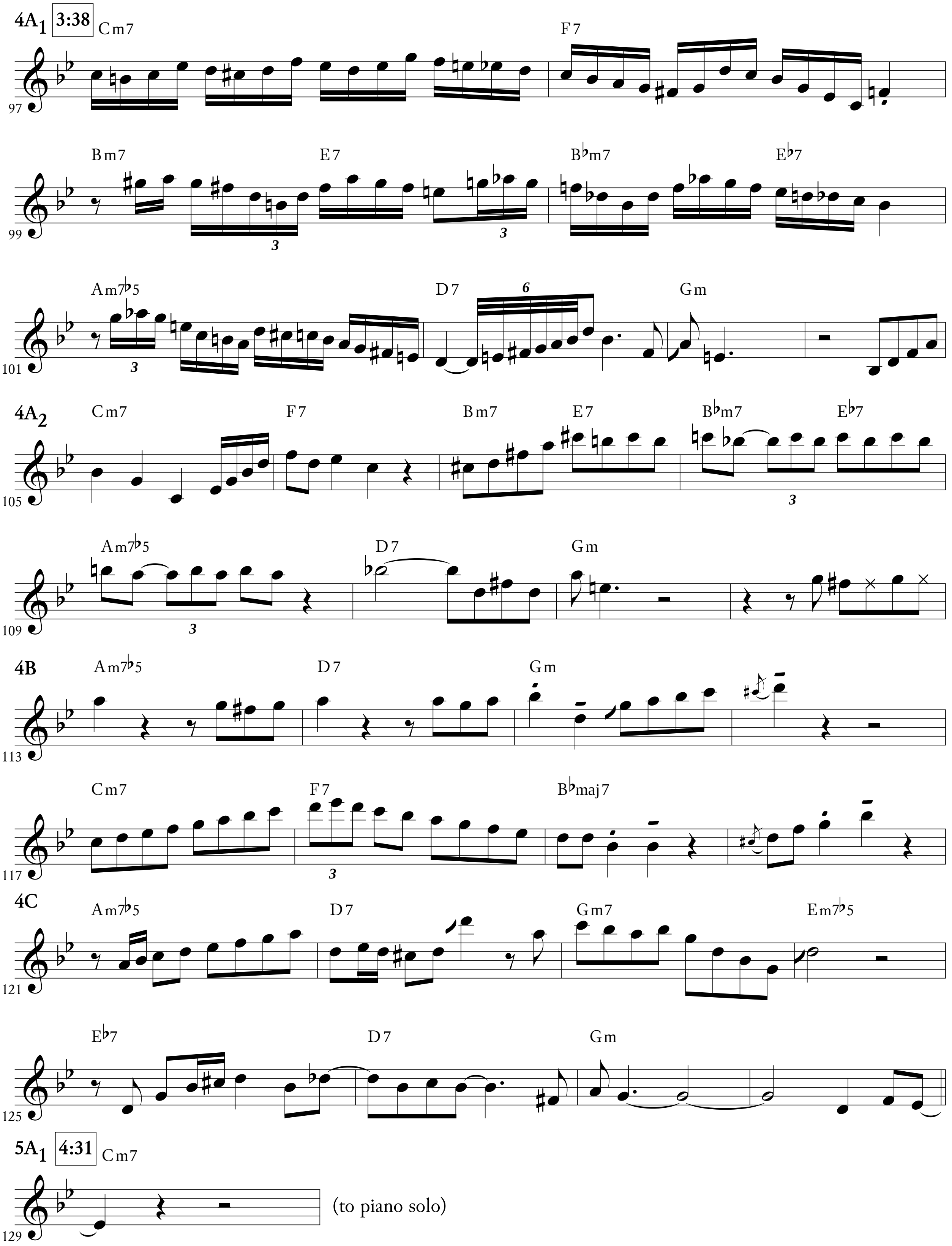

Ammons
1A1
1:02
Tenor Sax
Cm7
F7
Bm7
E7
B♭m7
E♭7
3
5
Am7♭5
D7
Gm
1A2
Cm7
F7
Bm7
E7
B♭m7
E♭7
9
3
3
Am7♭5
D7
Gm
13
1B
Am7♭5
D7
Gm
17
Cm7
F7
21
B♭maj7
23
1C
Am7♭5
D7
Gm7
Em7♭5
3
3
25

The head changes for this recording are as given in Figure 1.6, but the solo changes shown here are slightly different. The third and fourth bars of each A section consistently substitute Bm7–E7–B♭m7–E♭7 for B♭maj7–E♭maj7, and the progression Gm7–C7–Fm7–B♭7 in the third and fourth bars of the C sections appear here as Gm7–Em7♭5.



E♭7
D7
Gm
29
3
2A1
1:53
Cm7
F7
Bm7
E7
B♭m7
E♭7
33
3
Am7♭5
D7
Gm
37
3
2A2
Cm7
F7
Bm7
E7
B♭m7
E♭7
41
3
Am7♭5
D7
Gm
45
3
2B
Am7♭5
D7
Gm
49
3
Cm7
F7
B♭maj7
53
2C
Am7♭5
D7
Gm7
Em7♭5
57
3
E♭7
D7
Gm
Stitt
5
61
(rhythm unclear)

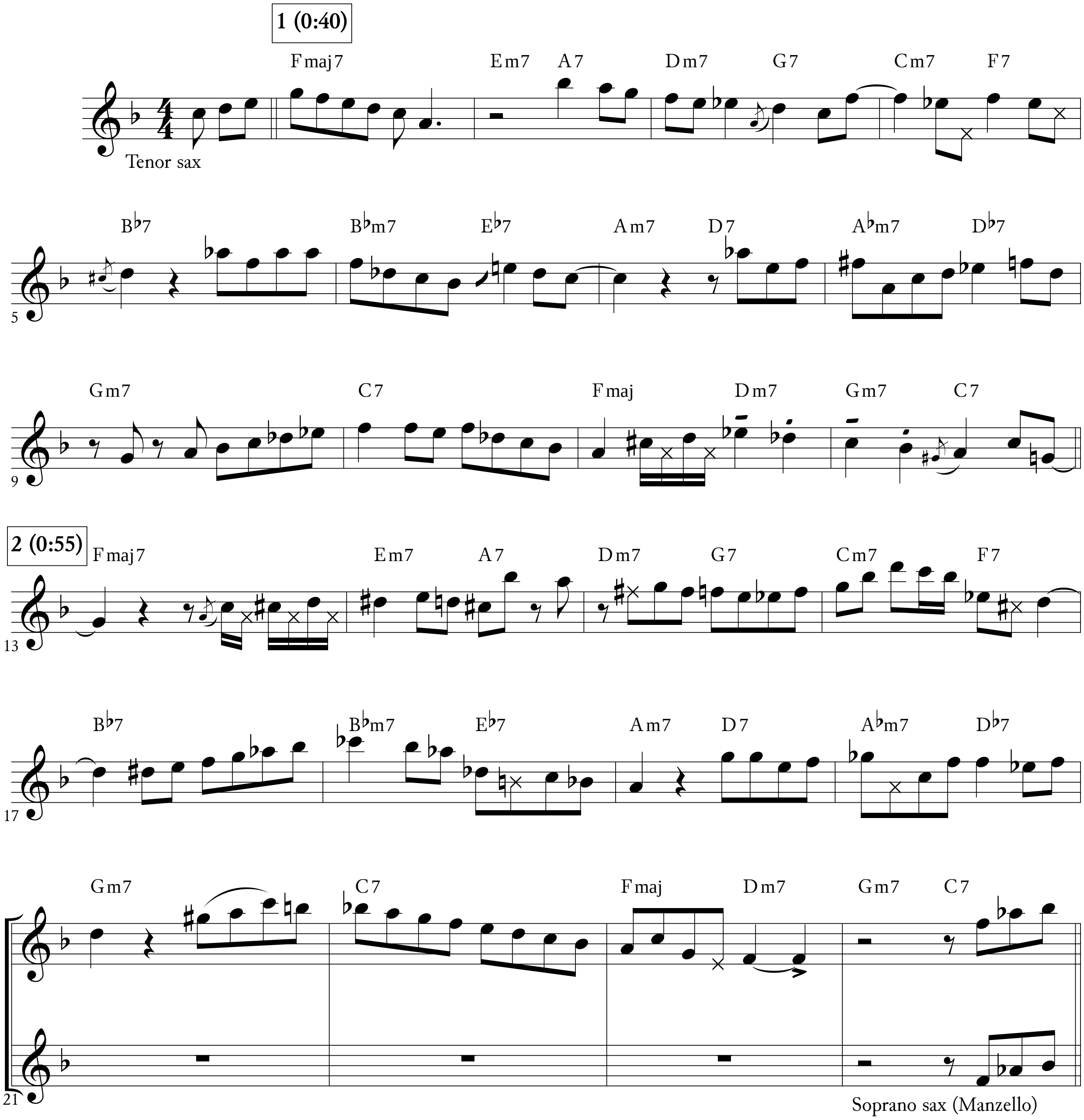

3A1
2:46
Cm7
F7
Bm7
E7
B♭m7
E♭7
(out)
65
Am7♭5
D7
Gm
69
3A2
Cm7
F7
73
Bm7
E7
5
B♭m7
E♭7
3
75
Am7♭5
D7
2
Gm
77
3B
Am7♭5
D7
Gm
81
Cm7
F7
B♭maj7
5
5
5
85
3C
Am7♭5
D7
Gm7
Em7♭5
89
E♭7
D7
Gm
93


4A1
3:38
Cm7
F7
97
Bm7
E7
B♭m7
E♭7
99
3
3
Am7♭5
D7
6
Gm
101
3
4A2
Cm7
F7
Bm7
E7
B♭m7
E♭7
105
3
Am7♭5
D7
Gm
109
3
4B
Am7♭5
D7
Gm
113
Cm7
F7
B♭maj7
117
3
4C
Am7♭5
D7
Gm7
Em7♭5
121
E♭7
D7
Gm
125
5A1
4:31
Cm7
129
(to piano solo)

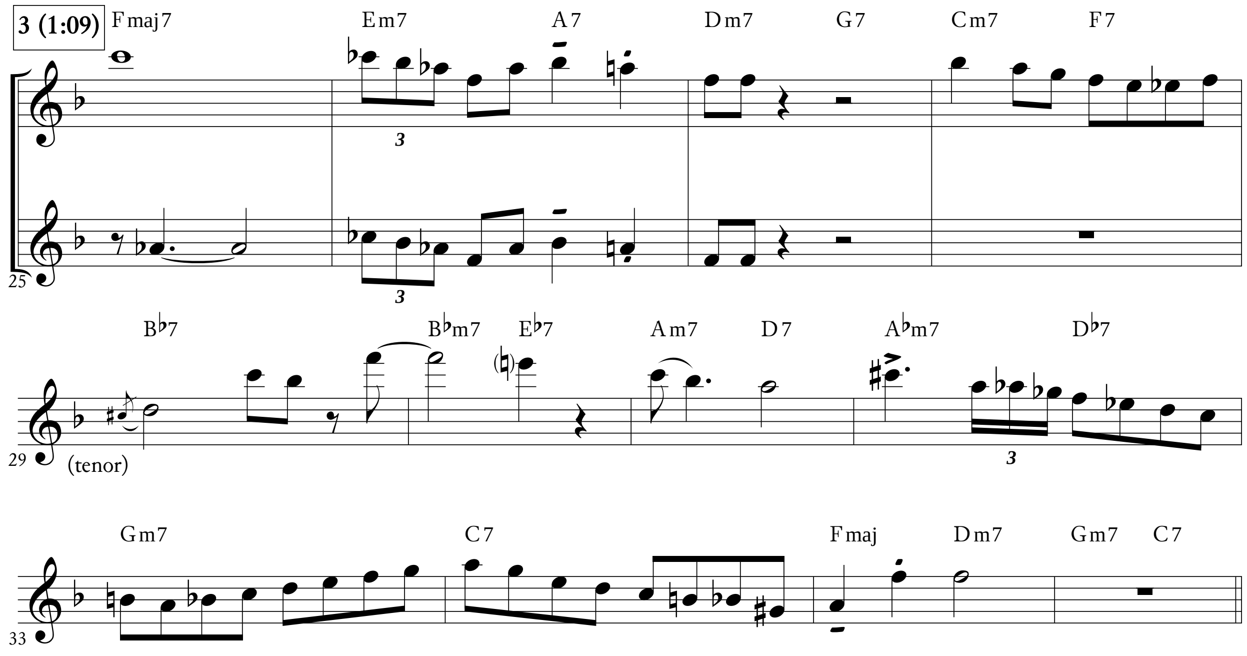
Blues for Alice

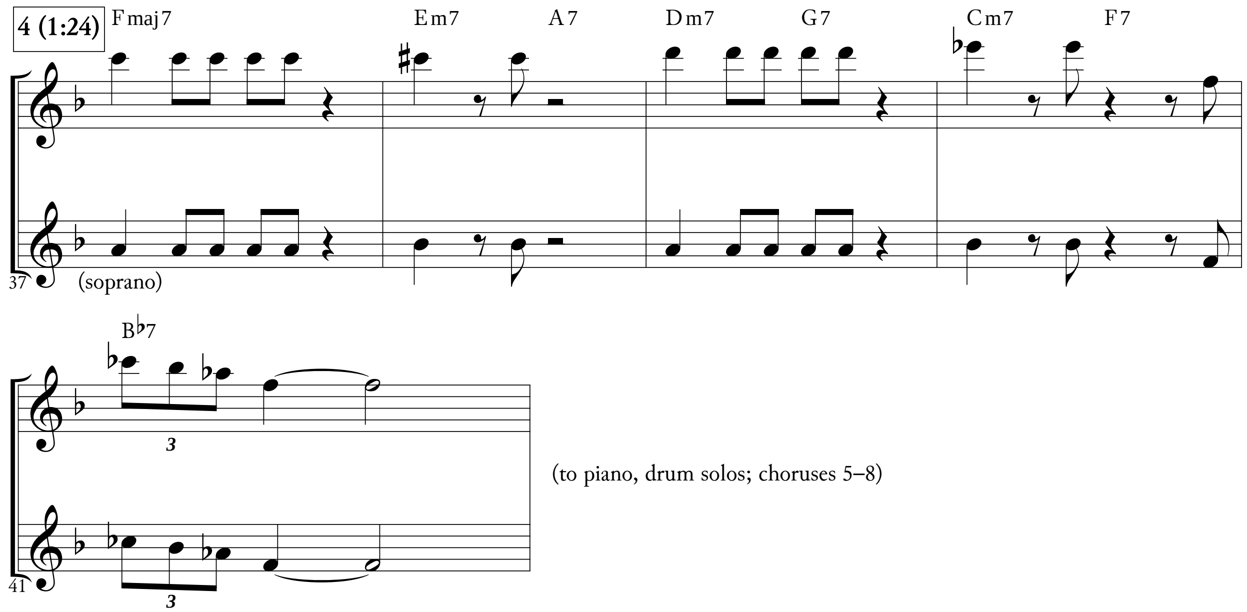
As played on *We Free Kings* (1961)
Rahsaan Roland Kirk, saxophones

Charlie Parker
trans. Michael McClimon

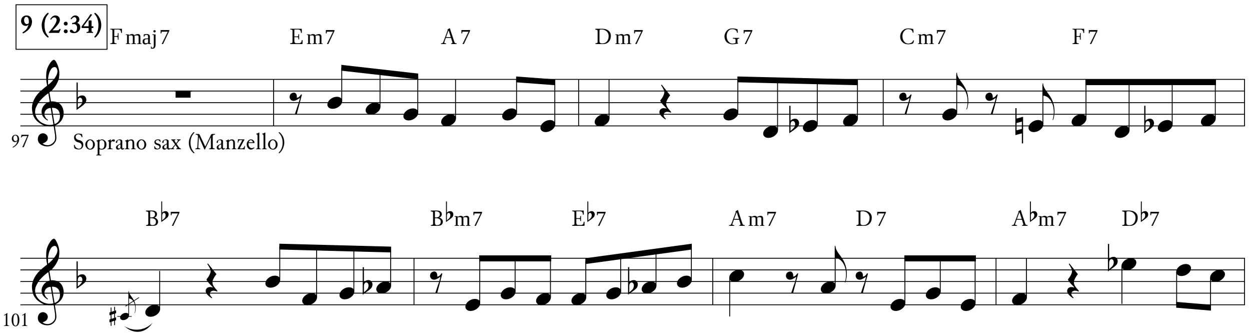

1 (0:40)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
Tenor sax
5
B♭7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
9
Gm7
C7
Fmaj
Dm7
Gm7
C7
2 (0:55)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
13
17
B♭7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
21
Gm7
C7
Fmaj
Dm7
Gm7
C7
Soprano sax (Manzello)

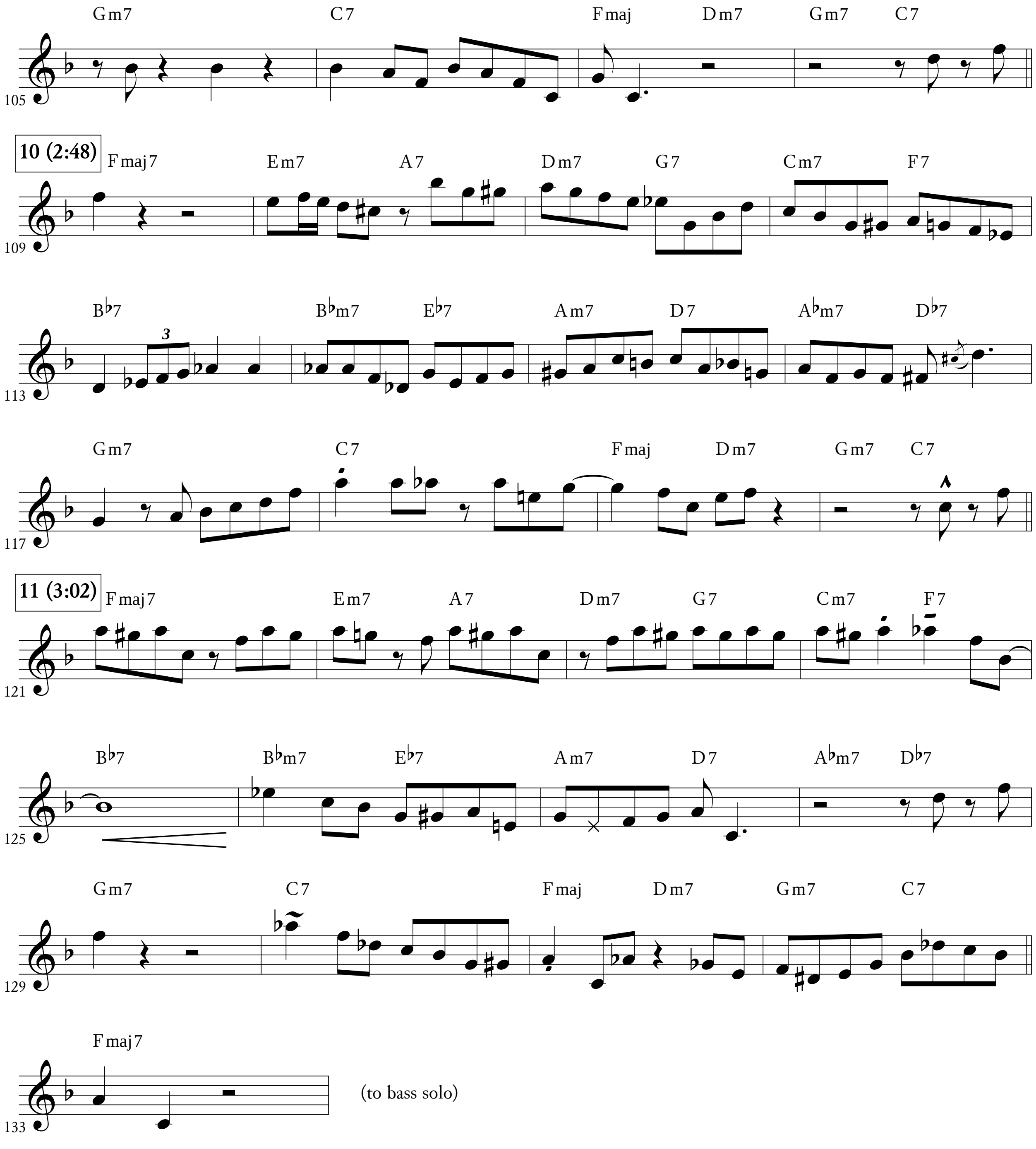
The album gives Kirk's name only as Roland Kirk; he added Rahsaan to his name in 1969. Kirk often played multiple instruments simultaneously; the transcription tries to make this clear, providing a separate staff for each instrument. The "manzello" is a modified soprano saxophone (both names are given here).


3 (1:09)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
3
25
B♭7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
29
(tenor)
Gm7
C7
Fmaj
Dm7
Gm7
C7
33


4 (1:24)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
37
(soprano)
B♭7
3
41

(to piano, drum solos; choruses 5–8)


9 (2:34)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
97
Soprano sax (Manzello)
B♭7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
101

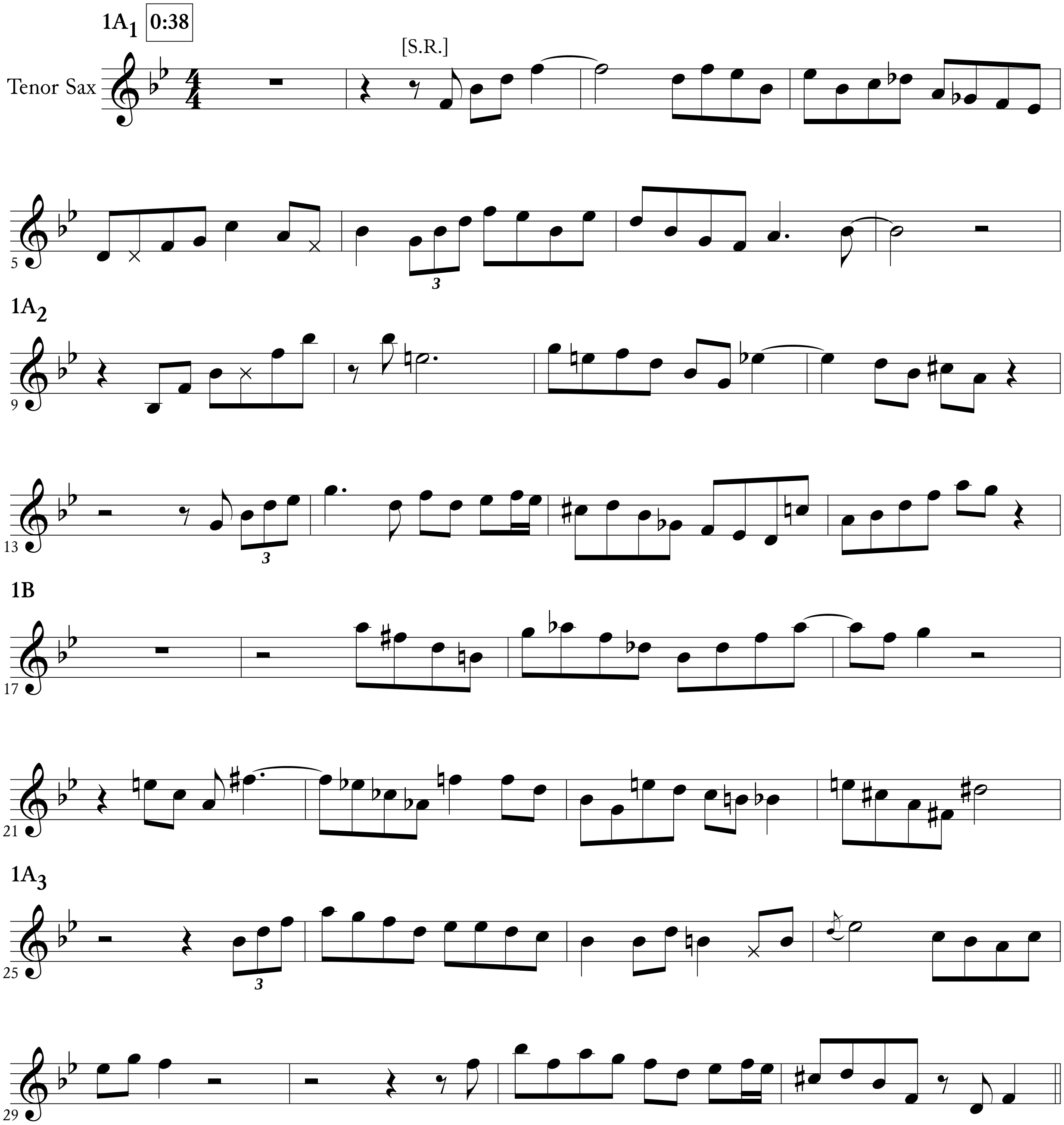

Gm7
C7
Fmaj
Dm7
Gm7
C7
105
10 (2:48)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
109
B♭7
3
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
113
Gm7
C7
Fmaj
Dm7
Gm7
C7
117
11 (3:02)
Fmaj7
Em7
A7
Dm7
G7
Cm7
F7
121
B♭7
B♭m7
E♭7
Am7
D7
A♭m7
D♭7
125
Gm7
C7
Fmaj
Dm7
Gm7
C7
129
Fmaj7
133

(to bass solo)

The Eternal Triangle

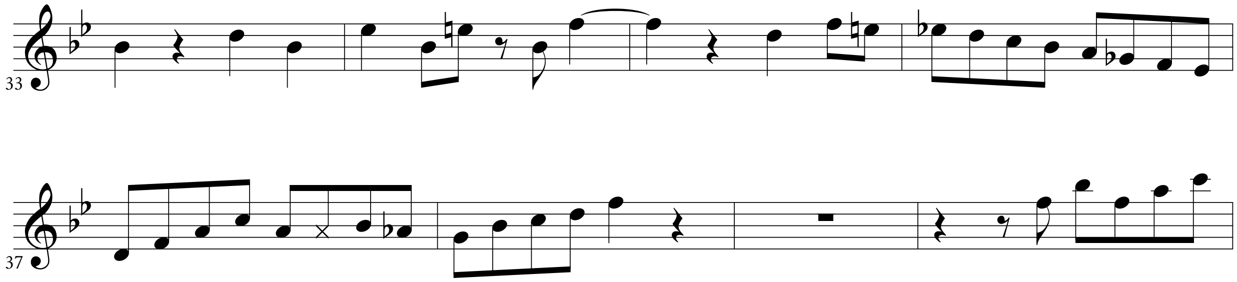
As played on *Sonny Side Up* (1958)
Sonny Rollins/Sonny Stitt, tenor saxophones

Sonny Stitt
trans. Michael McClimon

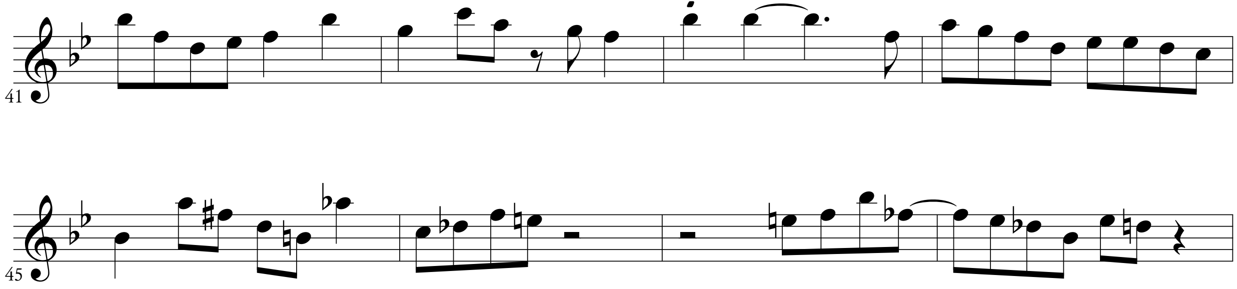

1A₁
0:38
[S.R.]
Tenor Sax
1A₂
1B
1A₃
3
5
9
13
17
21
25
29

Sonny Rollins is indicated in the transcription with [S.R], and Sonny Stitt with [S.S.].

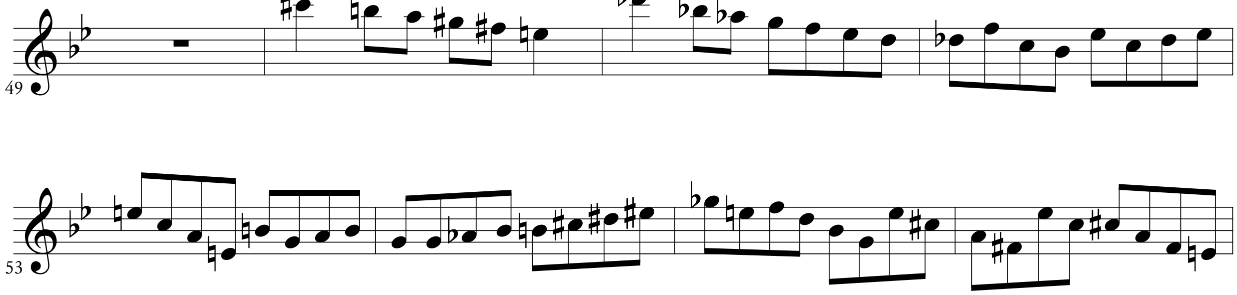
2A$_1$ 1:05



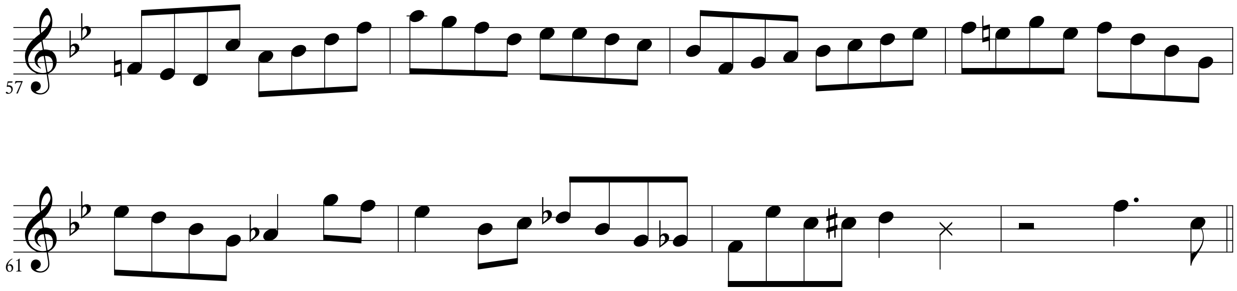
2A$_2$



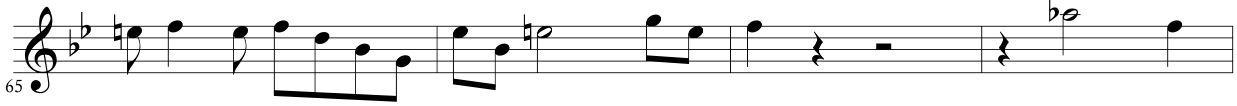
2B

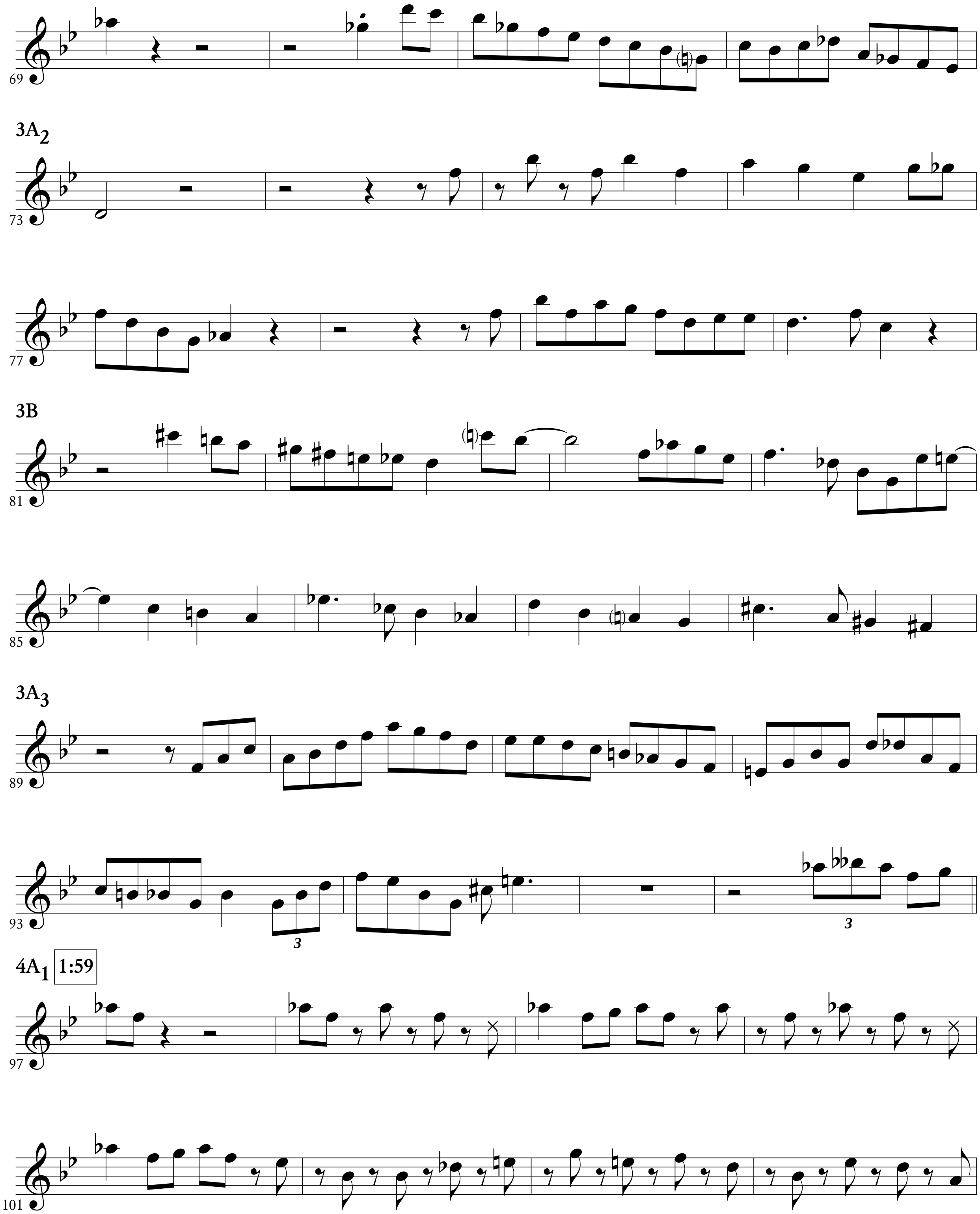


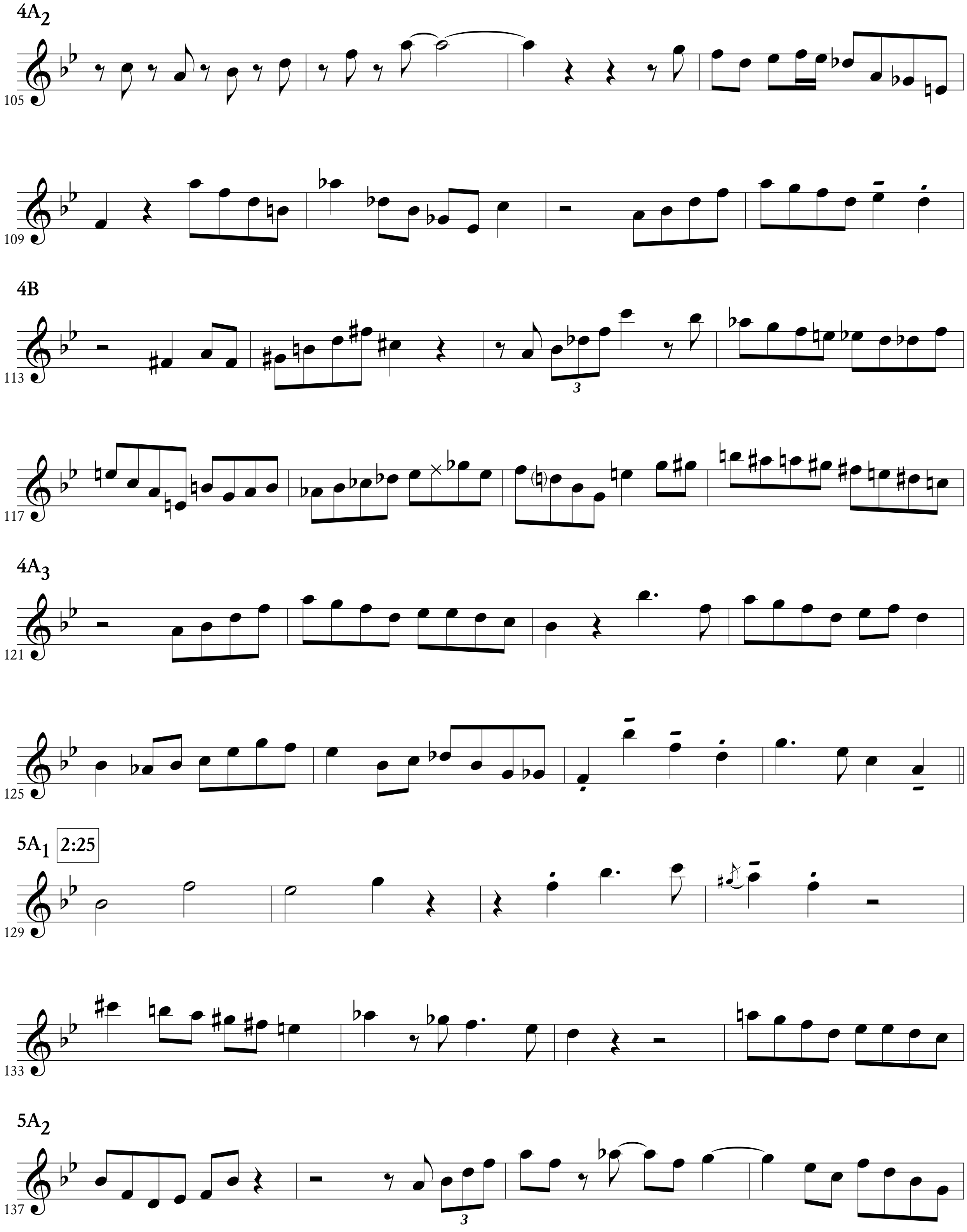
2A$_3$

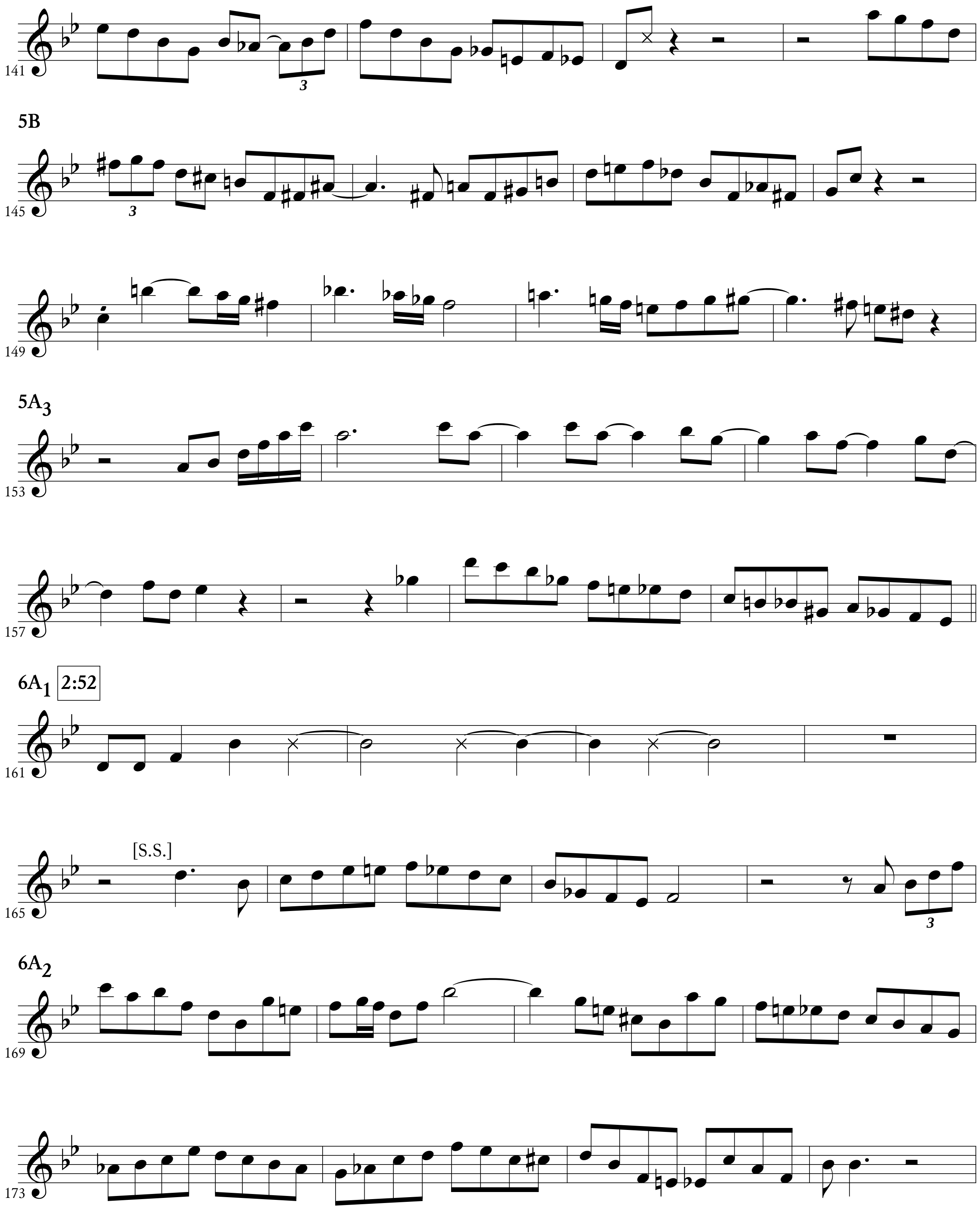


3A$_1$ 1:32

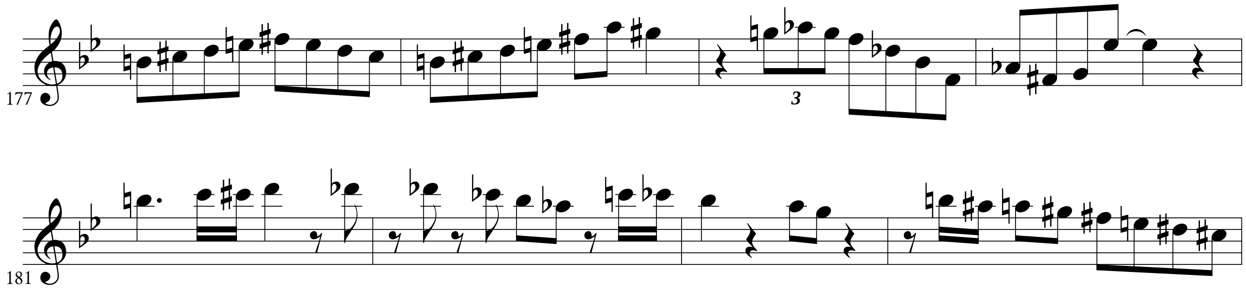



3A2
3B
3A3
4A1
1:59

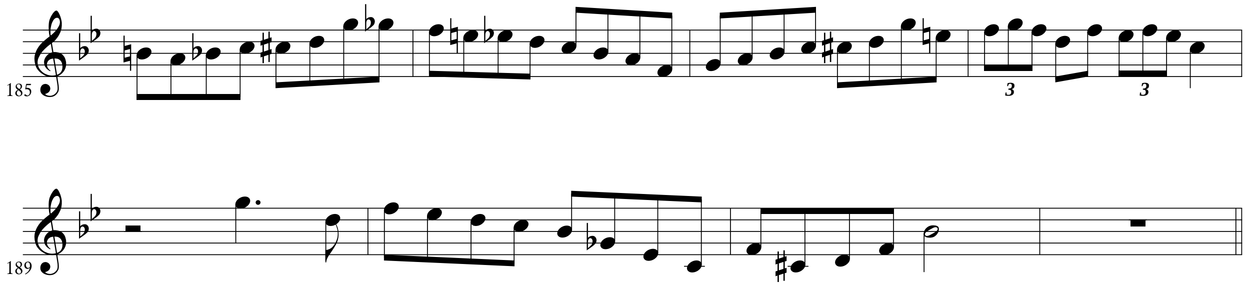

4A₂
4B
4A₃
5A₁
2:25
5A₂


5B
5A3
6A1
2:52
[S.S.]
6A2

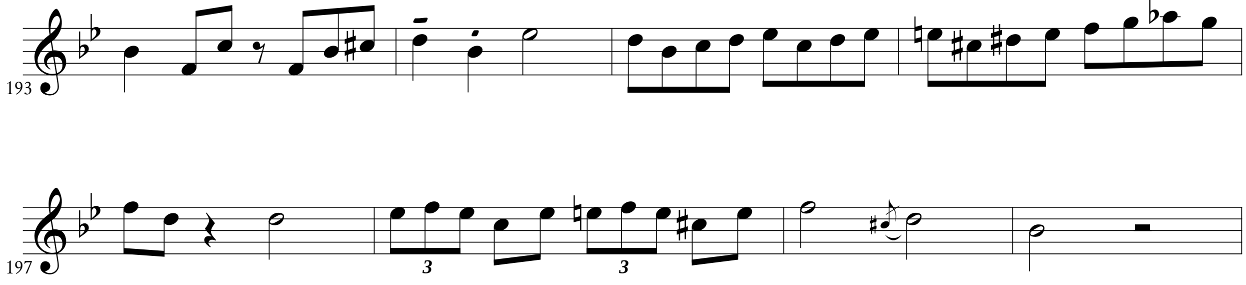
6B



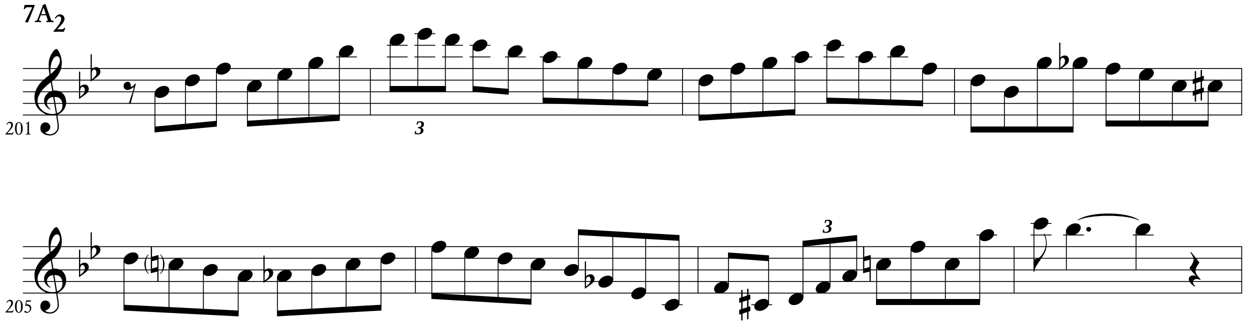
6A$_3$



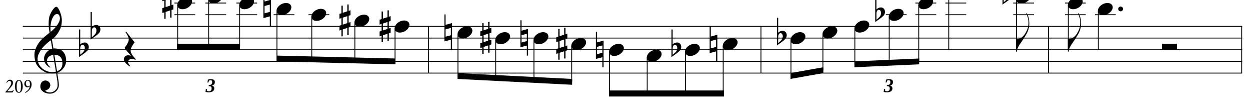
7A$_1$ 3:18

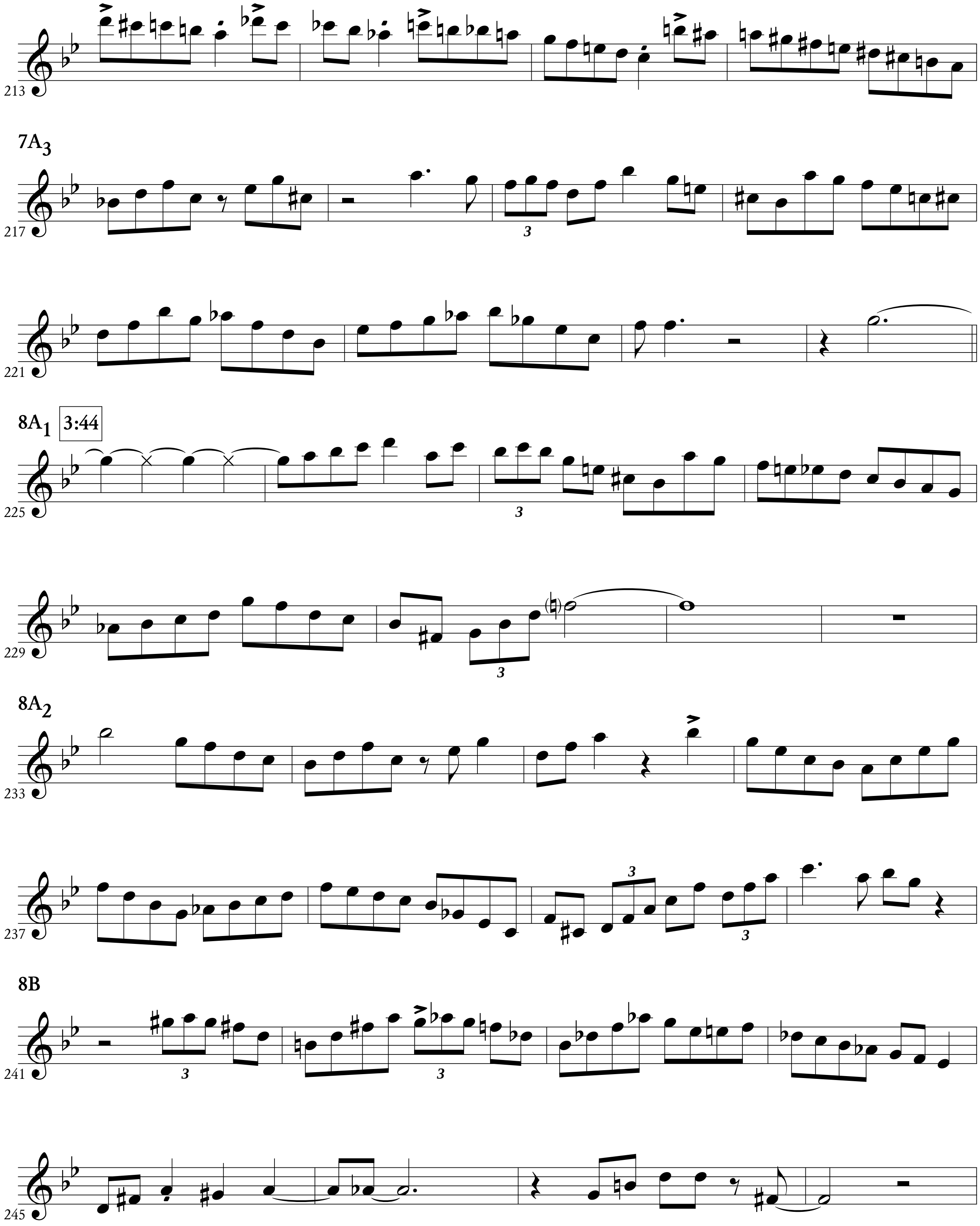


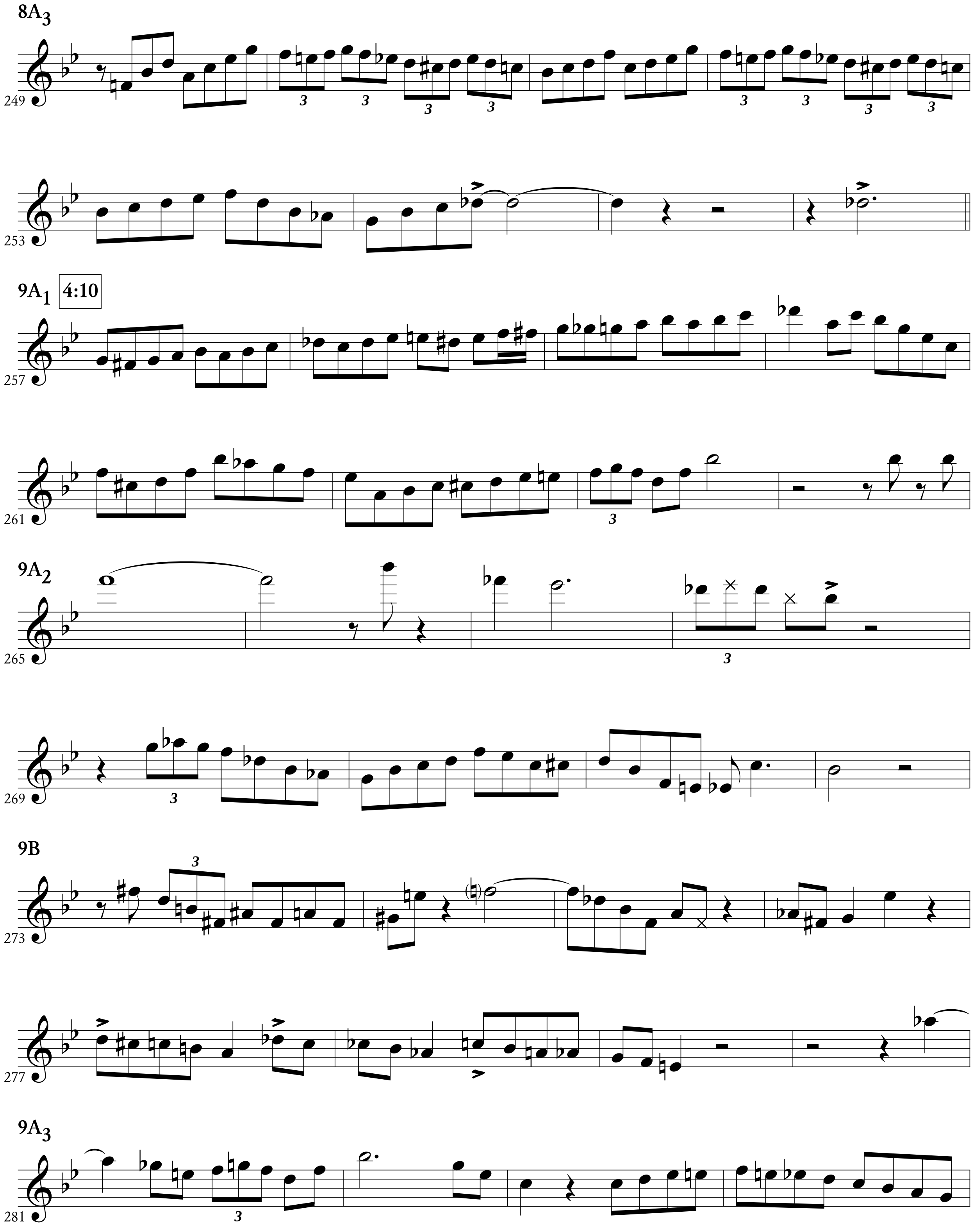
7A$_2$

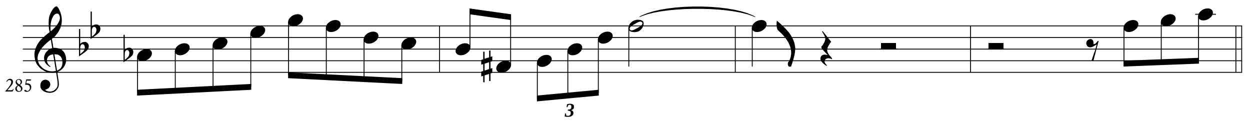


7B

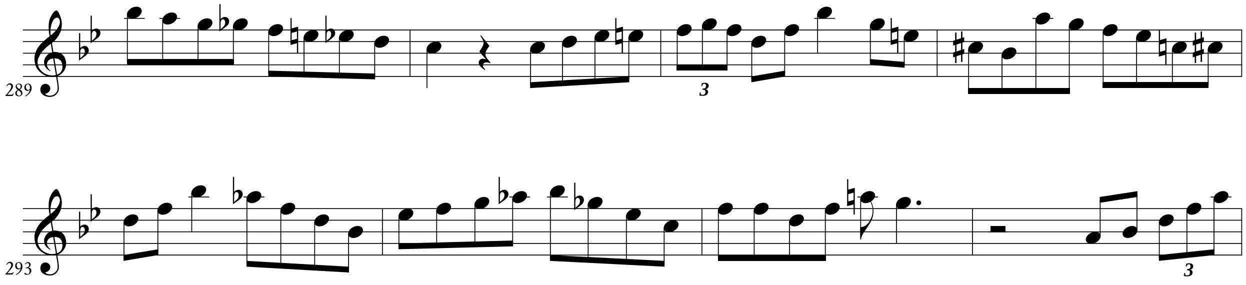



7A3
8A1
3:44
8A2
8B

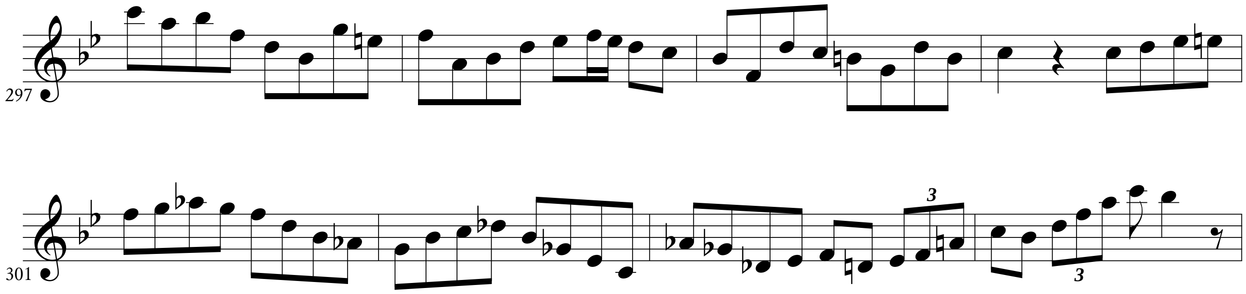

8A₃
9A₁
4:10
9A₂
9B
9A₃



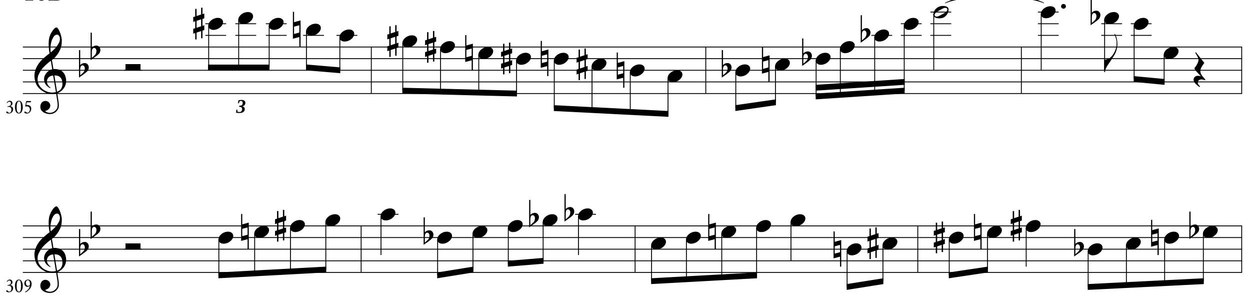
10A$_1$ $\boxed{4:37}$



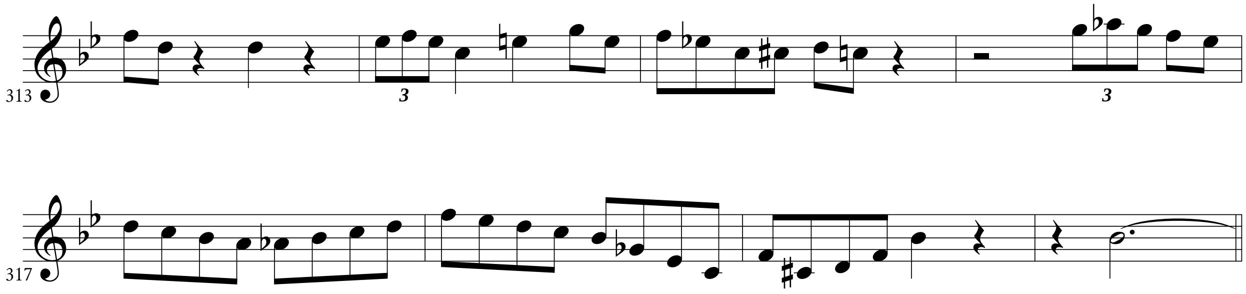
10A$_2$

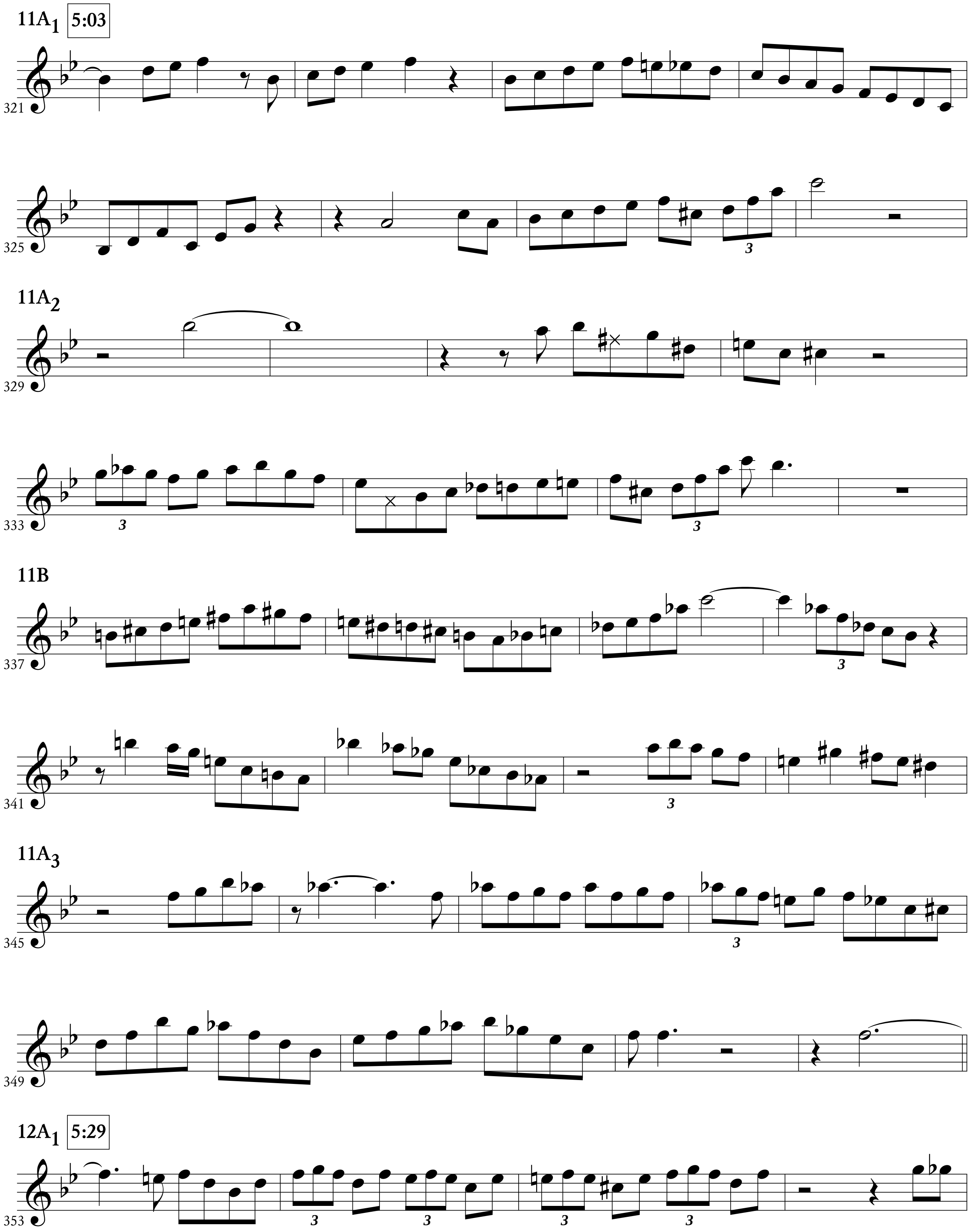


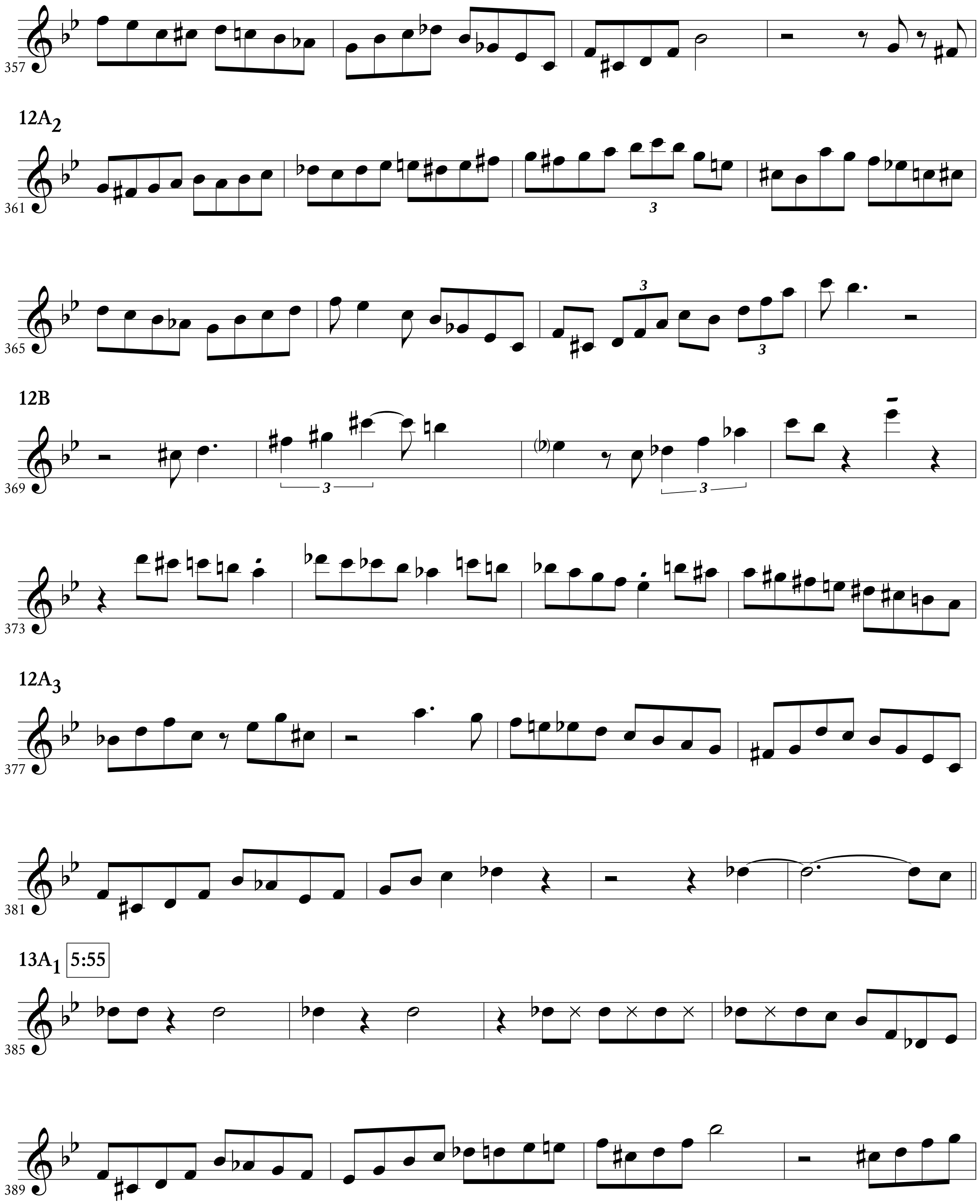
10B

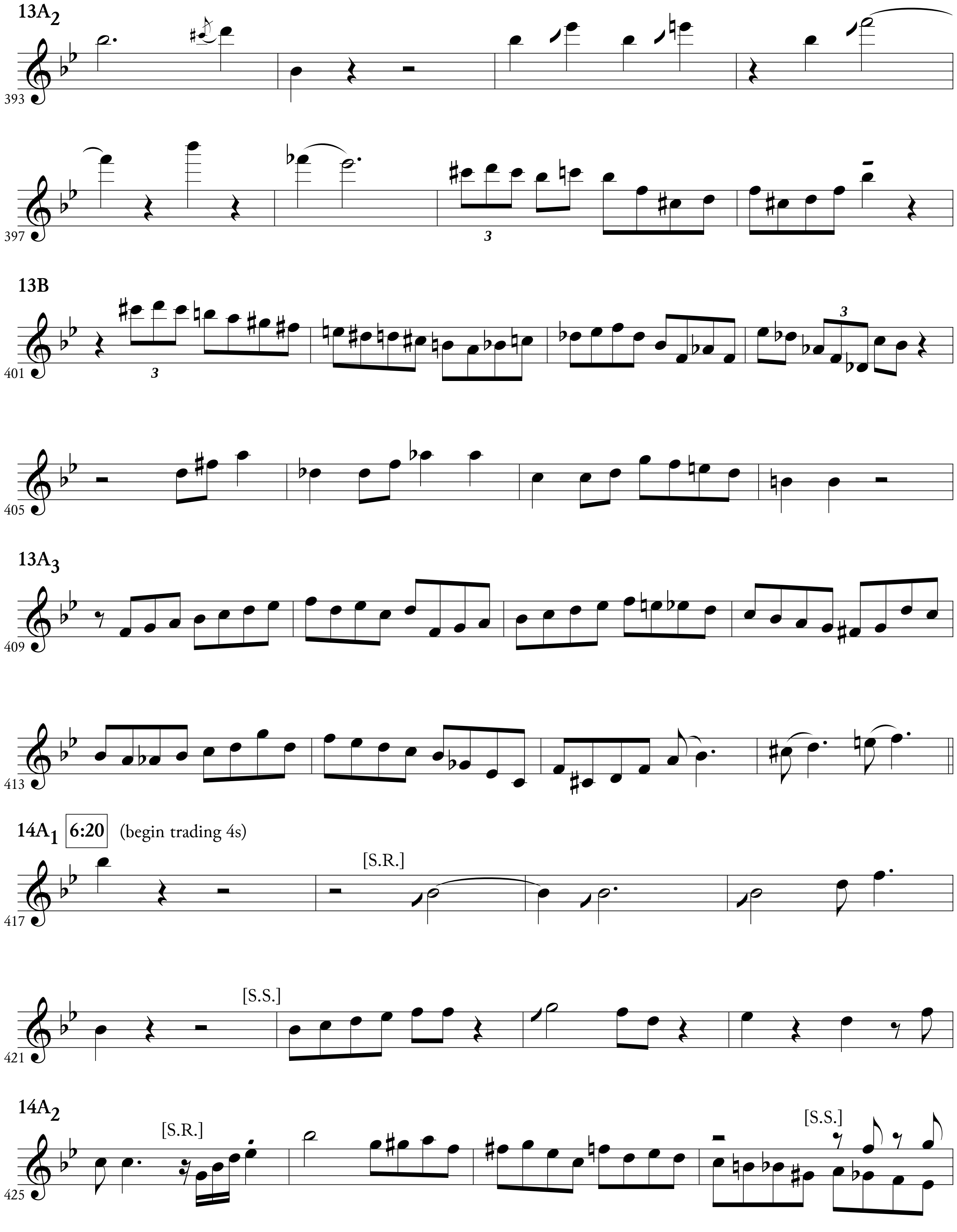


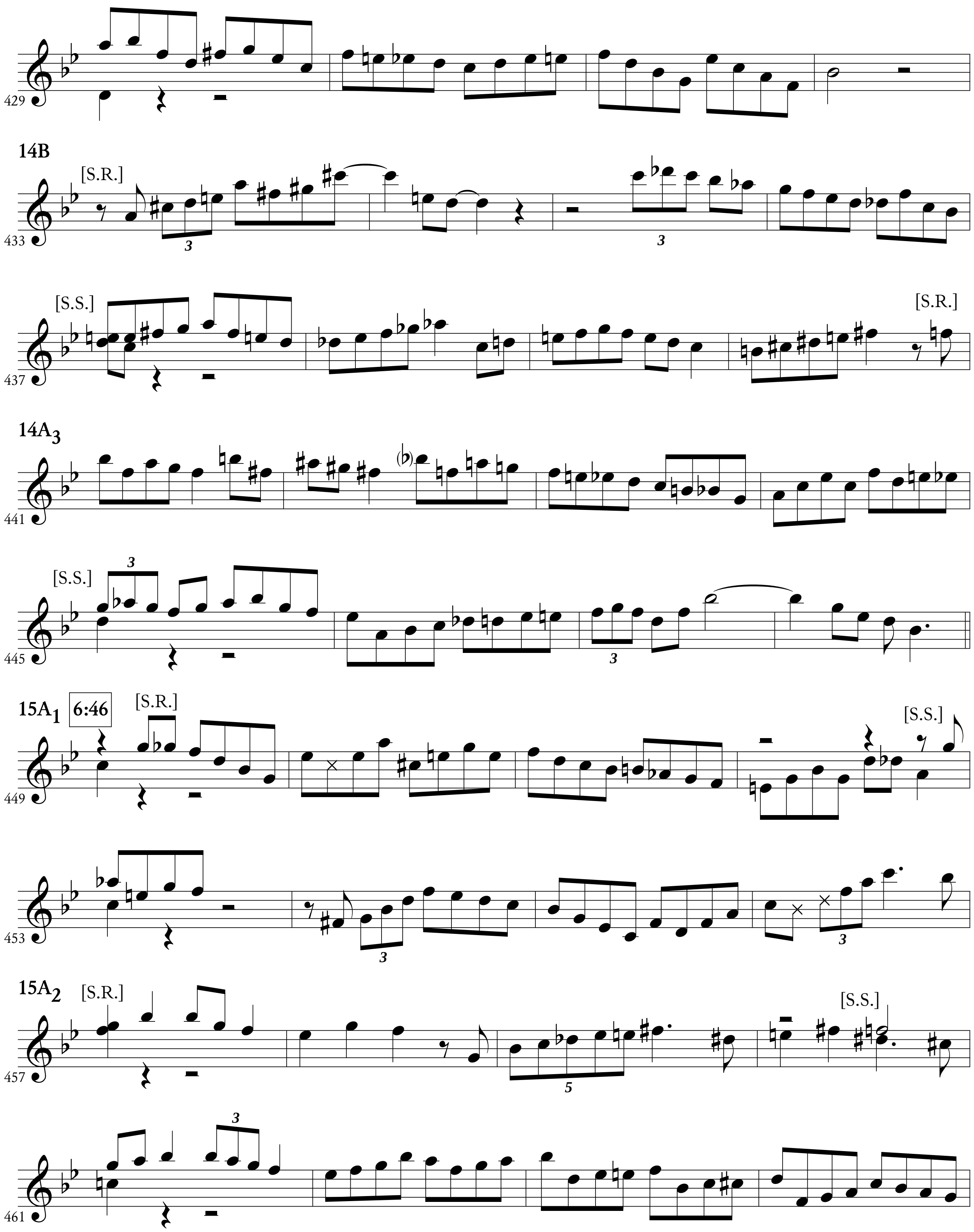
10A$_3$



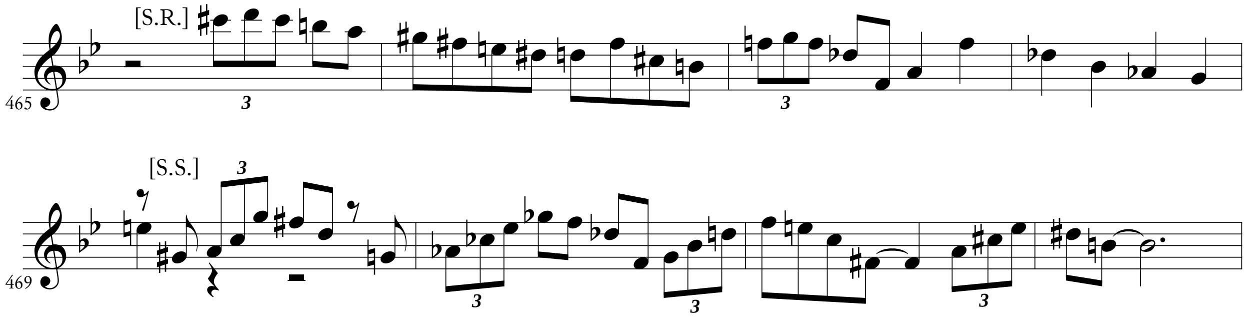

11A1
5:03
11A2
11A3
12A1
5:29


12A2
12A3
13A1
5:55

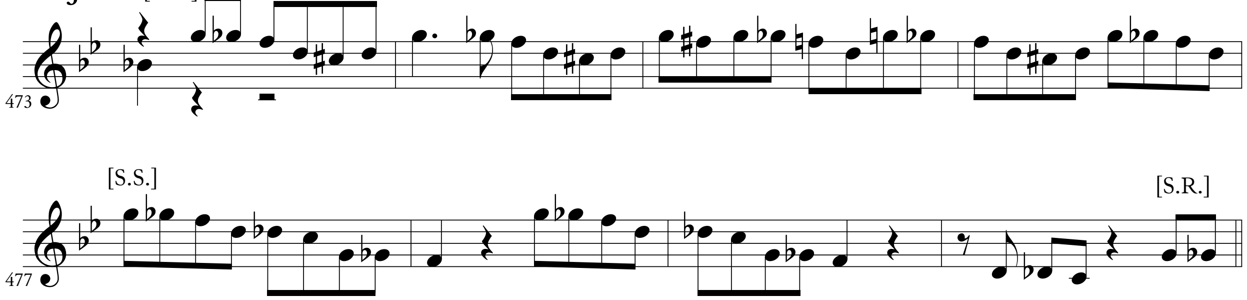

13A2
393
397
3
13B
401
3
3
405
13A3
409
413
14A1
6:20
(begin trading 4s)
417
[S.R.]
421
[S.S.]
14A2
425
[S.R.]
[S.S.]


429
14B
[S.R.]
3
3
433
[S.S.]
[S.R.]
437
14A3
(♭)
441
[S.S.]
3
3
445
15A1
6:46
[S.R.]
[S.S.]
449
3
3
453
15A2
[S.R.]
5
[S.S.]
457
3
461

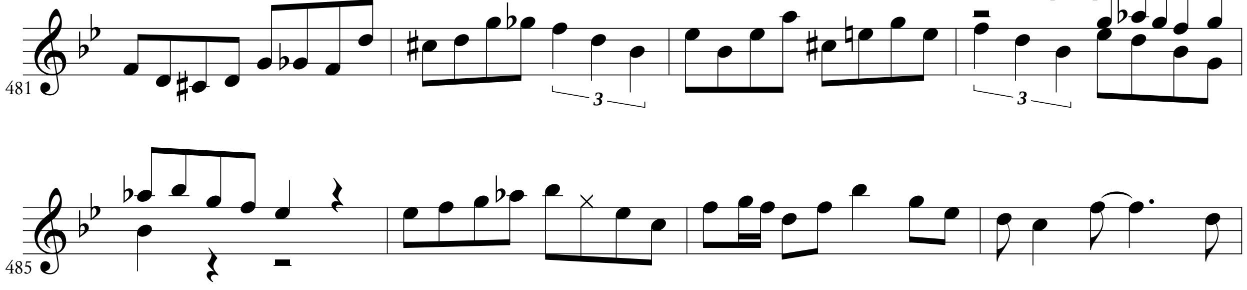
15B


[S.R.]
465
[S.S.]
469

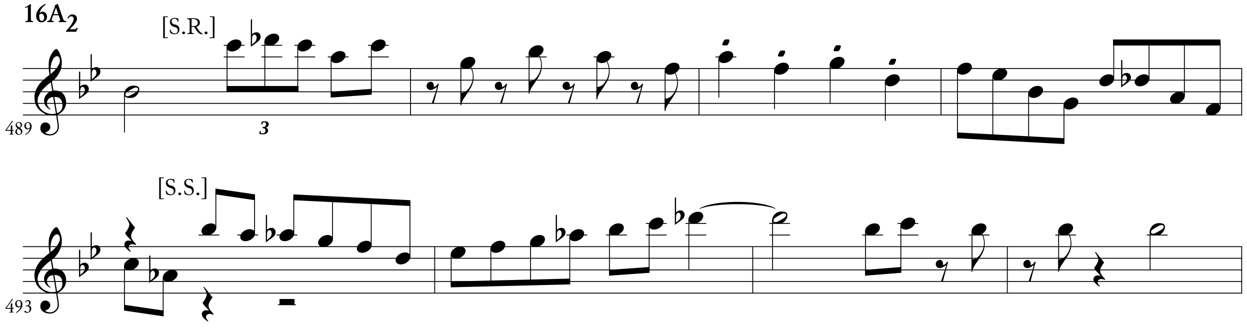
$15A_3$


[S.R.]
473
[S.S.]
[S.R.]
477

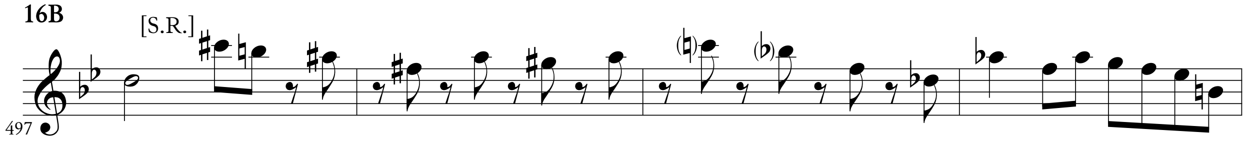
$16A_1$ 7:12

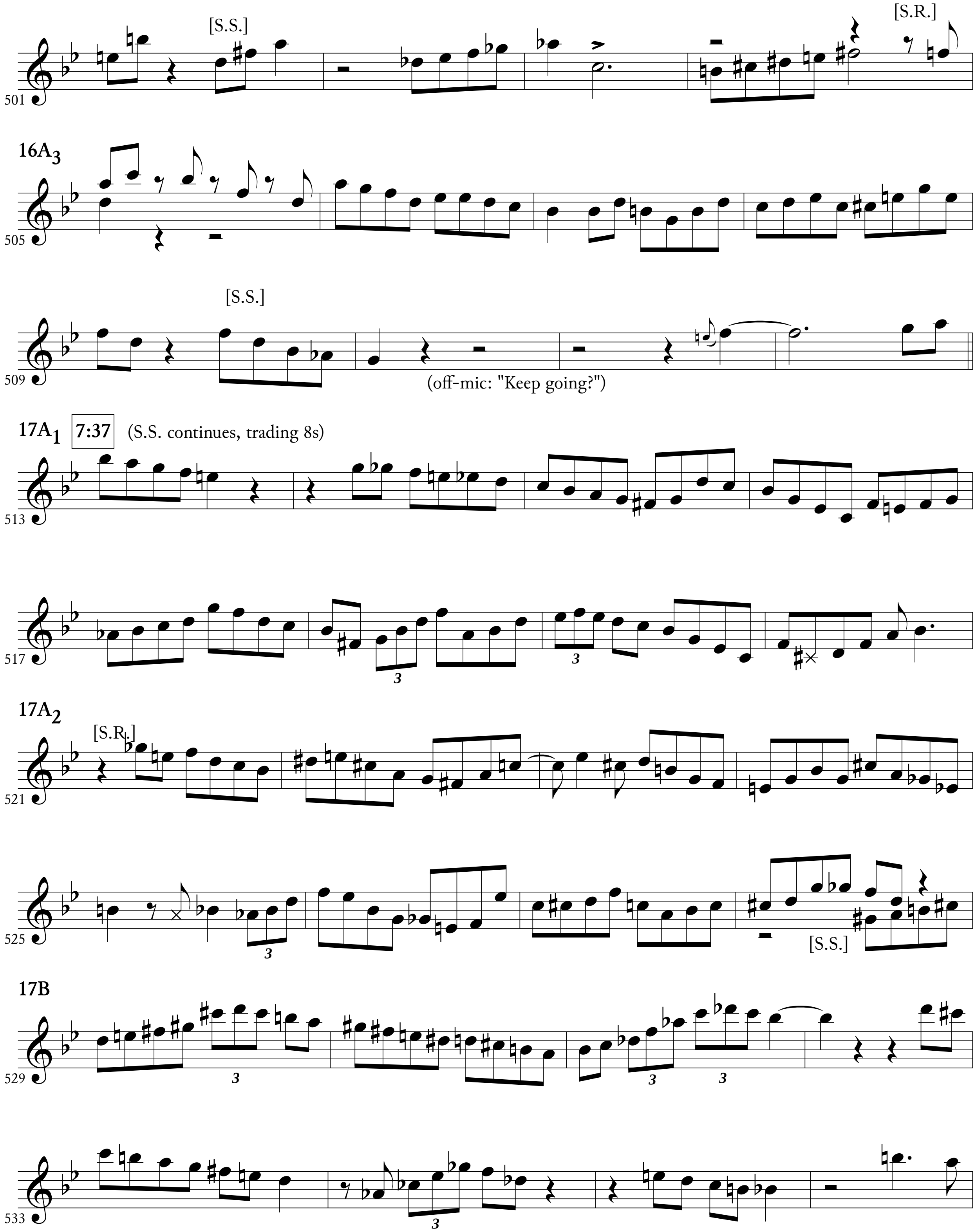


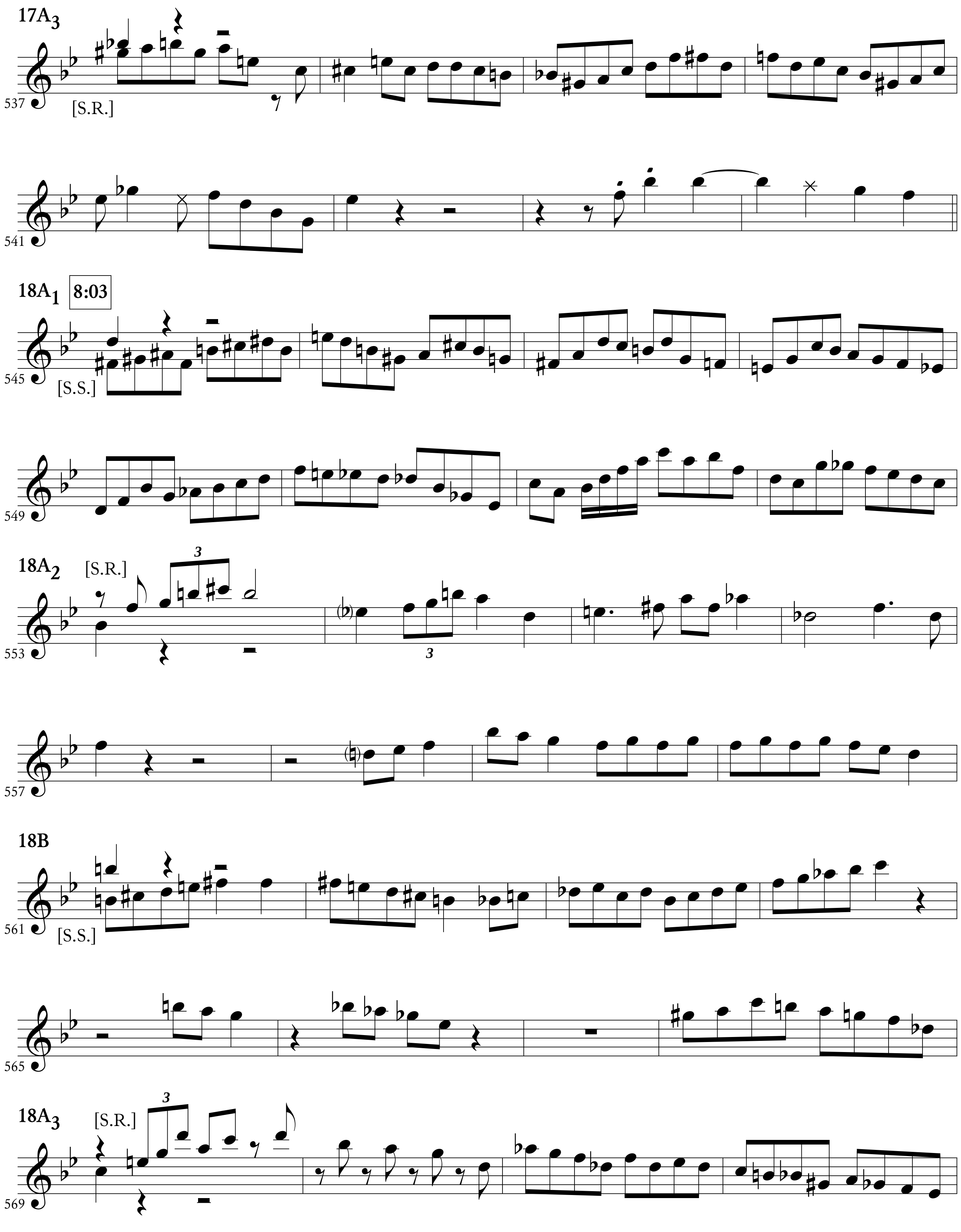
$16A_2$

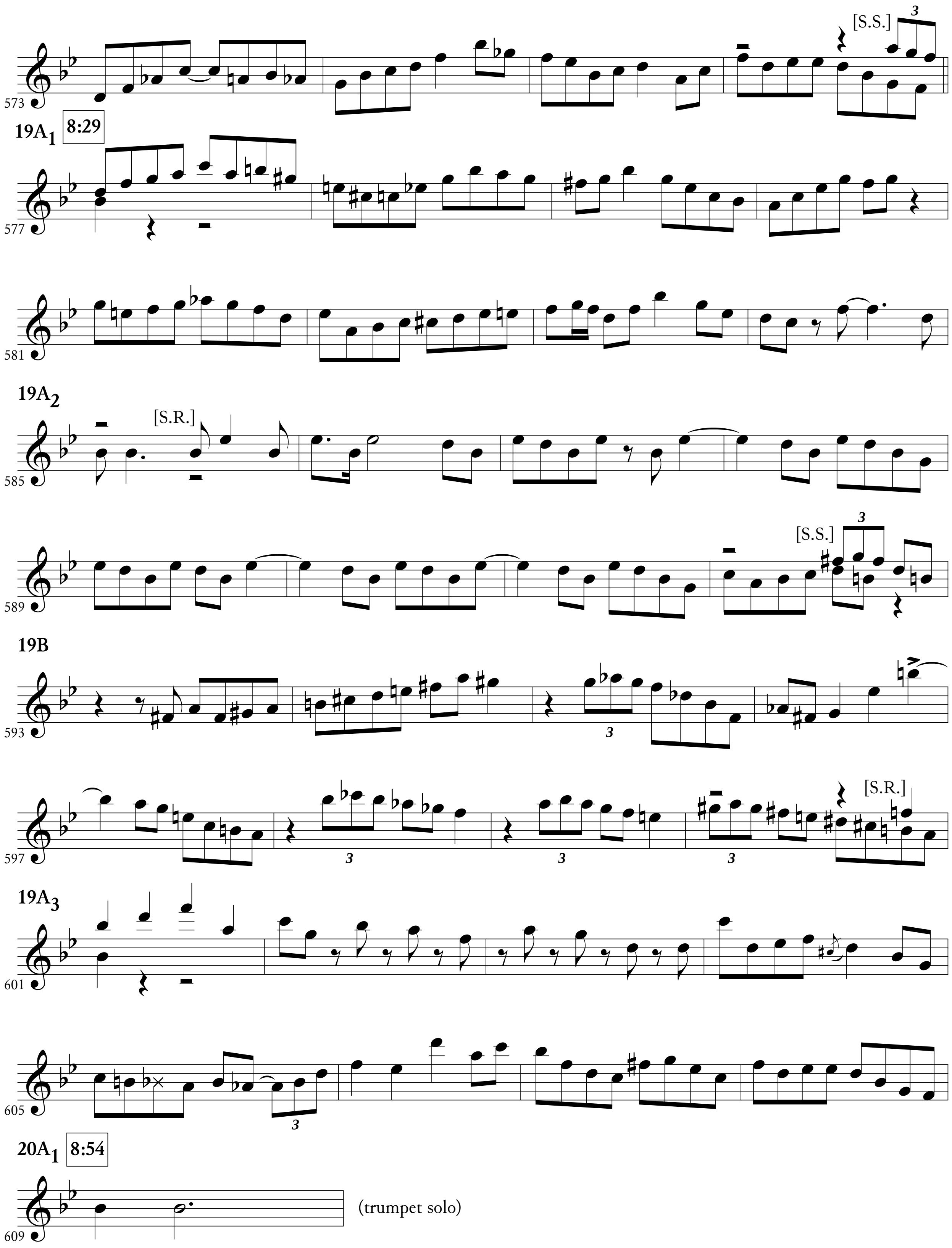

[S.R.]
489
[S.S.]
493

16B




[S.S.]
[S.R.]
501
16A3
505
[S.S.]
509
(off-mic: "Keep going?")
17A1
7:37
(S.S. continues, trading 8s)
513
3
517
17A2
[S.R.]
521
525
[S.S.]
17B
529
533


17A$_3$
[S.R.]
537
541
18A$_1$
8:03
[S.S.]
545
549
18A$_2$
[S.R.]
553
557
18B
[S.S.]
561
565
18A$_3$
[S.R.]
569

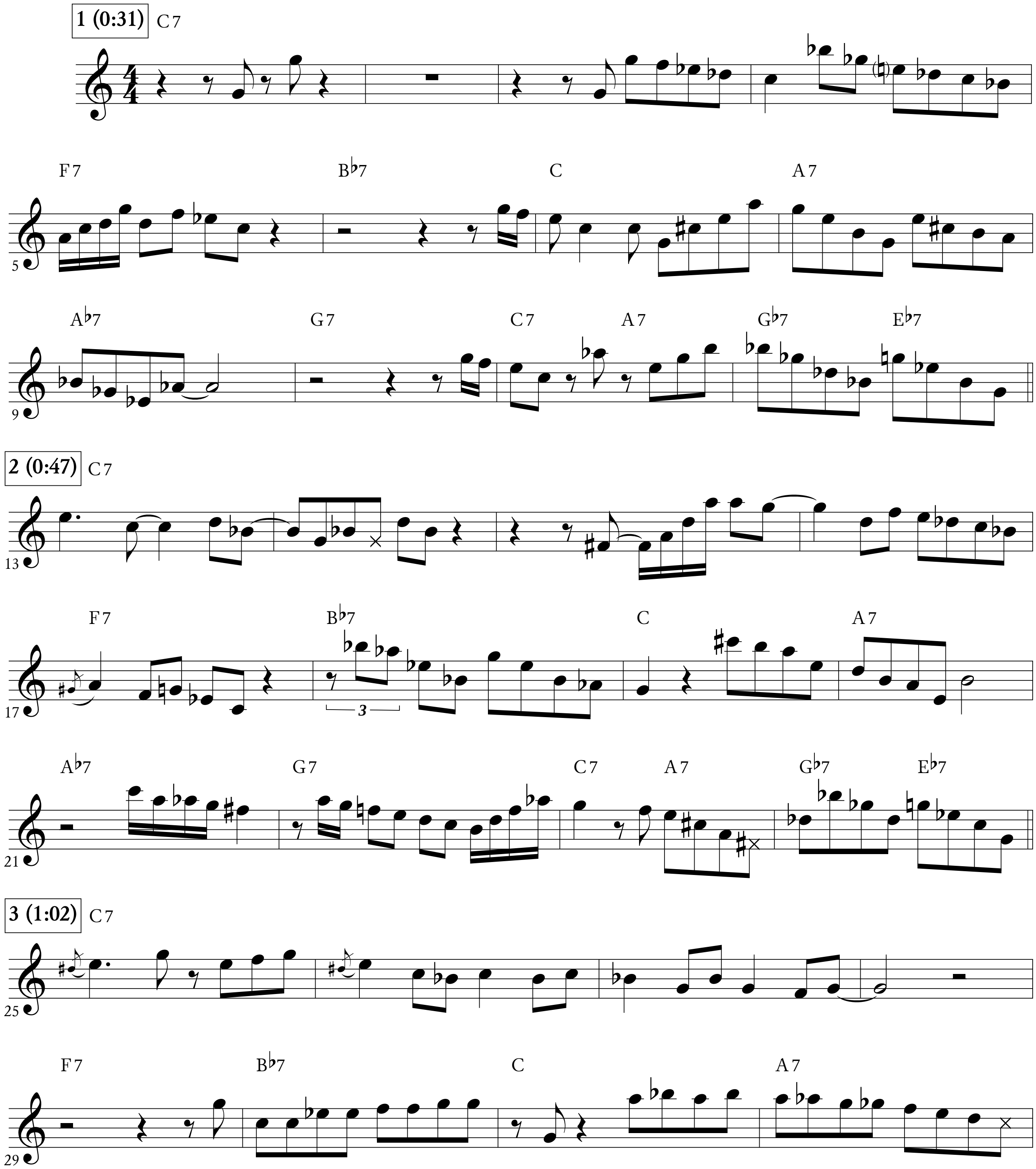


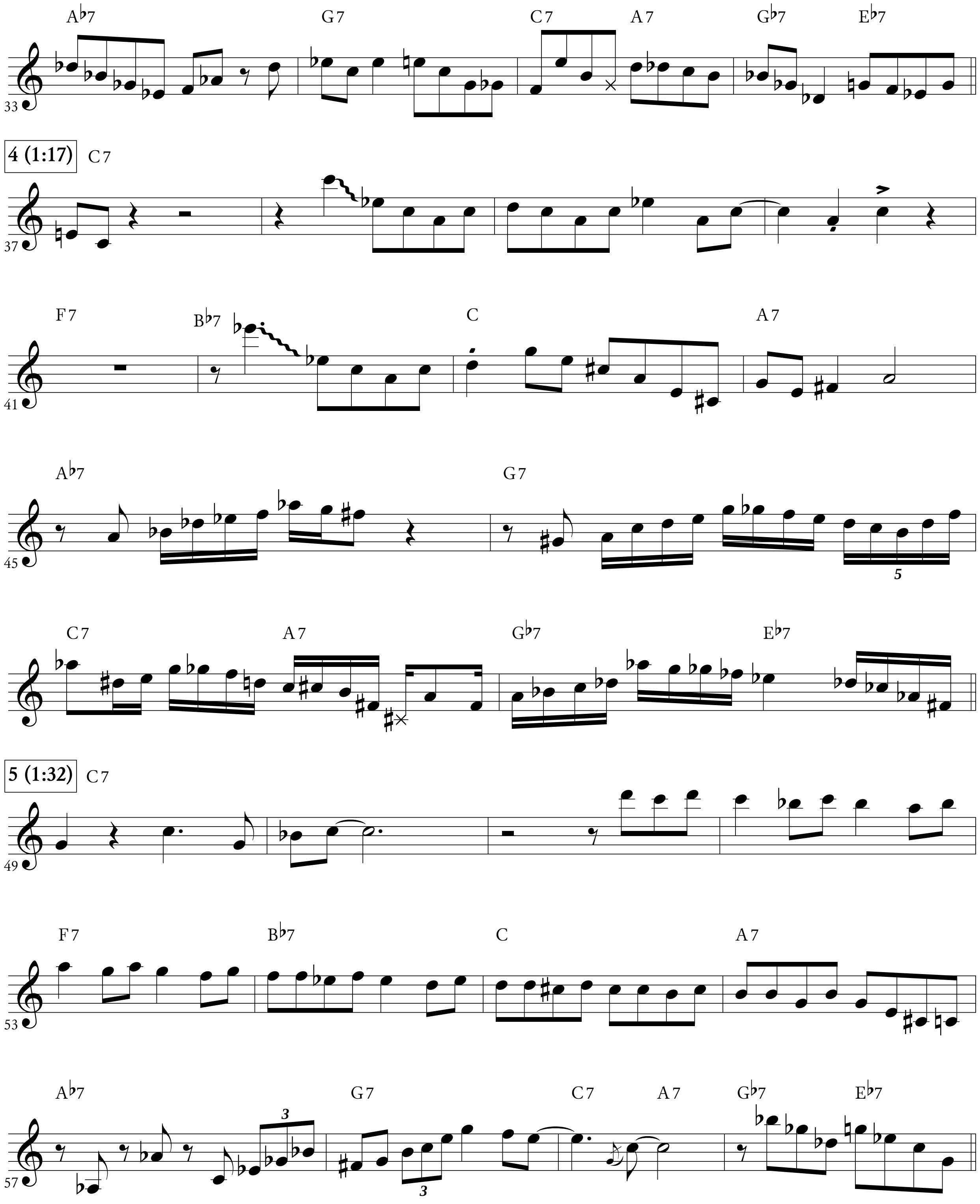
[S.S.]
19A1
8:29
19A2
[S.R.]
[S.S.]
[S.R.]
19A3
20A1
8:54
(trumpet solo)

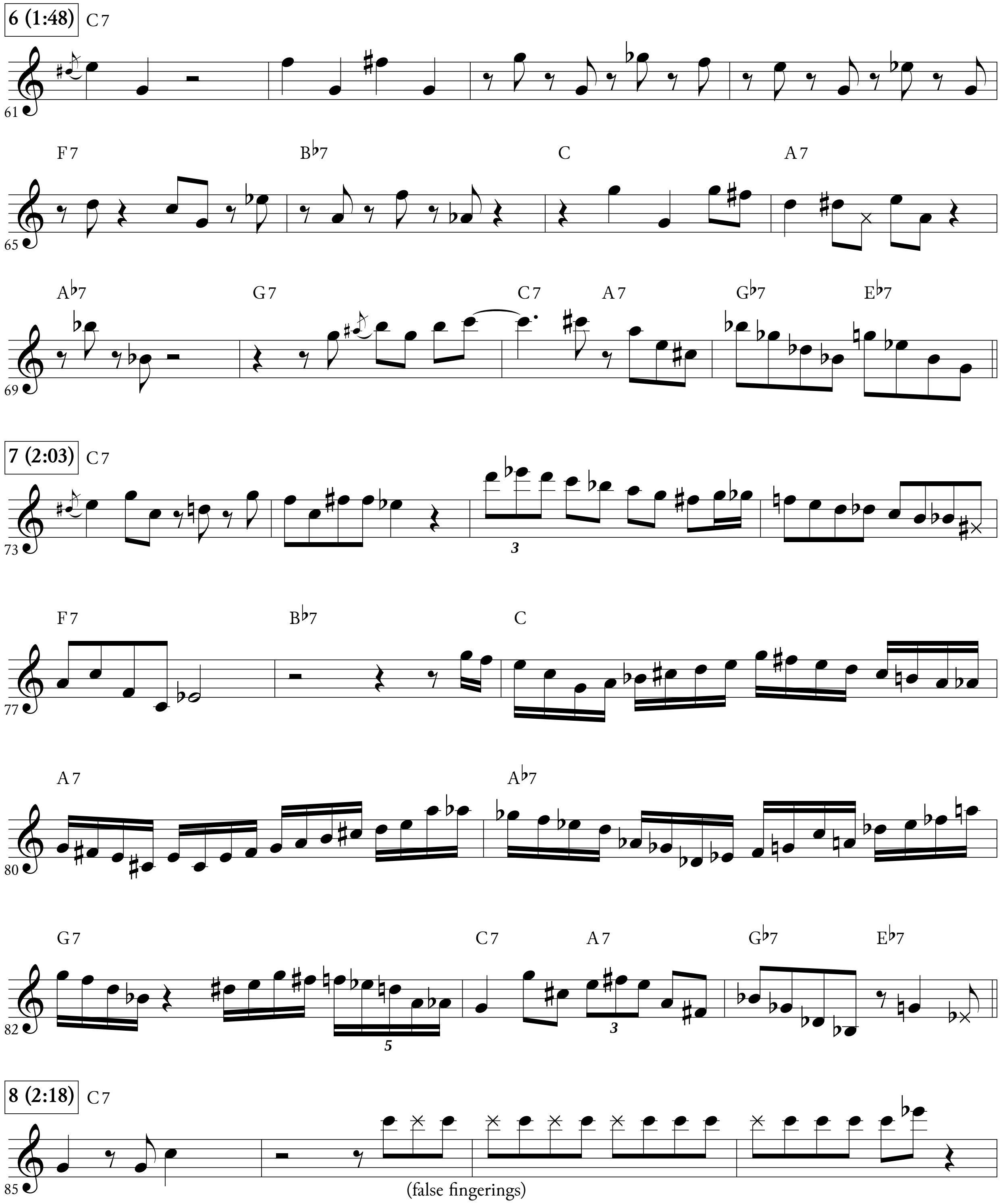
Isotope

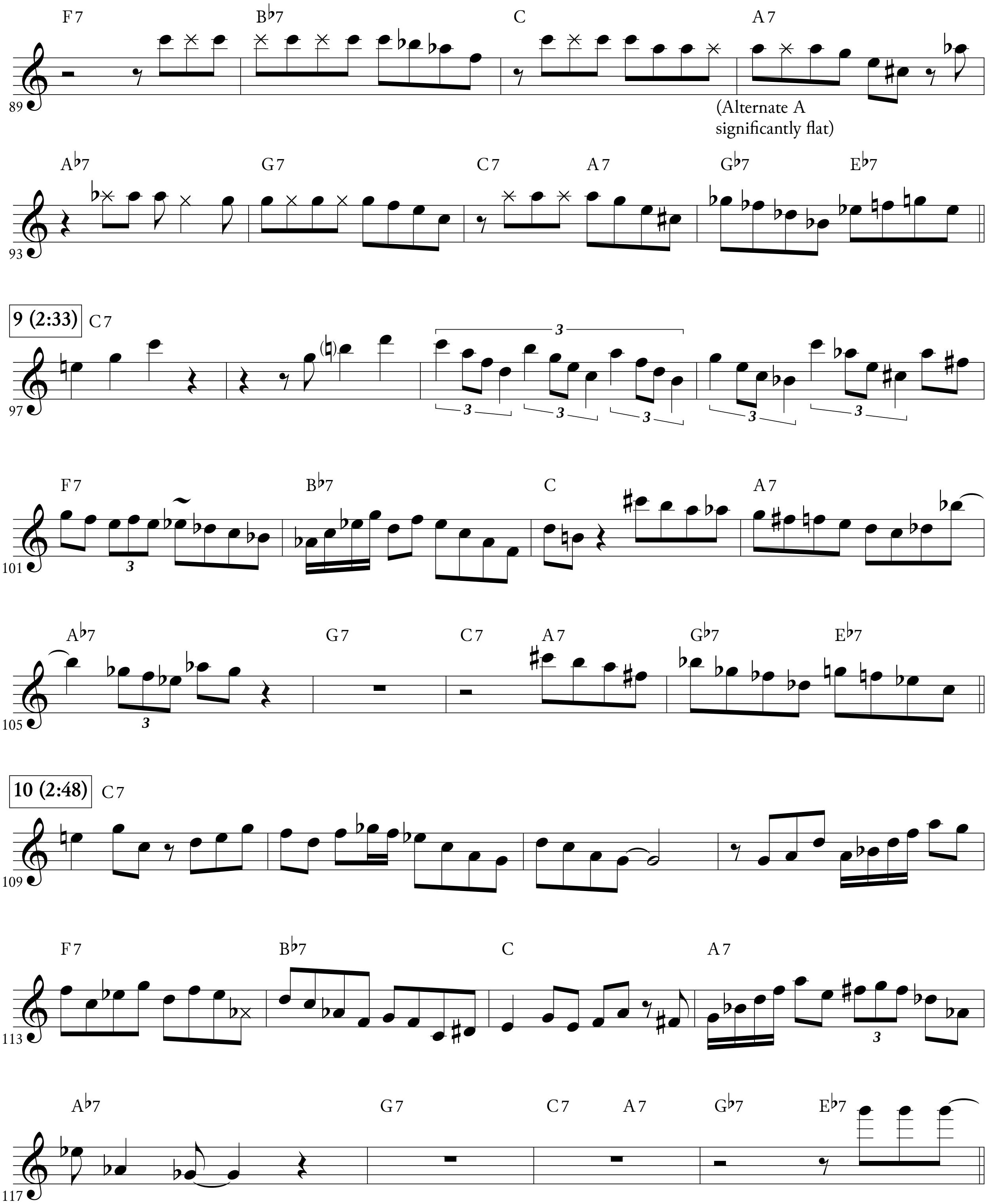
As played on *Inner Urge* (1965)
Joe Henderson, tenor sax

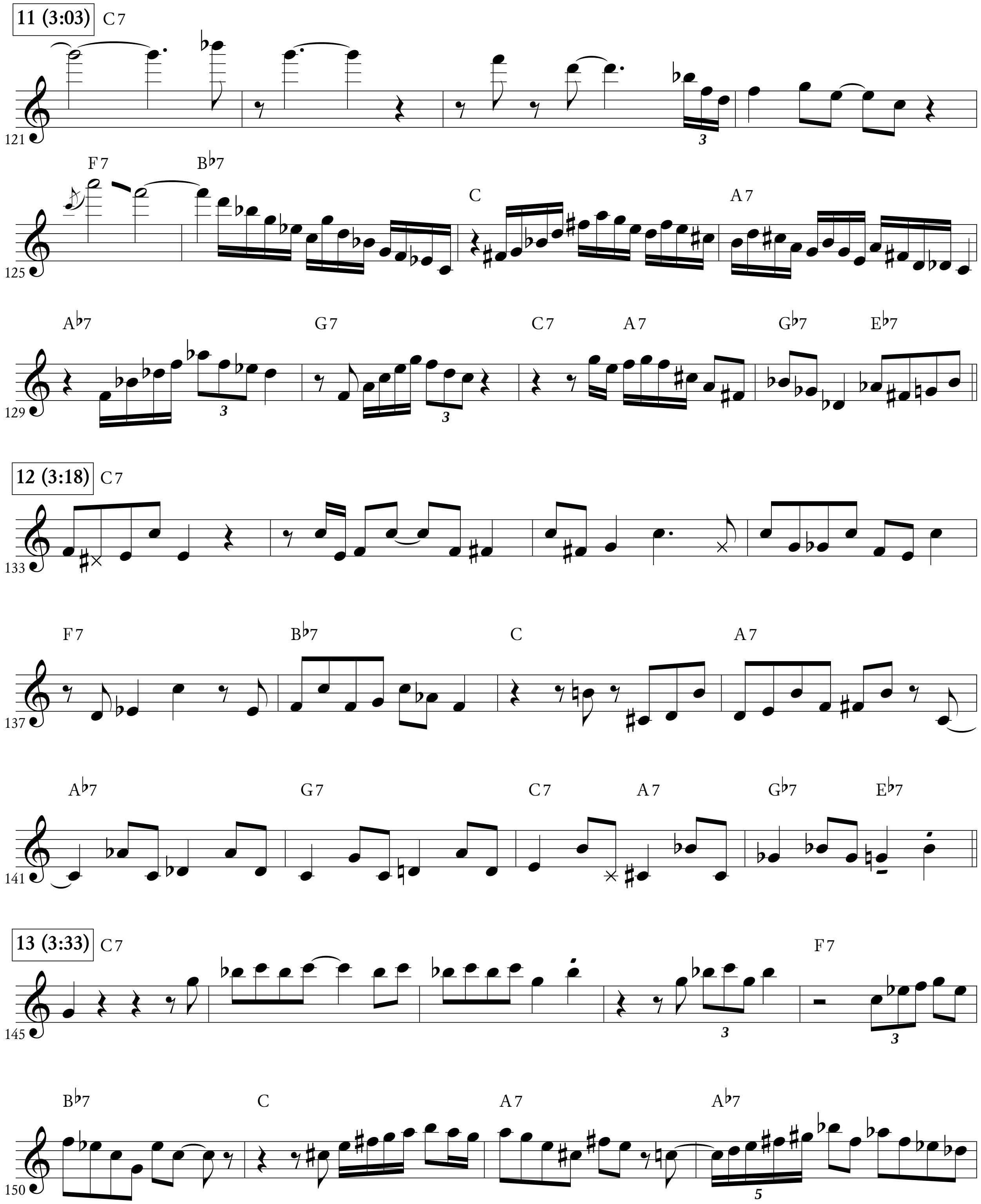
Joe Henderson
trans. Michael McClimon

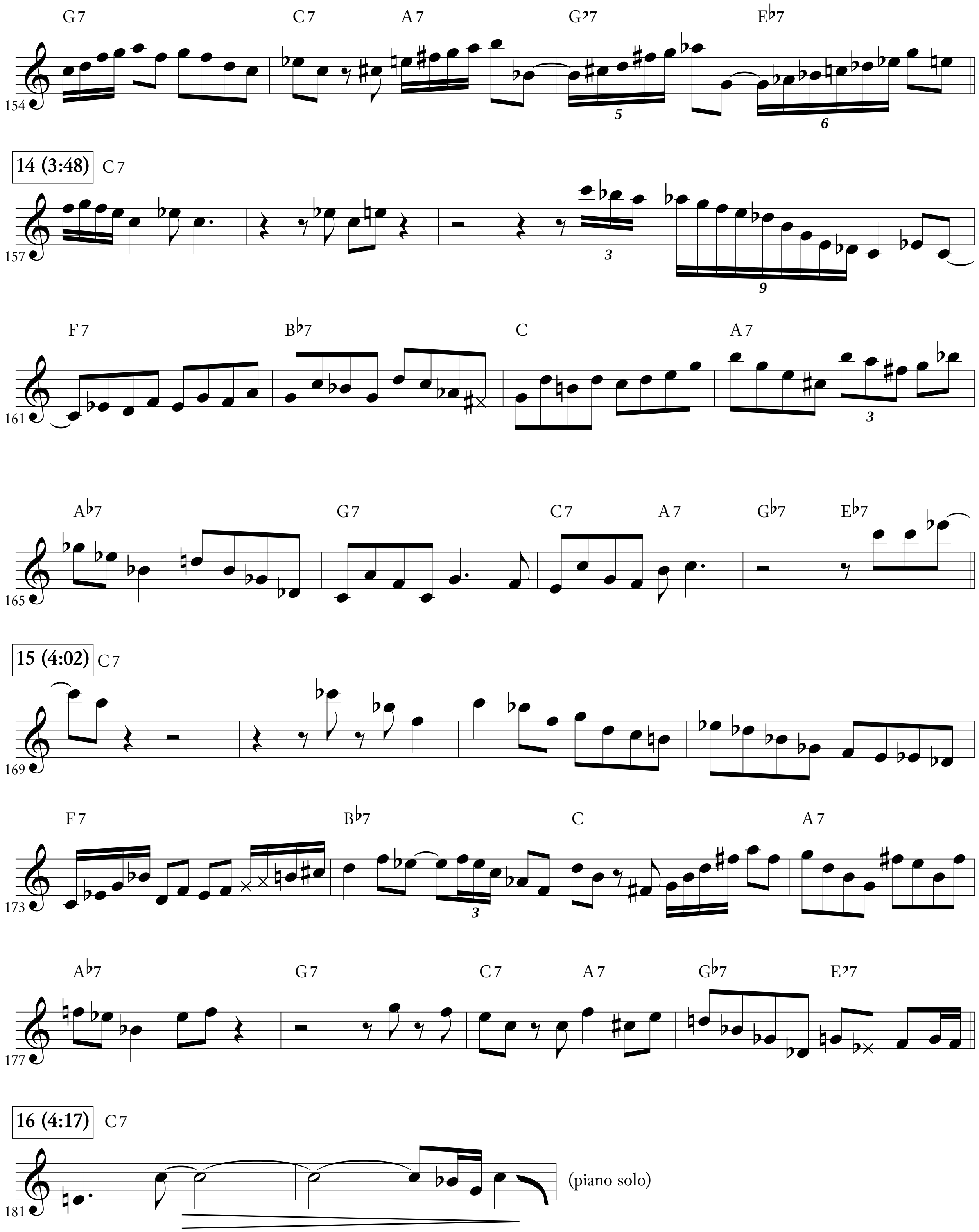

1 (0:31)
C7
5
F7
B♭7
C
A7
9
A♭7
G7
C7
A7
G♭7
E♭7
2 (0:47)
C7
13
17
F7
B♭7
3
C
A7
21
A♭7
G7
C7
A7
G♭7
E♭7
3 (1:02)
C7
25
29
F7
B♭7
C
A7


A♭7
G7
C7
A7
G♭7
E♭7
33
4 (1:17)
C7
37
F7
B♭7
C
A7
41
A♭7
G7
45
5
C7
A7
G♭7
E♭7
5 (1:32)
C7
49
F7
B♭7
C
A7
53
A♭7
G7
C7
A7
G♭7
E♭7
57
3
3


6 (1:48)
C7
61
F7
B♭7
C
A7
65
A♭7
G7
C7
A7
G♭7
E♭7
69
7 (2:03)
C7
3
73
F7
B♭7
C
77
A7
A♭7
80
G7
C7
A7
G♭7
E♭7
5
3
82
8 (2:18)
C7
85
(false fingerings)


F7
B♭7
C
A7
89
(Alternate A significantly flat)
A♭7
G7
C7
A7
G♭7
E♭7
93
9 (2:33)
C7
97
3
F7
B♭7
C
A7
101
A♭7
G7
C7
A7
G♭7
E♭7
105
10 (2:48)
C7
109
F7
B♭7
C
A7
113
A♭7
G7
C7
A7
G♭7
E♭7
117

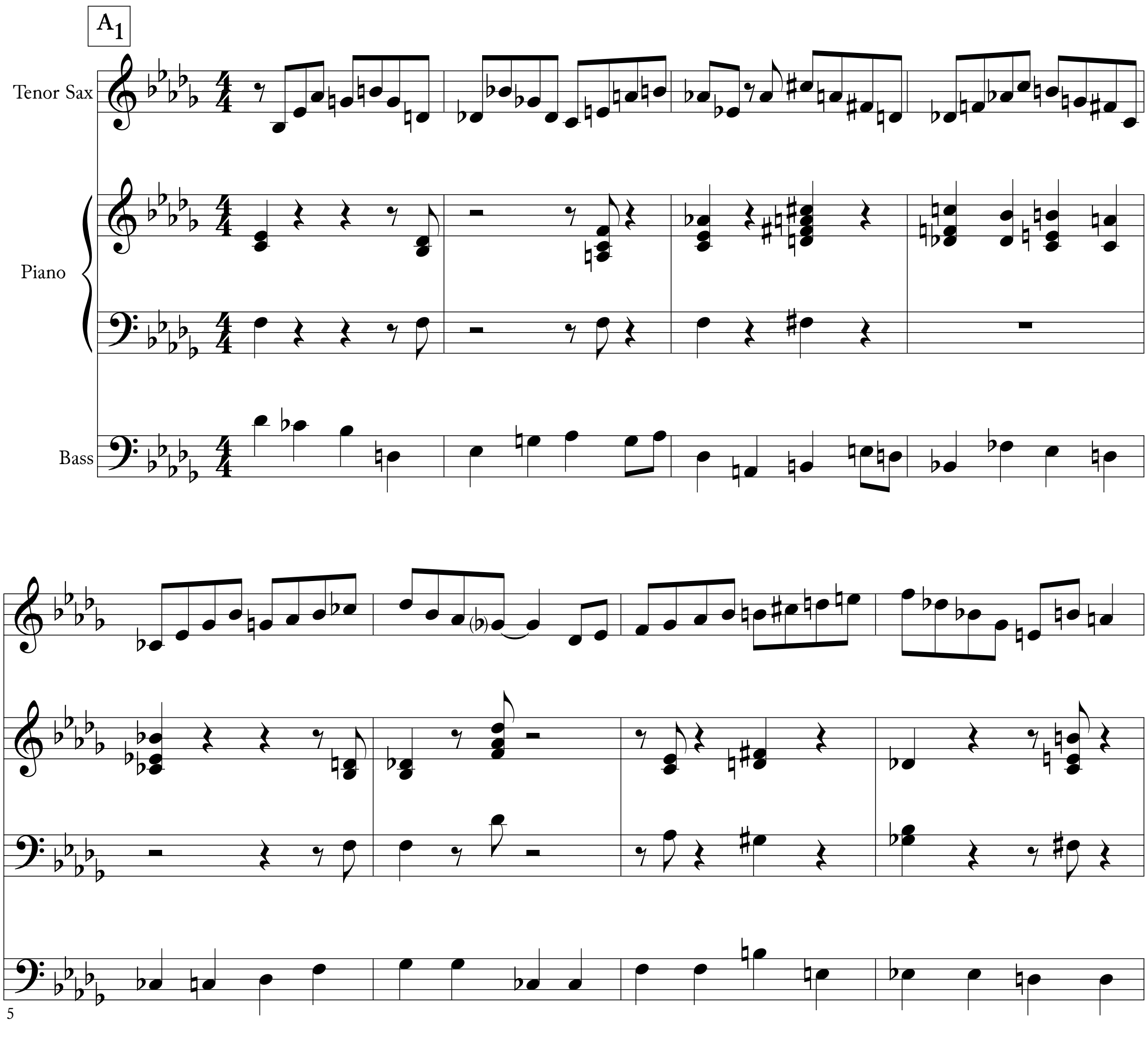

11 (3:03)
C7
121
3
F7
B♭7
125
C
A7
A♭7
G7
C7
A7
G♭7
E♭7
129
3
3
12 (3:18)
C7
133
F7
B♭7
C
A7
137
A♭7
G7
C7
A7
G♭7
E♭7
141
13 (3:33)
C7
145
3
F7
3
B♭7
C
A7
A♭7
150
5


G7
C7
A7
G♭7
E♭7
154
14 (3:48)
C7
157
F7
B♭7
C
A7
161
A♭7
G7
C7
A7
G♭7
E♭7
165
15 (4:02)
C7
169
F7
B♭7
C
A7
173
A♭7
G7
C7
A7
G♭7
E♭7
177
16 (4:17)
C7
181
(piano solo)

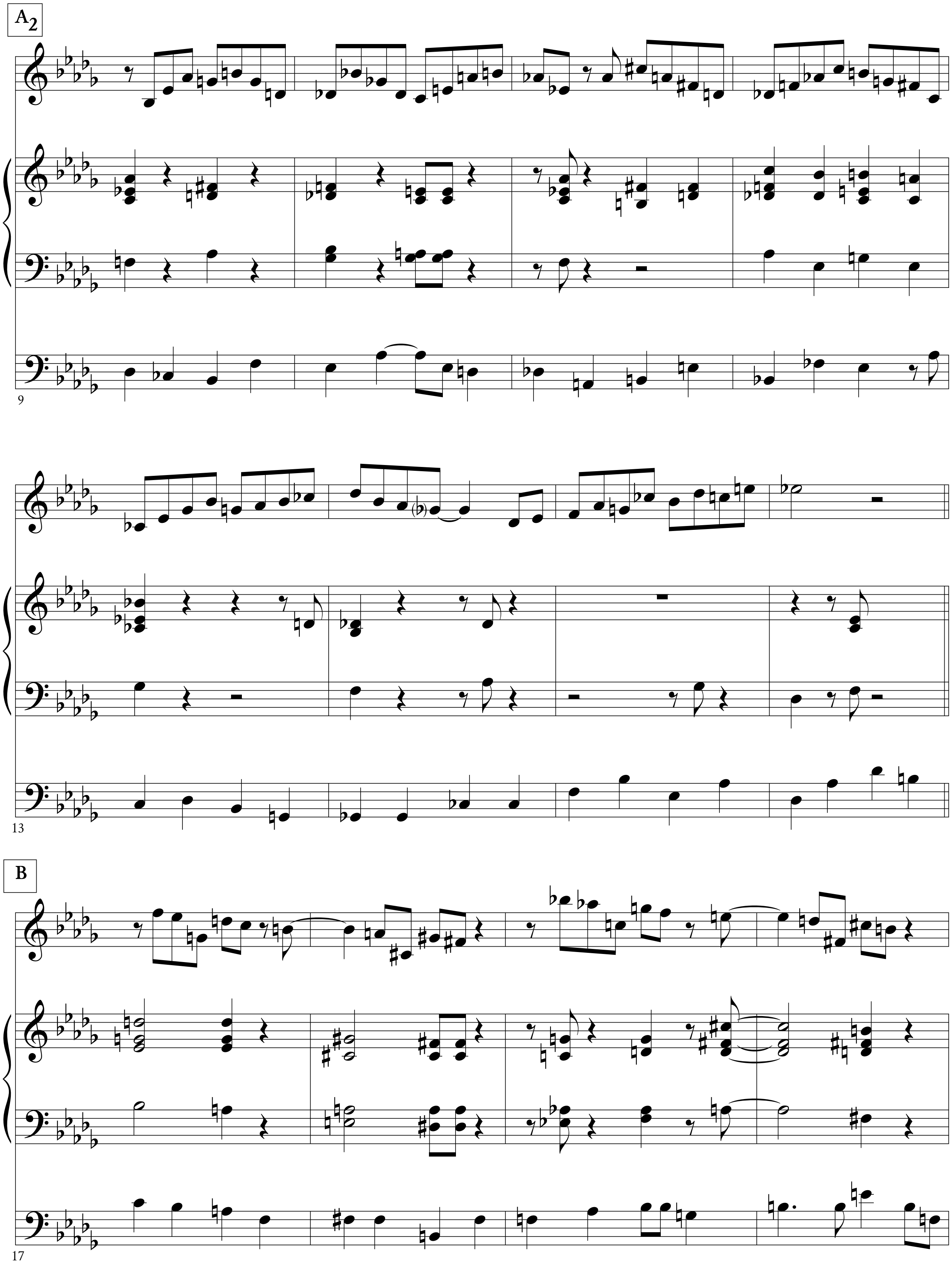
Lo-Joe

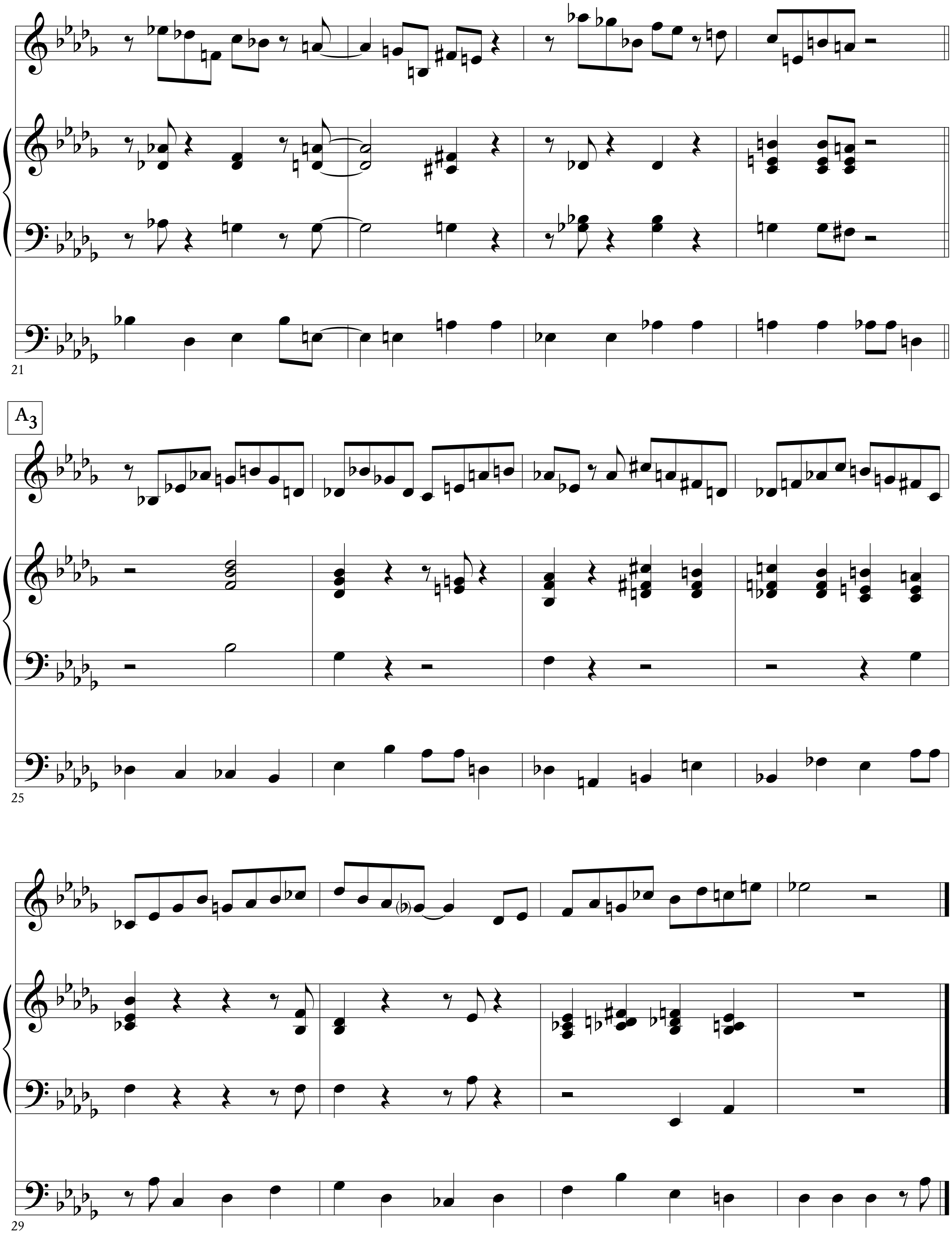
As played on *Amsterdam After Dark* (1979)
Coleman, tenor sax; Hilton Ruiz, piano; Sam Jones, bass

George Coleman
trans. Michael McClimon



This transcription contains only the head of the tune; the bass line sounds down an octave. In many cases, the piano voicings are difficult to hear in the recording. I have tried to keep the transcription as faithful as possible, but the disclaimer on using the transcription as a supplement to (not a replacement for) the recording applies more strongly than usual in this case.

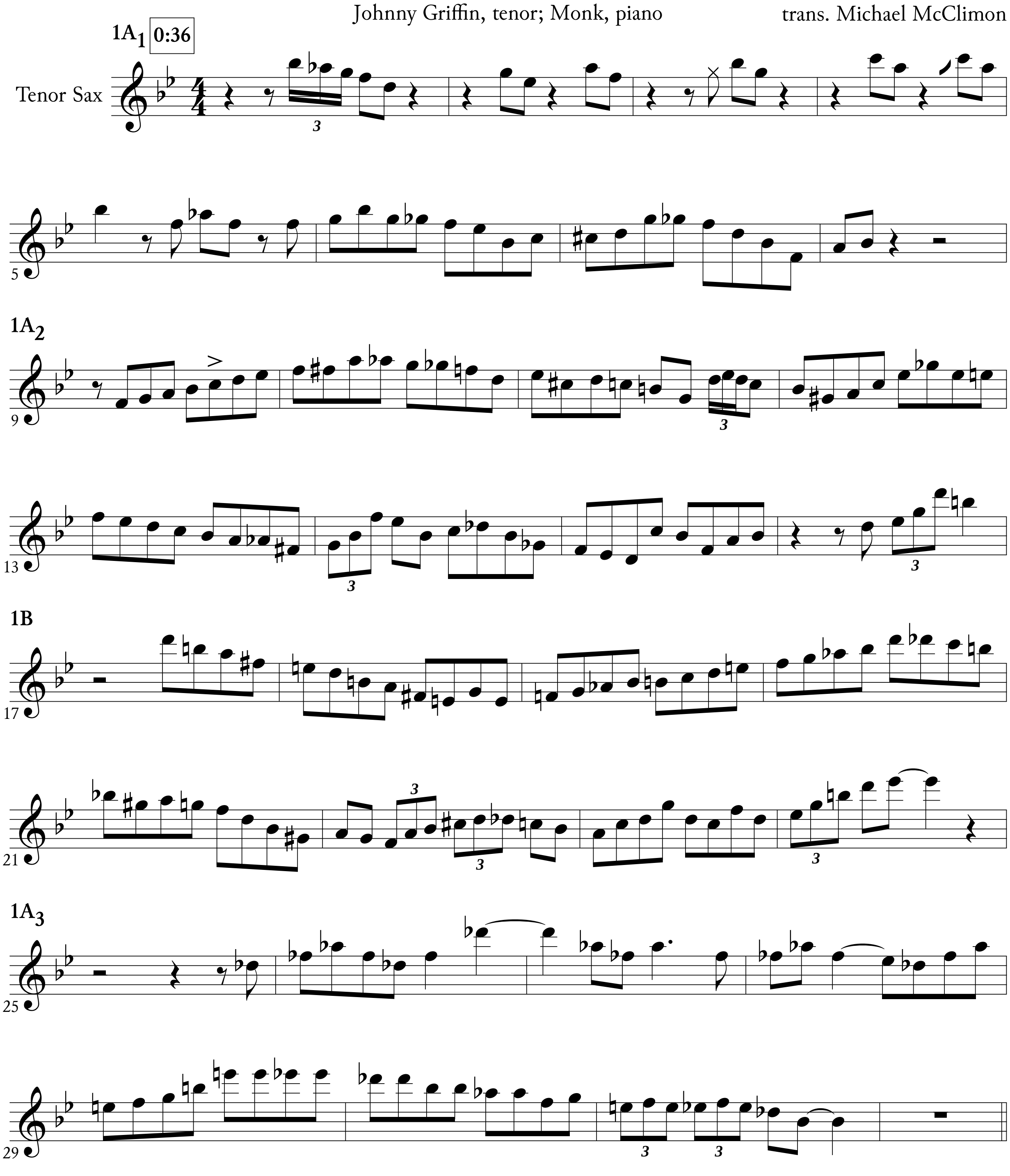

A2
B


A3

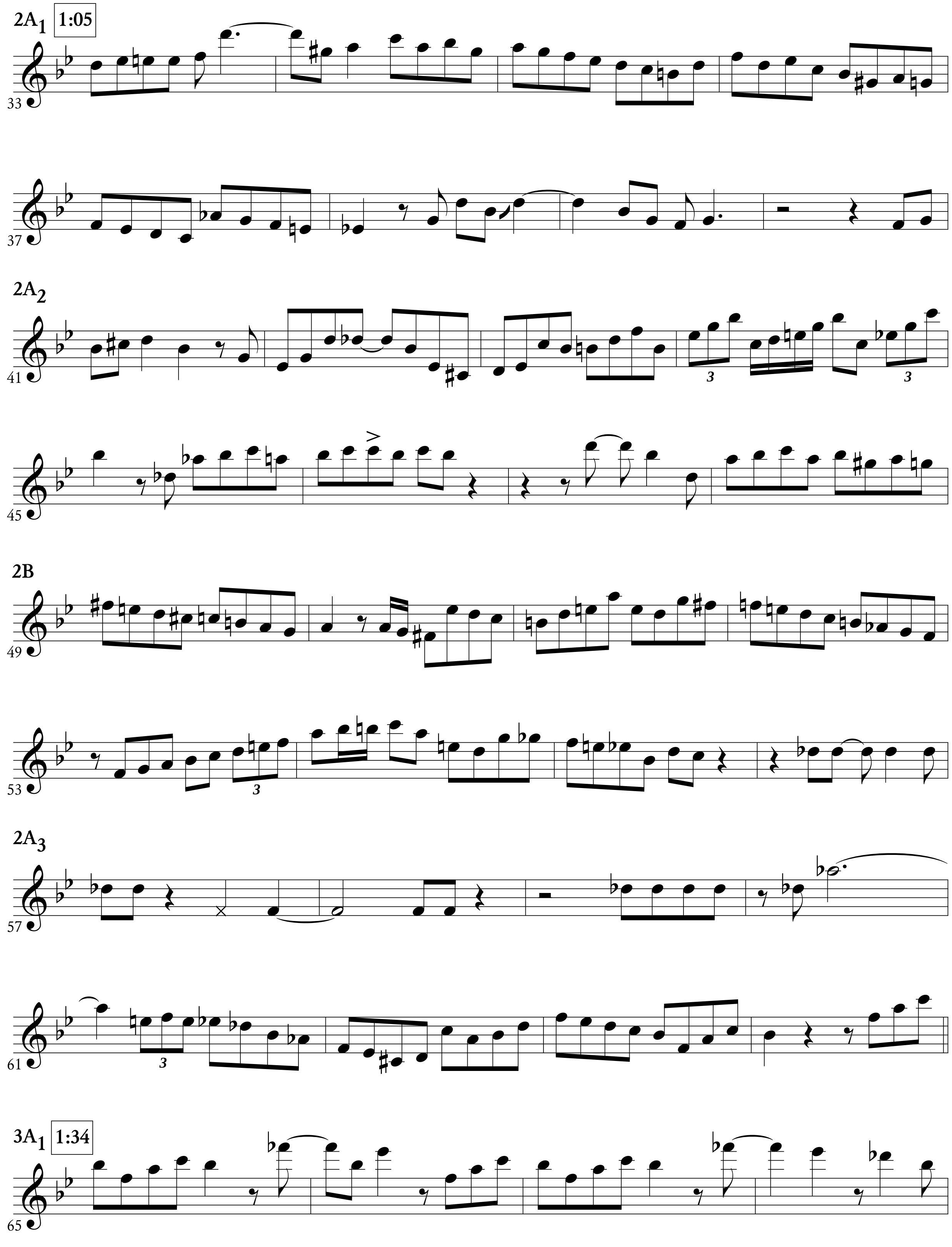
Rhythm-a-ning

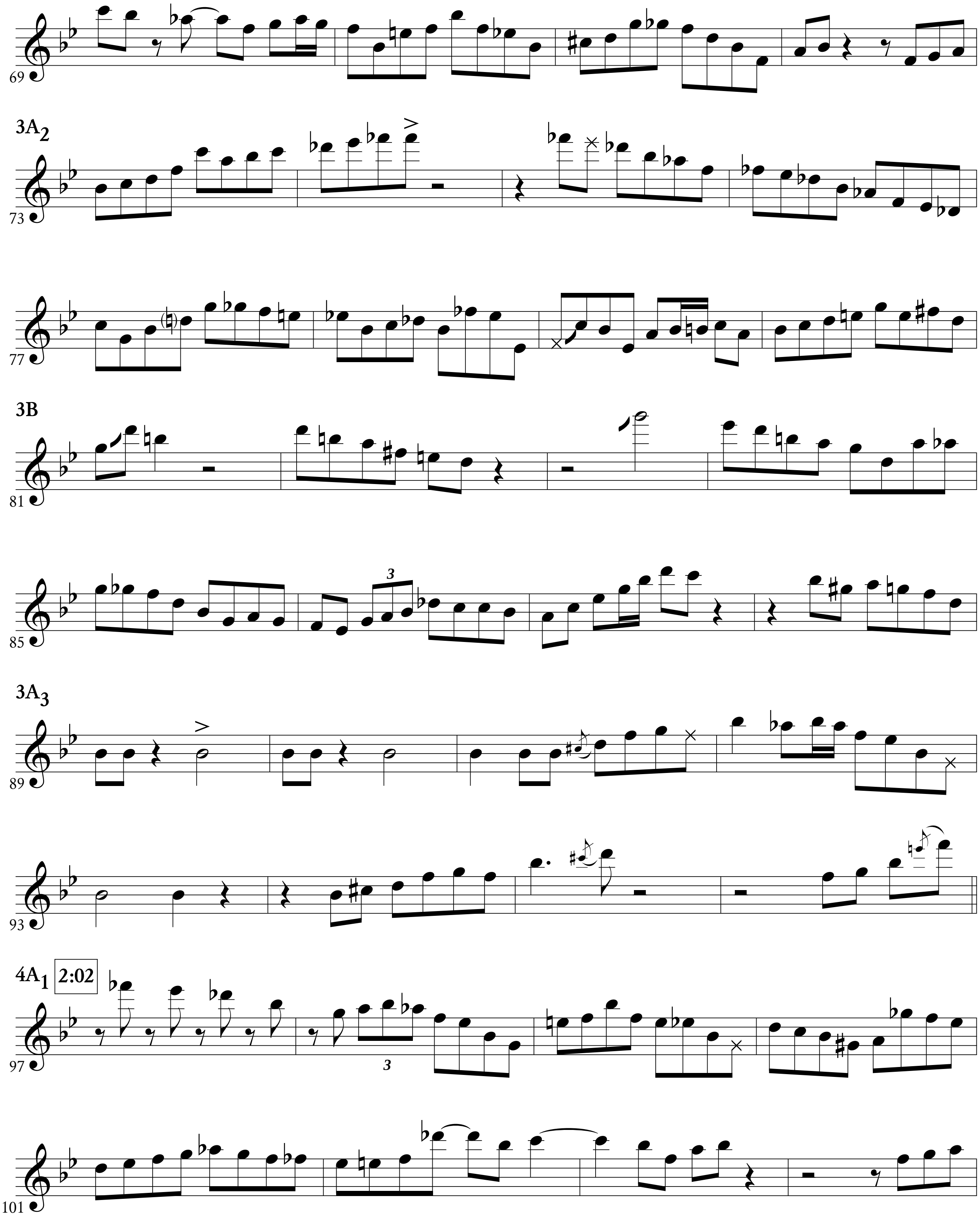
As played on *Thelonious in Action* (1958)
Johnny Griffin, tenor; Monk, piano

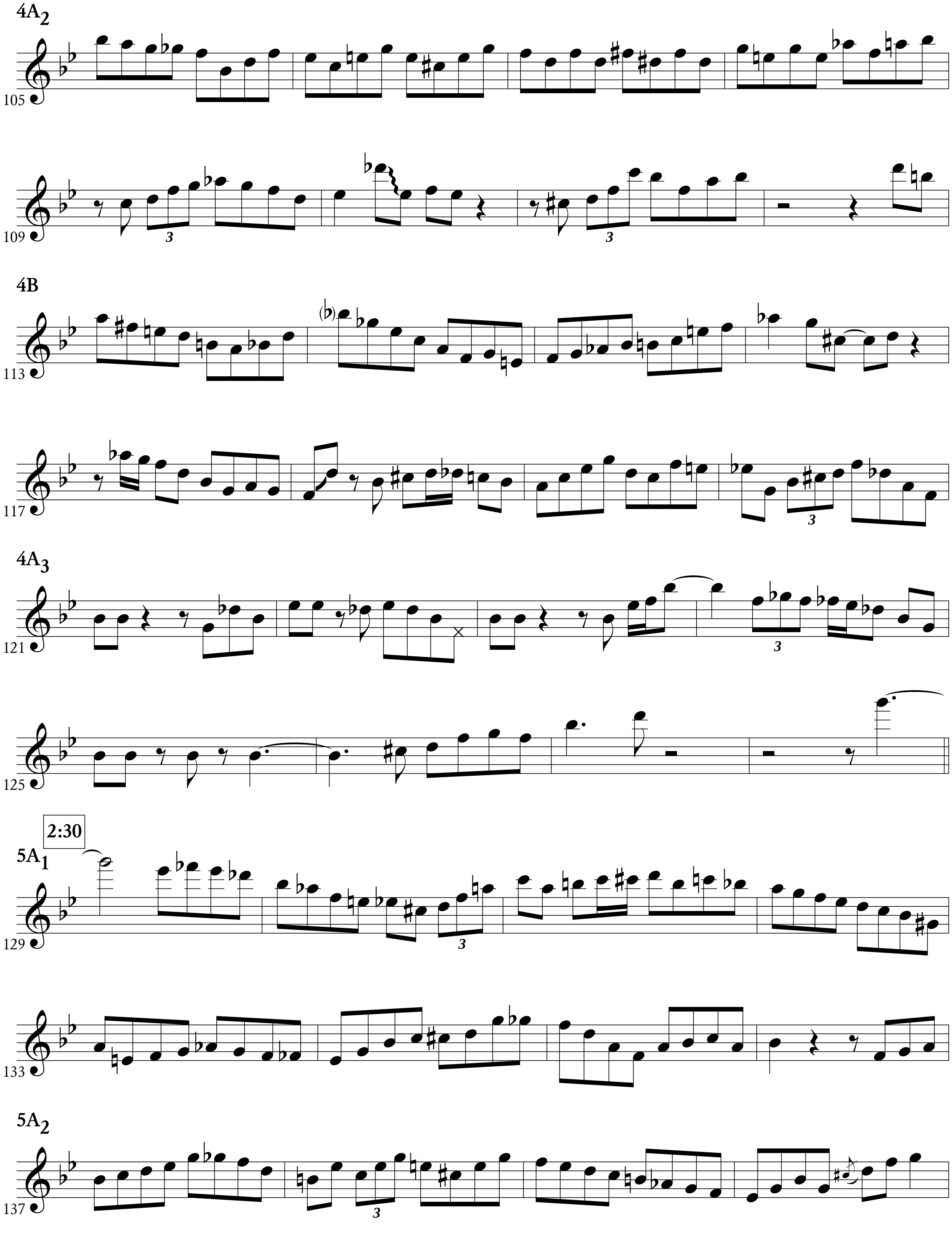
Thelonious Monk
trans. Michael McClimon

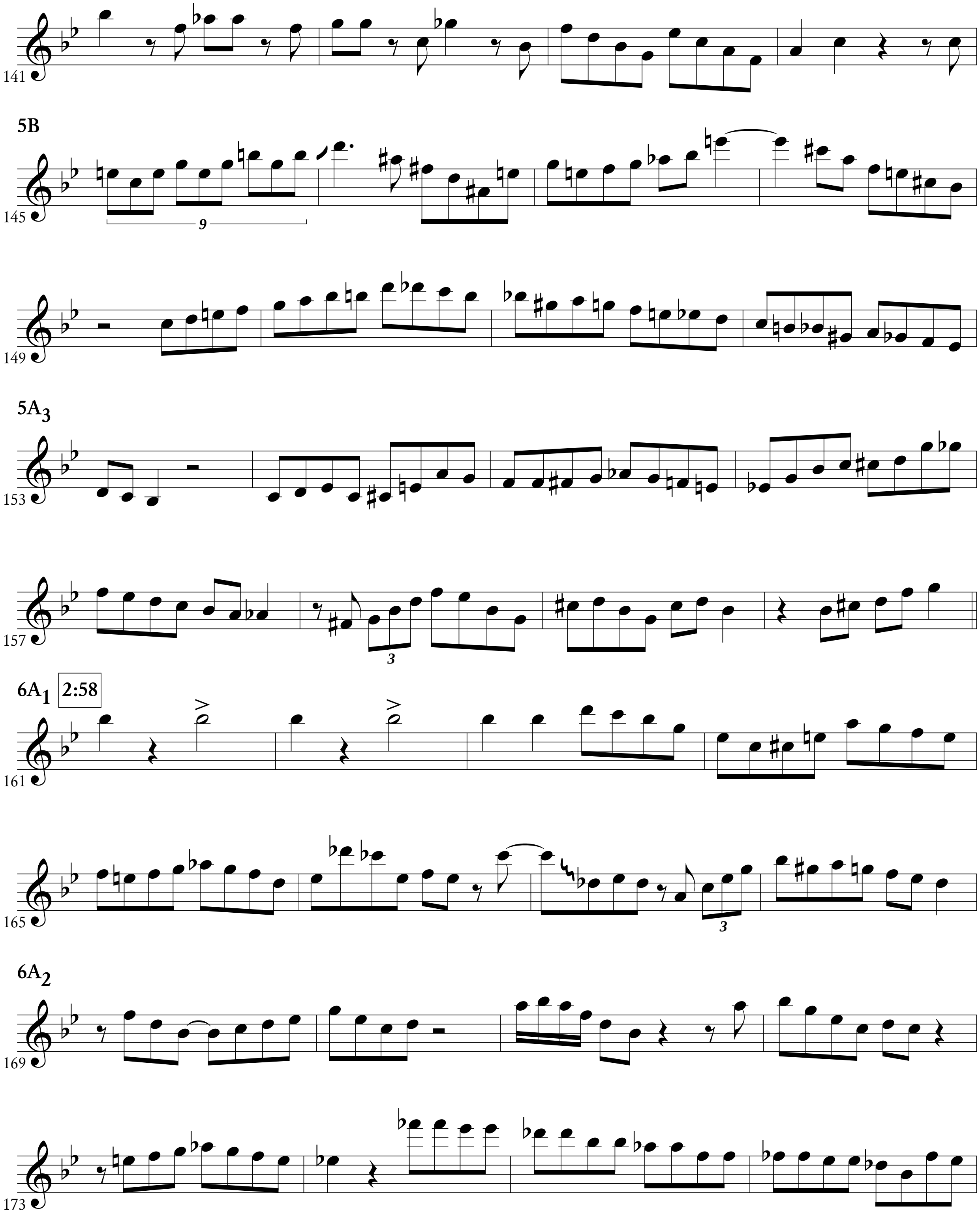


1A₁
0:36
Tenor Sax
3
5
9
1A₂
3
13
3
3
1B
17
21
3
3
3
1A₃
25
29
3
3

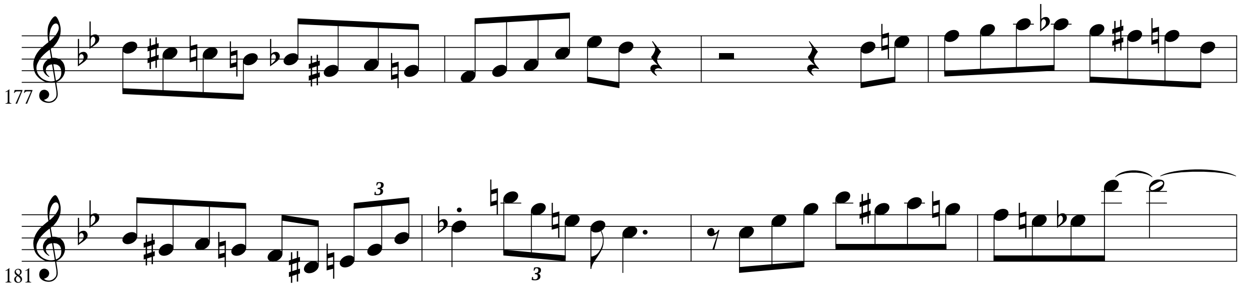

2A1 1:05
2A2
2B
2A3
3A1 1:34


3A2
3B
3A3
4A1
2:02

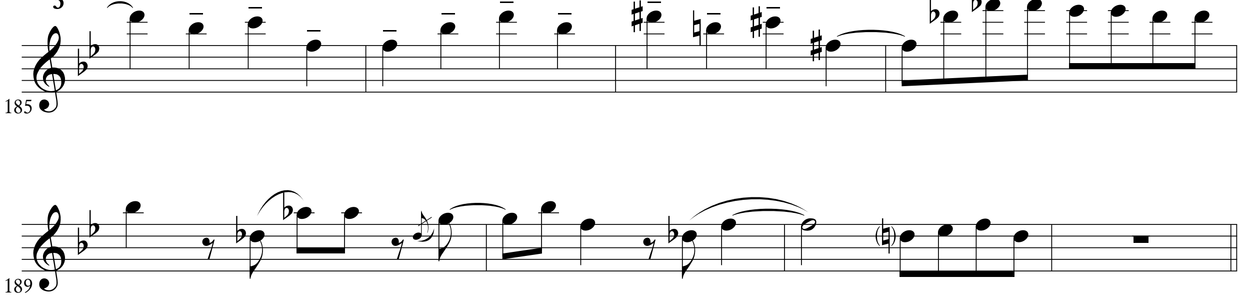

4A2
4B
4A3
2:30
5A1
5A2


5B
5A3
6A1
2:58
6A2

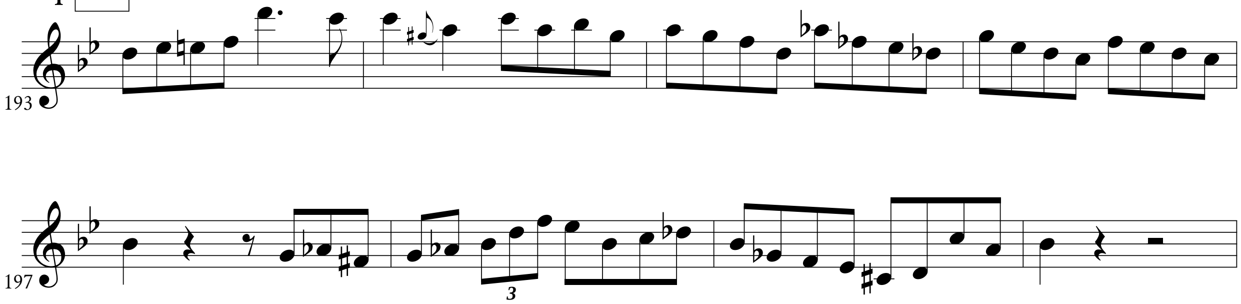
6B



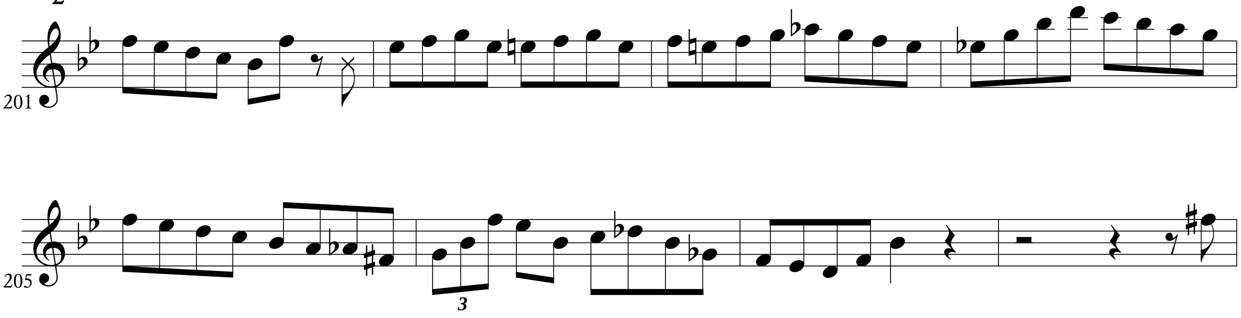
6A$_3$



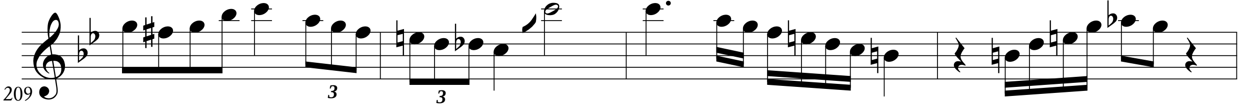
7A$_1$ **3:26**

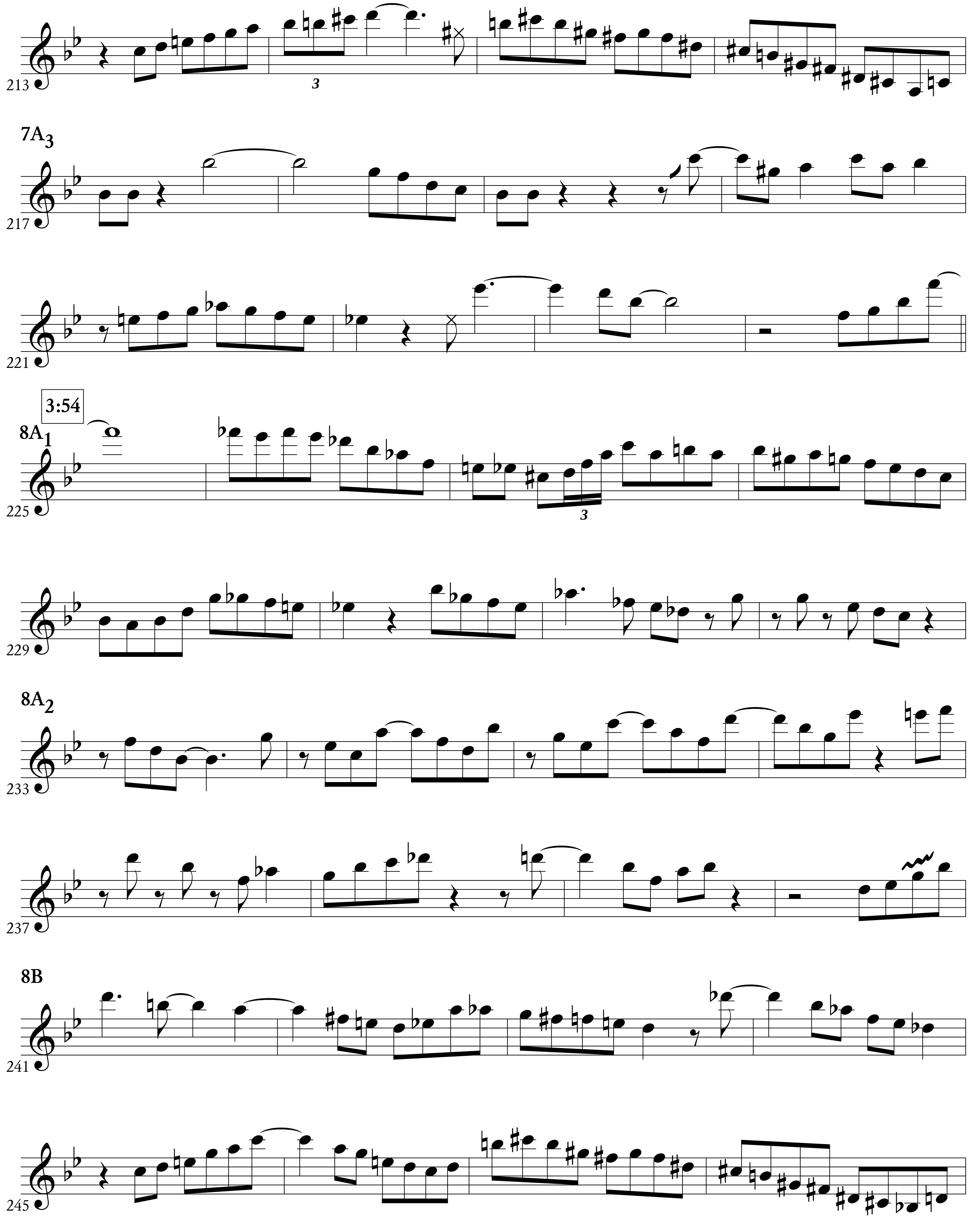


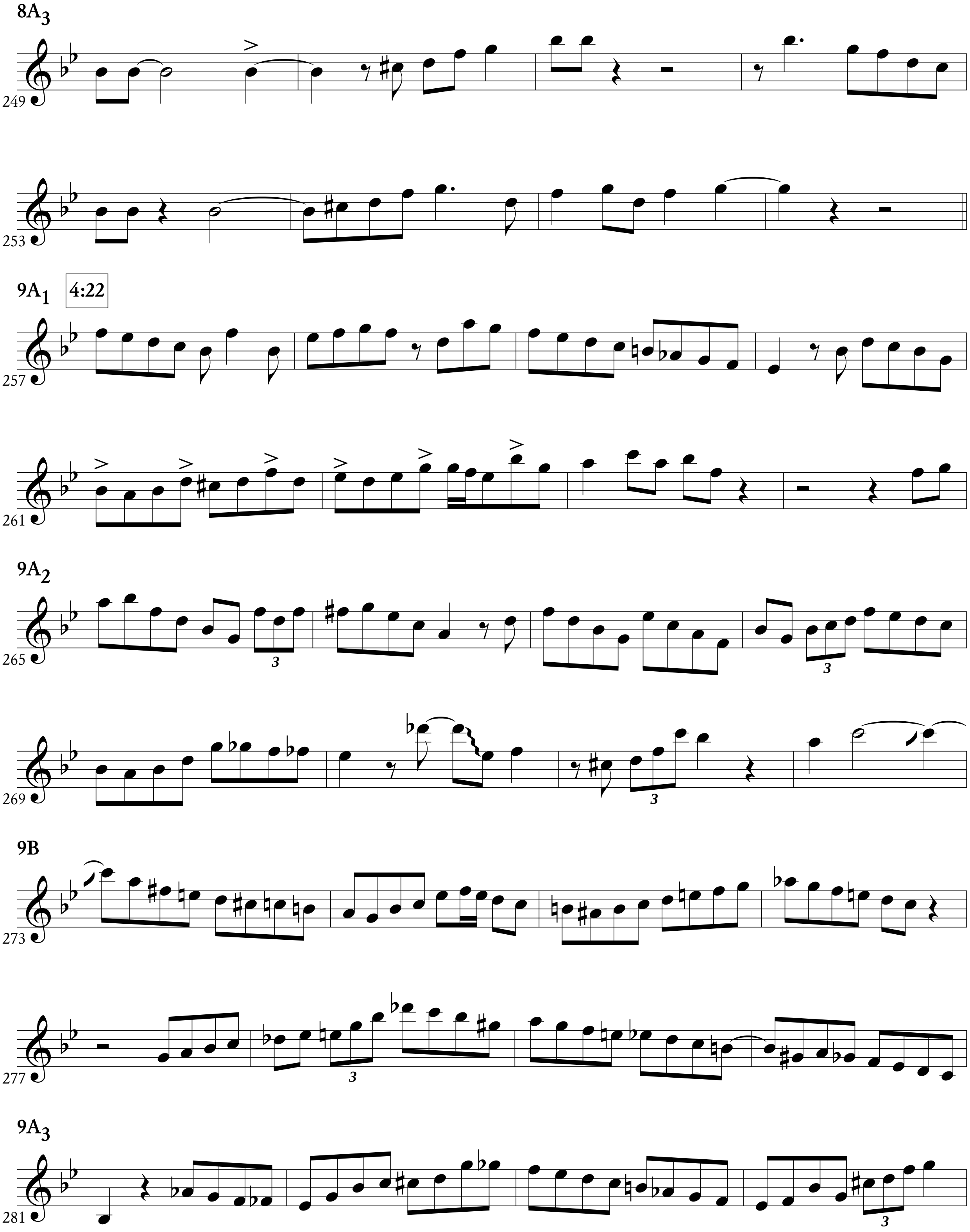
7A$_2$

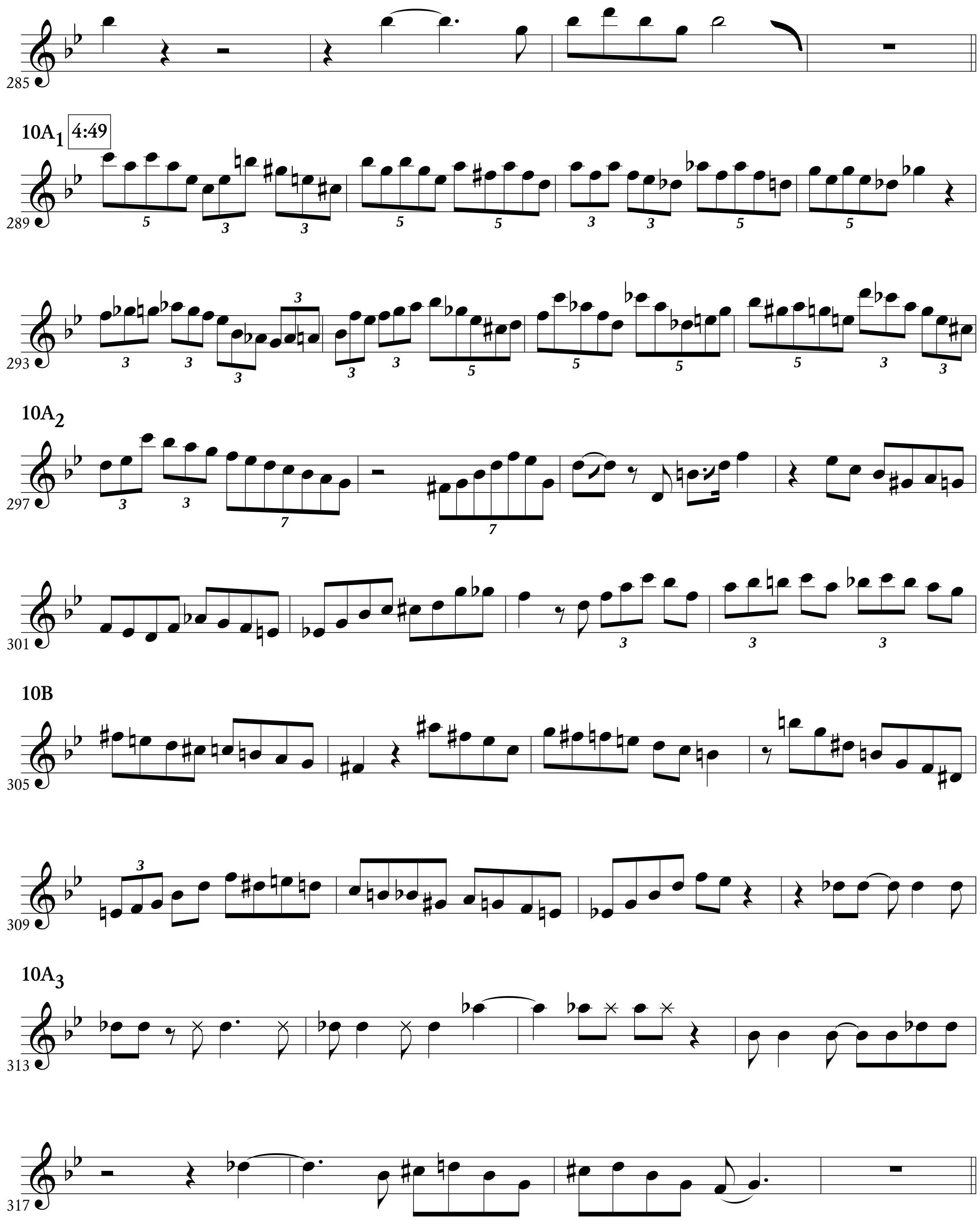


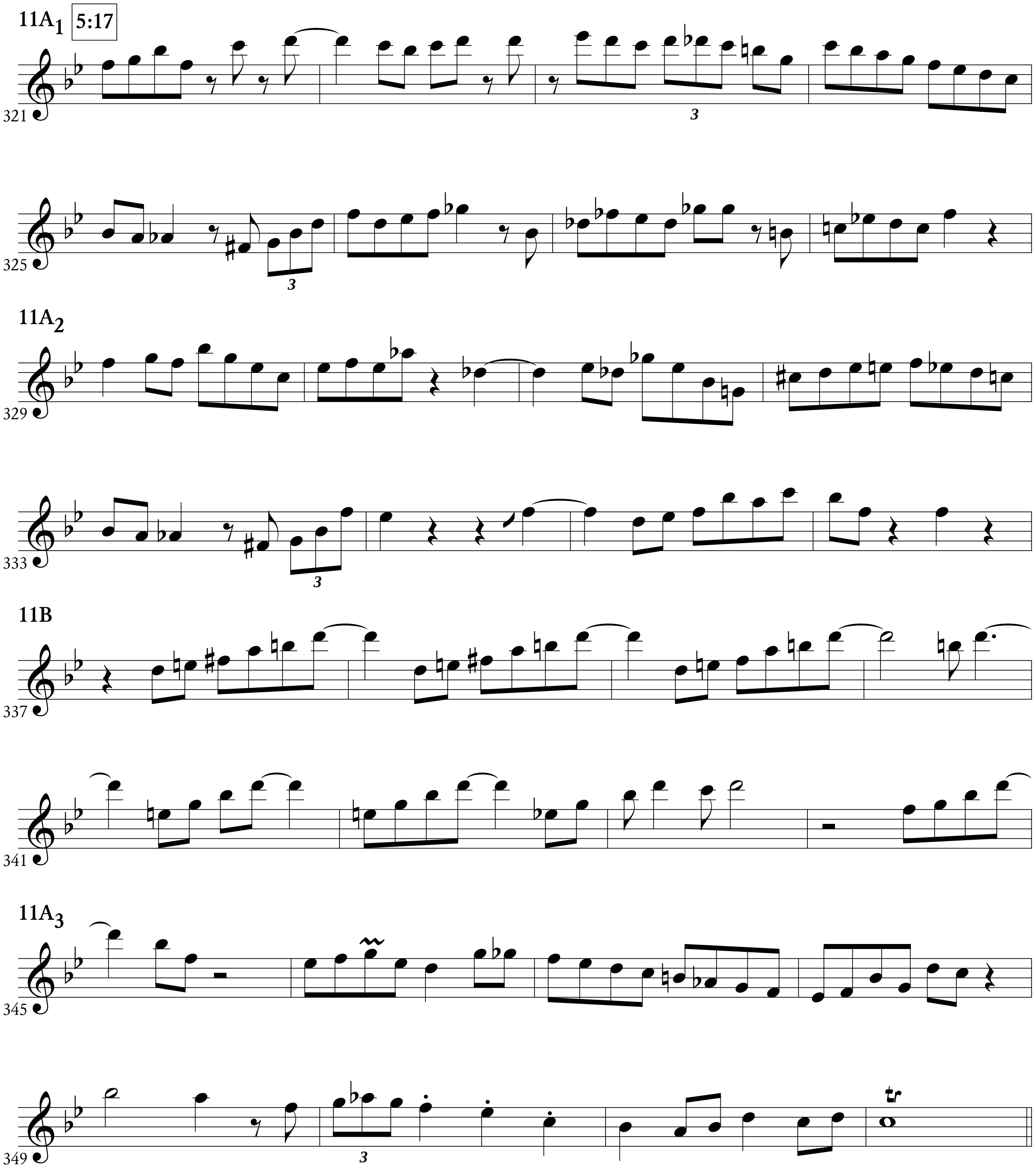
7B



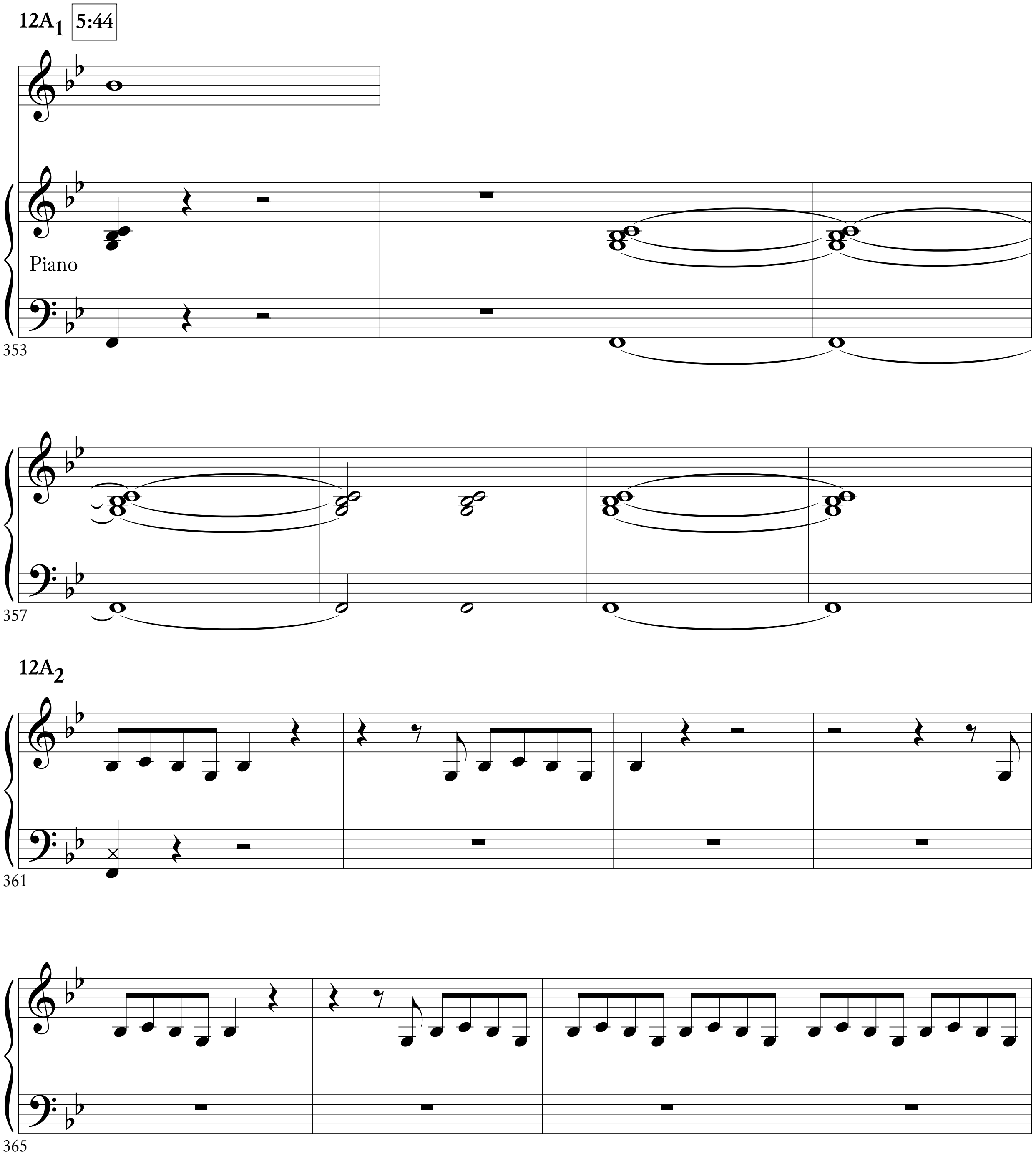

7A3
3:54
8A1
8A2
8B


8A₃
9A₁
4:22
9A₂
9B
9A₃

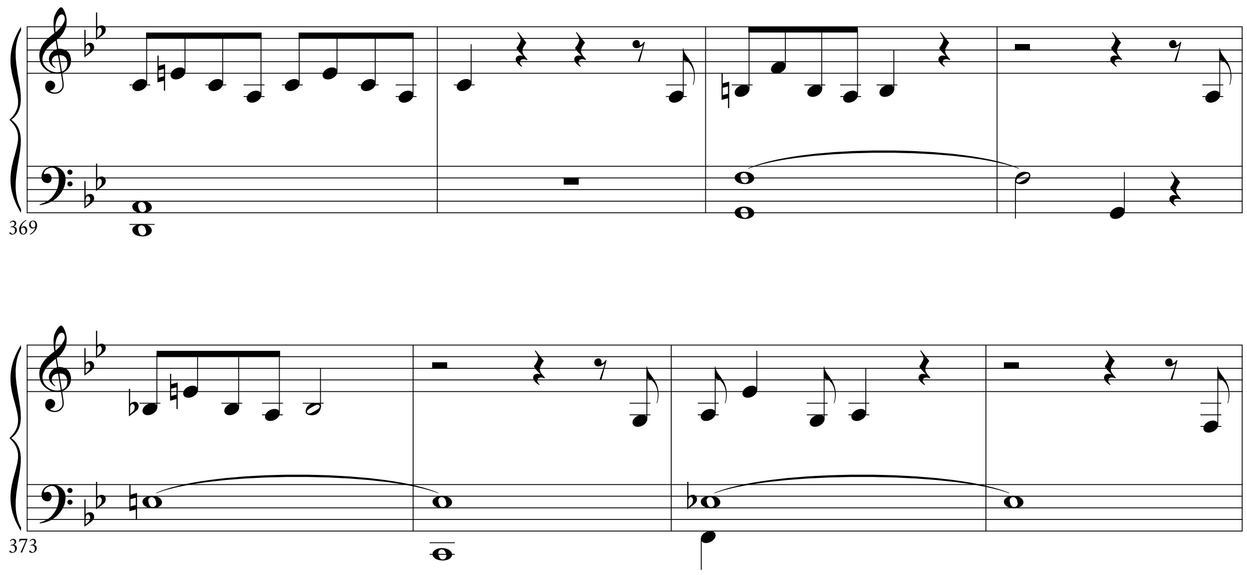

10A1
4:49
10A2
10A3



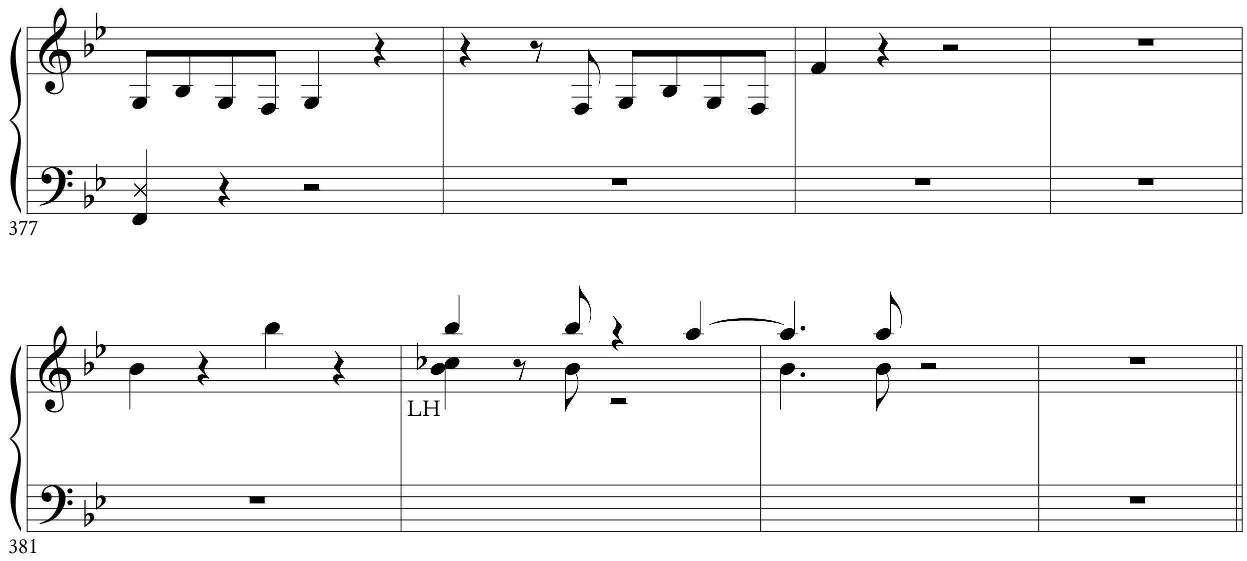
11A1
5:17
11A2
11A3


$12A_1$
5:44
Piano
353
357
$12A_2$
361
365

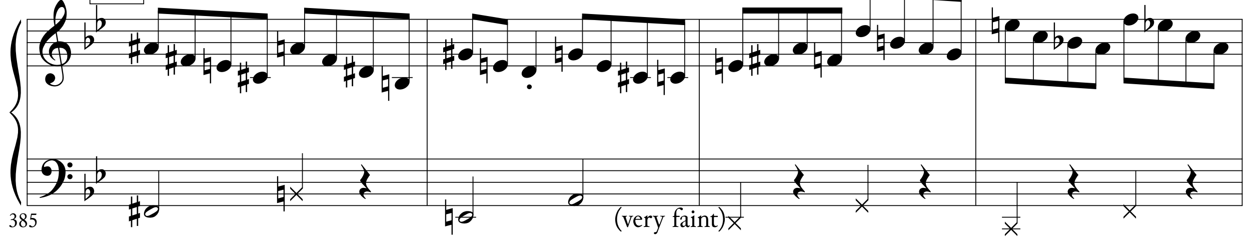
12B

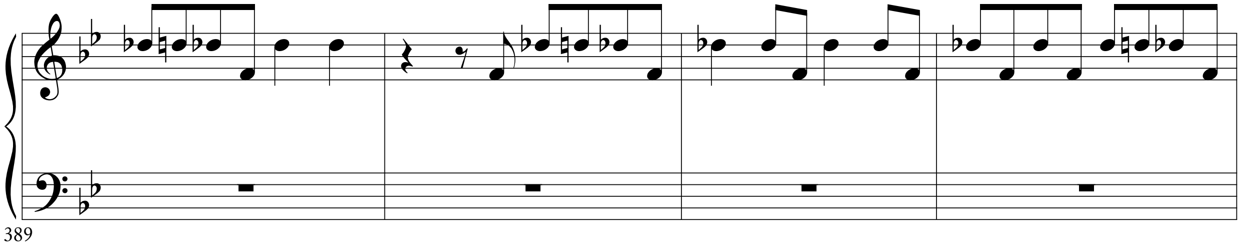


12A$_3$

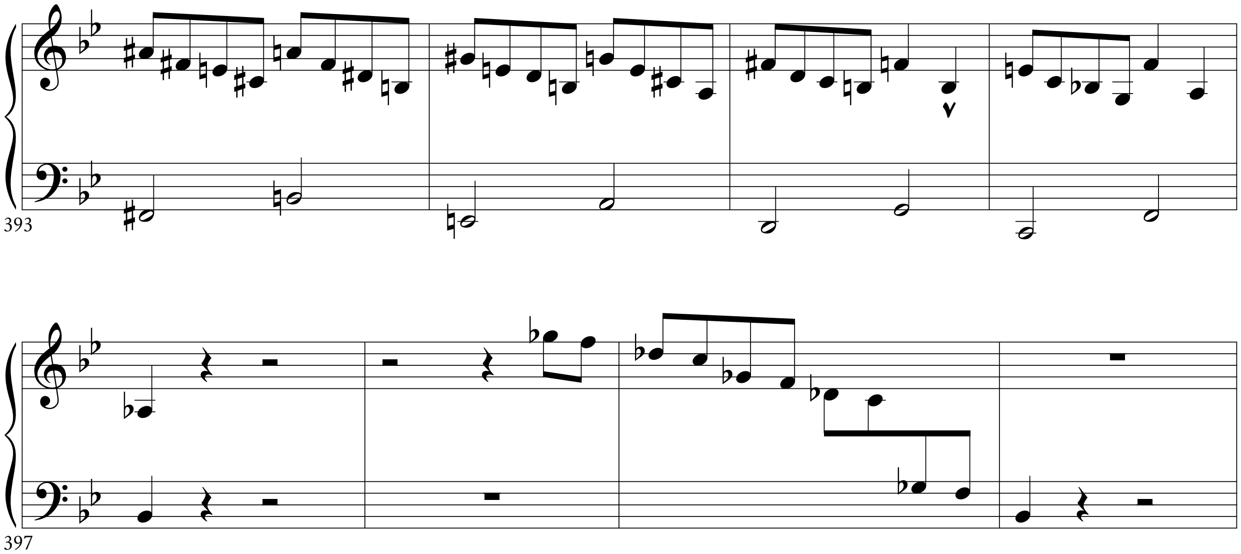


13A$_1$ $\boxed{6:12}$

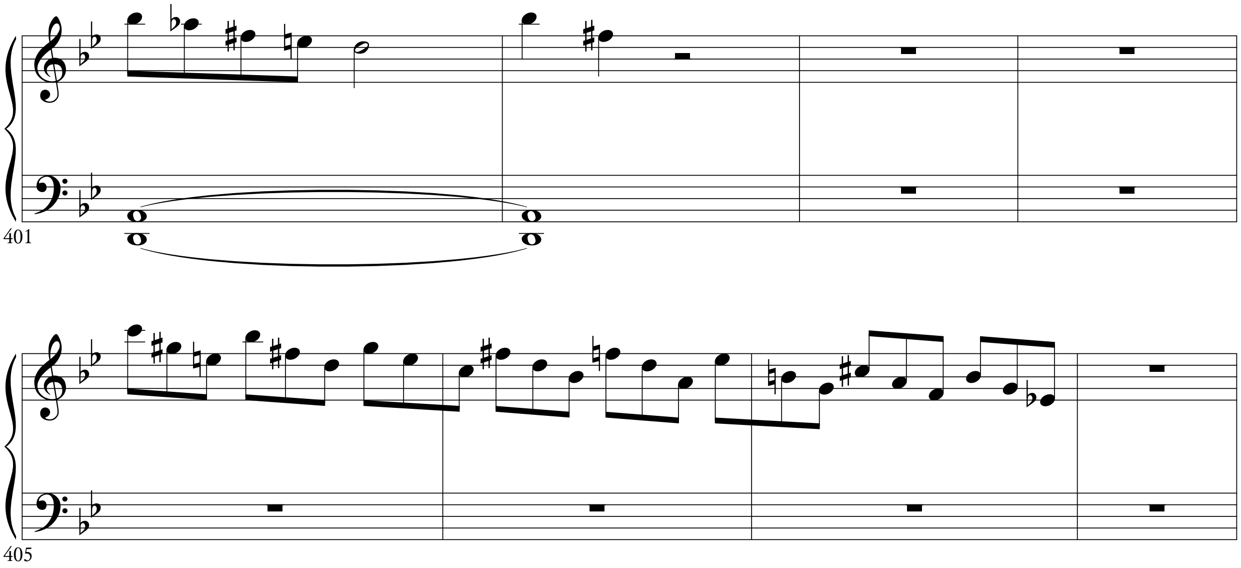

(very faint)
385



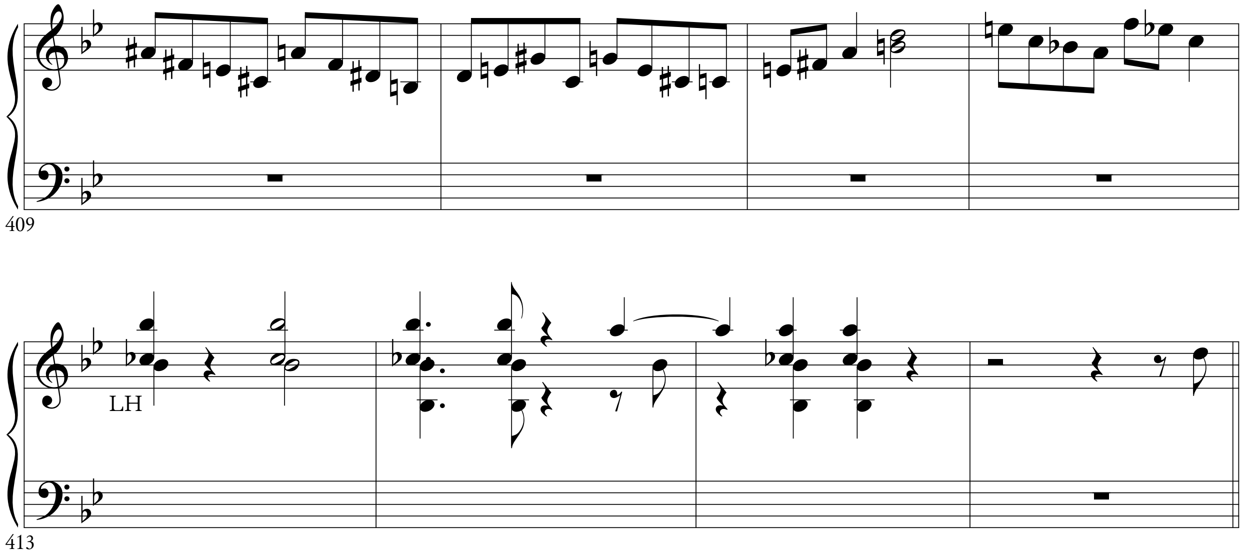
13A$_2$



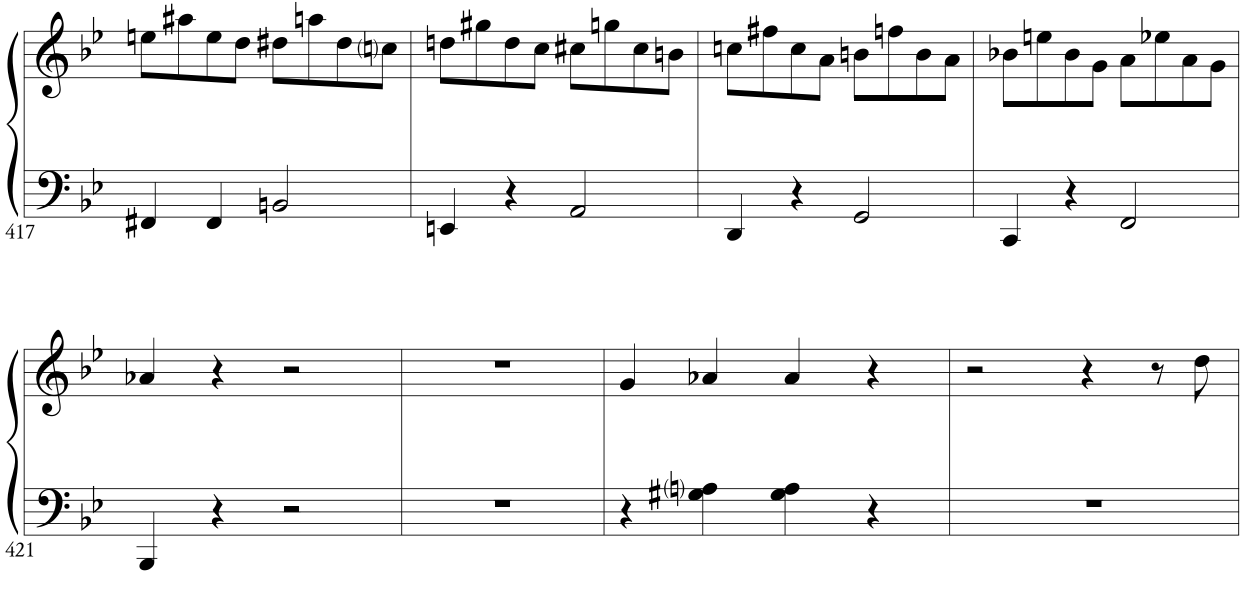
13B



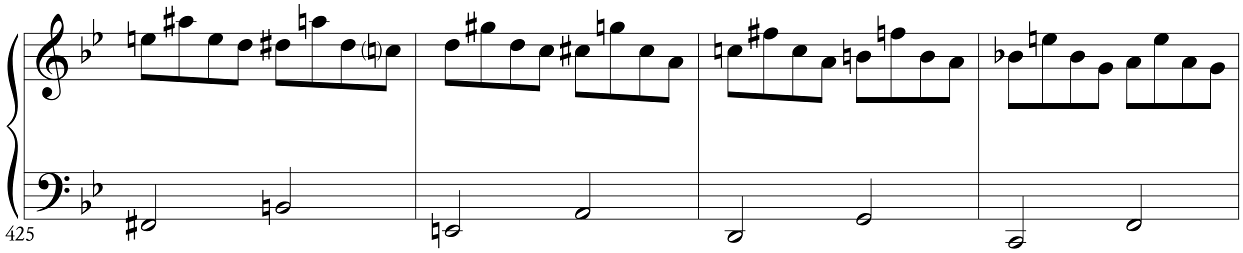
13A$_3$

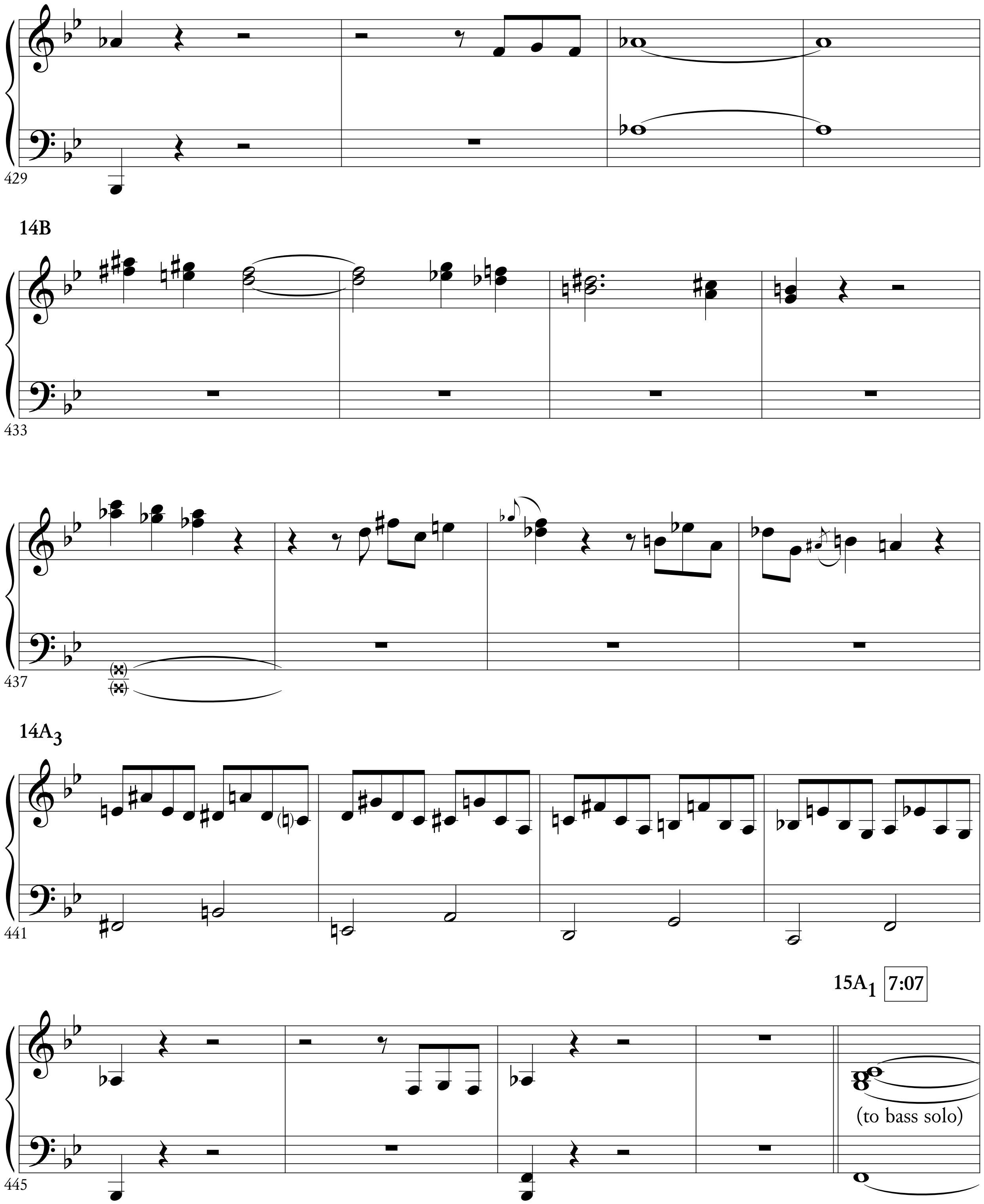


14A$_1$ 6:39



14A$_2$




14A$_3$
15A$_1$
7:07
(to bass solo)

Bibliography

Adams, Kyle. "When Does the Present Become the Past? A Re-examination of 'Presentism' and 'Historicism.'" Paper presented at the annual meeting of the Society for Music Theory, Charlotte, NC, November 2013.

Aebersold, Jamey, ed. *Charlie Parker Omnibook*. Atlantic Music Corp., 1978.

———. *"I Got Rhythm" Changes in All Keys*. Jamey Aebersold Play-A-Long Series, vol. 47. New Albany, IN: Jamey Aebersold Jazz, 1991.

———. *Jazz Handbook*. New Albany, IN: Jamey Aebersold Jazz, 2010. http://www.jazzbooks.com/mm5/download/FQBK-handbook.pdf.

———. *The II–V*^{7}*–I Progression*. Jamey Aebersold Play-A-Long Series, vol. 3. New Albany, IN: Jamey Aebersold Jazz, 1974.

Arthurs, Daniel. "Reconstructing Tonal Principles in the Music of Brad Mehldau." PhD diss., Indiana University, 2011.

Baker, David. *How to Learn Tunes*. New Albany, IN: Jamey Aebersold Jazz, 1997.

———. *How to Play Bebop, Vol. 1: The Bebop Scales and Other Scales in Common Use*. Van Nuys, CA: Alfred, 1985.

Berendt, Joachim-Ernst, and Günther Huesmann. *The Jazz Book: From Ragtime to the 21st Century*. 7th ed. Translated by H. and B. Bredigkeit, Dan Morgenstern, Tim Nevill, and Jeb Bishop. Chicago: Lawrence Hill, 2009.

Berliner, Paul. *Thinking in Jazz: The Infinite Art of Improvisation*. Chicago: University of Chicago Press, 1994.

Biamonte, Nicole. "Augmented-Sixth Chords vs. Tritone Substitutes." *Music Theory Online* 14, no. 2 (June 2008).

Bishop, John. "A Permutational Triadic Approach to Jazz Harmony and the Chord/Scale Relationship." PhD diss., Louisiana State University, 2012.

Bregman, Albert S. *Auditory Scene Analysis*. Cambridge, MA: MIT Press, 1990.

Bribitzer-Stull, Matthew. "The A♭–C–E Complex: The Origin and Function of Chromatic Major Third Collections in Nineteenth-Century Music." *Music Theory Spectrum* 28, no. 2 (Fall 2006): 167–90.

Burt, Peter. "Takemitsu and the Lydian Chromatic Concept of George Russell." *Contemporary Music Review* 21, no. 4 (2002): 79–109.

Callender, Clifton. "Voice-Leading Parsimony in the Music of Alexander Scriabin." *Journal of Music Theory* 42, no. 2 (1998): 219–33.

Callender, Clifton, Ian Quinn, and Dmitri Tymoczko. "Generalized Voice-Leading Spaces." *Science* 320, no. 5874 (April 2008): 346–48.

Capuzzo, Guy. "Neo-Riemannian Theory and the Analysis of Pop-Rock Music." *Music Theory Spectrum* 26, no. 2 (October 2004): 177–99.

———. "Pat Martino's *The Nature of the Guitar*: An Intersection of Jazz Theory and Neo-Riemannian Theory." *Music Theory Online* 12, no. 1 (February 2006).

Carter, Nathan. *Visual Group Theory*. Washington, D.C.: Mathematical Association of America, 2009.

Childs, Adrian P. "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords." *Journal of Music Theory* 42, no. 2 (October 1998): 181–93.

Clement, Brett. "A New Lydian Theory for Frank Zappa's Modal Music." *Music Theory Spectrum* 36, no. 1 (Spring 2014).

Clough, John, Nora Engebretsen, and Jonathan Kochavi. "Scales, Sets, and Interval Cycles: A Taxonomy." *Music Theory Spectrum* 21, no. 1 (Spring 1999): 47–101.

Clough, John, and Gerald Myerson. "Variety and Multiplicity in Diatonic Systems." *Journal of Music Theory* 29, no. 2 (Autumn 1985): 249–70.

Cohn, Richard. *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature*. New York: Oxford University Press, 2012.

———. "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations." *Journal of Music Theory* 41, no. 1 (Spring 1997): 1–66.

Coker, Jerry. *Elements of the Jazz Language for the Developing Improvisor*. Miami: Belwin, 1991.

Cook, Nicholas. "Schenker's Theory of Music as Ethics." *Journal of Musicology* 7, no. 4 (October 1989): 415–39.

Cook, Stephen. Review of Dizzy Gillespie, *Sonny Side Up*. *AllMusic.com*. Accessed July 13, 2015. http://www.allmusic.com/album/sonny-side-up-mw0000188698.

Coolman, Todd. *The Bottom Line: The Ultimate Bass Line Book*. New Albany, IN: Jamey Aebersold Jazz, 1990.

De Cosmo, Emile. *The Diatonic Cycle*. North Bergen, N.J.: EDC Publications, 1970.

Demsey, David. “Chromatic Third Relations in the Music of John Coltrane.” *Annual Review of Jazz Studies* 5 (1991): 145–80.

Deveaux, Scott. *The Birth of Bebop: A Social and Musical History*. Berkeley: University of California Press, 1997.

Douthett, Jack, and Peter Steinbach. “Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition.” *Journal of Music Theory* 42, no. 2 (1998): 241–63.

Folio, Cynthia. “An Analysis of Polyrhythm in Selected Improvised Jazz Solos.” In *Concert Music, Rock, and Jazz since 1945: Essays and Analytical Studies*, edited by Elizabeth West Marvin and Richard Hermann, 103–35. Rochester: University of Rochester Press, 1995.

Ford, Philip. “Somewhere/Nowhere: Hipness as an Aesthetic.” *The Musical Quarterly* 86, no. 1 (April 2002): 49–81.

Forte, Allen. *The American Popular Ballad of the Golden Era, 1924–1950*. Princeton University Press, 1995.

Gjerdingen, Robert. *Music in the Galant Style*. New York: Oxford University Press, 2007.

Gollin, Edward. “Some Aspects of Three-Dimensional *Tonnetze*.” *Journal of Music Theory* 42, no. 2 (1998): 195–206.

Goodheart, Matthew. “The ‘Giant Steps’ Fragment.” *Perspectives of New Music* 39, no. 2 (July 2001): 63–95.

Graf, Richard, and Barrie Nettles. *The Chord Scale Theory and Jazz Harmony*. Advance Music, 1997.

Grossman, Israel, and Wilhelm Magnus. *Groups and Their Graphs*. Washington, D.C.: Mathematical Association of America, 1964.

Hall, Rachel Wells. Review of David Lewin, *Generalized Musical Intervals and Transformations* and *Musical Form and Transformation: Four Analytic Essays*. *Journal of the American Musicological Society* 62, no. 1 (Spring 2009): 205–22.

Heyer, David J. “Applying Schenkerian Theory to Mainstream Jazz: A Justification for an Orthodox Approach.” *Music Theory Online* 18, no. 3 (September 2012).

———. “Vocabulary, Voice Leading, and Motivic Coherence in Chet Baker’s Improvisations.” PhD diss., University of Oregon, 2011.

Hodson, Robert. *Interaction, Improvisation, and Interplay in Jazz*. New York: Routledge, 2007.

Hook, Julian. “Contemporary Methods in Mathematical Music Theory: A Comparative Case Study.” *Journal of Mathematics and Music* 7, no. 2 (2013): 89–102.

———. “Cross-Type Transformations and the Path Consistency Condition.” *Music Theory Spectrum* 29, no. 1 (April 2007): 1–40.

———. “David Lewin and the Complexity of the Beautiful.” *Intégral* 21 (2007): 155–90.

———. *Exploring Musical Spaces*. New York: Oxford University Press, forthcoming.

———. Review of Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (Oxford University Press, 2011). *Music Theory Online* 17, no. 3 (October 2011).

———. “Signature Transformations.” In *Music Theory and Mathematics: Chords, Collections, and Transformations*, edited by Jack Douthett, Martha M. Hyde, and Charles J. Smith, 137–60. Rochester: University of Rochester Press, 2008.

———. “Spelled Heptachords.” In *Mathematics and Computation in Music*, Proceedings of the Third International Conference of the Society for Mathematics and Computation in Music, Paris, June 2011, edited by Carlos Agon, Emmanuel Amiot, Moreno Andreatta, Gérard Assayag, Jean Bresson, and John Mandereau, 84–97. Heidelberg: Springer, 2011.

———. “Uniform Triadic Transformations.” *Journal of Music Theory* 46, nos. 1/2 (Spring–Autumn 2002): 57–126.

Hyer, Brian. “What Is a Function?” In *The Oxford Handbook of Neo-Riemannian Music Theories*, edited by Edward Gollin and Alexander Rehding, 92–139. New York: Oxford University Press, 2011.

Jaffe, Andy. *Jazz Harmony*. Tübingen: Advance Music, 1996.

Julien, Patricia. “The Structural Function of Harmonic Relations in Wayne Shorter’s Early Compositions: 1959–1963.” PhD diss., University of Maryland, 2003.

Kelley, Robin D. G. *Thelonious Monk: The Life and Times of an American Original*. New York: Free Press, 2009.

Kernfeld, Barry. *The Story of Fake Books: Bootlegging Songs to Musicians*. Lanham, MD: Scarecrow Press, 2006.

Klumpenhouwer, Henry. “In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin’s *Generalized Musical Intervals and Transformations*.” *Music Theory Spectrum* 28, no. 2 (Fall 2006): 277–89.

Larson, Steve. *Analyzing Jazz: A Schenkerian Approach*. Hillsdale, NY: Pendragon Press, 2009.

Larson, Steve. "Composition versus Improvisation?" *Journal of Music Theory* 49, no. 2 (October 2005): 241–75.

———. "Schenkerian Analysis of Modern Jazz: Questions about Method." *Music Theory Spectrum* 20, no. 2 (October 1998): 209–41.

———. "Strict Use of Analytic Notation." *Journal of Music Theory Pedagogy* 10 (1996): 31–71.

Lerdahl, Fred. *Tonal Pitch Space*. New York: Oxford University Press, 2004.

Lester, Joel. *Compositional Theory in the Eighteenth Century*. Cambridge, MA: Harvard University Press, 1992.

Levine, Mark. *The Jazz Piano Book*. Petaluma, CA: Sher Music, 1989.

———. *The Jazz Theory Book*. Petaluma, CA: Sher Music, 1995.

Lewin, David. *Generalized Musical Intervals and Transformations*. New York: Oxford University Press, [1987] 2007.

———. *Musical Form and Transformation: Four Analytic Essays*. New York: Oxford University Press, [1993] 2007.

Lewis, George. "Improvised Music after 1950: Afrological and Eurological Perspectives." *Black Music Research Journal* 16, no. 1 (Spring 1996): 91–122.

Love, Stefan. "A Model of Common-Tone Connections Among Jazz Scales." *Journal of Music Theory Pedagogy* 23 (2009): 155–69.

———. "Subliminal Dissonance or 'Consonance': Two Views of Jazz Meter." *Music Theory Spectrum* 35, no. 1 (Spring 2013): 48–61.

Martin, Henry. *Charlie Parker and Thematic Improvisation*. Lanham, MD: Scarecrow Press, 1996.

———. "Jazz Harmony." PhD diss., Princeton University, 1980.

———. "Jazz Harmony: A Syntactic Background." *Annual Review of Jazz Studies* 4 (1988): 9–30.

———. "Schenker and the Tonal Jazz Repertory." *Tijdschrift voor Muziektheorie* 16, no. 1 (2011): 1–20.

McClimon, Michael. "Reconceptualizing the *Lydian Chromatic Concept*: George Russell as Historical Theorist." Paper presented at the annual meeting of the Society for Music Theory, St. Louis, MO, October 2015.

McFarland, Mark. "Schenker and the Tonal Jazz Repertory: A Response to Martin." *Music Theory Online* 18, no. 3 (September 2012).

McGowan, James. "Dynamic Consonance in Selected Piano Performances of Tonal Jazz." PhD diss., Eastman School of Music, 2005.

———. "Psychoacoustic Foundations of Contextual Harmonic Stability in Jazz Piano Voicings." *Journal of Jazz Studies* 7, no. 2 (October 2011): 156–91.

Michaelsen, Garrett. "Analyzing Musical Interaction in Jazz Improvisations of the 1960s." PhD diss., Indiana University, 2013.

———. "Chord-Scale Networks in the Music of Wayne Shorter." Paper presented at the West Coast Conference of Music Theory and Analysis, Eugene, OR, March 2012.

Mulholland, Joe, and Tom Hojnacki. *The Berklee Book of Jazz Harmony*. Boston: Berklee Press, 2013.

Neumeyer, David. "Thematic Reading, Proto-backgrounds, and Registral Transformations." *Music Theory Spectrum* 31, no. 2 (October 2009): 284–324.

Owens, Thomas. *Bebop: The Music and its Players*. New York: Oxford University Press, 1995.

Pöhlert, Werner. *Basic Harmony*. Translated by Jürgen Krohn and Norman Bowie. Werner Pöhlert Publications, 1989.

Porter, Lewis. *John Coltrane: His Life and Music*. Ann Arbor: University of Michigan Press, 1998.

The Real Book. 6th ed. Milwaukee: Hal Leonard, n.d.

Rings, Steven. "Riemannian Analytical Values, Paleo- and Neo-." In *The Oxford Handbook of Neo-Riemannian Music Theories*, edited by Edward Gollin and Alexander Rehding, 487–511. New York: Oxford University Press, 2011.

———. *Tonality and Transformation*. New York: Oxford University Press, 2011.

Russell, George. *The Lydian Chromatic Concept of Tonal Organization*. New York: Concept, 1959.

———. *The Lydian Chromatic Concept of Tonal Organization*. 4th ed. *The Art and Science of Tonal Gravity*. Brookline, MA: Concept, 2001.

Samarotto, Frank. "Treading the Limits of Tonal Coherence: Transformation vs. Prolongation in Selected Works by Brahms." Paper presented at the annual meeting of the Society for Music Theory, Madison, WI, November 2003.

Sanguinetti, Giorgio. *The Art of Partimento: History, Theory, and Practice*. New York: Oxford University Press, 2012.

Santa, Matthew. "Nonatonic Progressions in the Music of John Coltrane." *Annual Review of Jazz Studies* 13 (2003): 13–25.

Satyendra, Ramon. “An Informal Introduction to Some Formal Concepts from Lewin’s Transformational Theory.” *Journal of Music Theory* 48, no. 1 (Spring 2004): 99–141.

Schenker, Heinrich. *Free Composition [Der freie Satz]*. Edited and translated by Ernst Oster. New York: Longman, [1935] 1979.

Sickler, Don, ed. *Thelonious Monk Fake Book*. Steve Cardenas, transcriber. Milwaukee: Hal Leonard, 2002.

Straus, Joseph. “Uniformity, Balance, and Smoothness in Atonal Voice Leading.” *Music Theory Spectrum* 25, no. 2 (October 2003): 305–52.

Strunk, Steven. “The Harmony of Early Bop: A Layered Approach.” *Journal of Jazz Studies* 6 (1979): 4–53.

———. “Linear Intervallic Patterns in Jazz Repertory.” *Annual Review of Jazz Studies* 8 (1996): 63–115.

———. “Notes on Harmony in Wayne Shorter’s Compositions, 1964–67.” *Journal of Music Theory* 49, no. 2 (Autumn 2005): 301–32.

Swift, Richard G. Letter to the editor. *Journal of Music Theory* 4, no. 1 (April 1960): 128.

Tufte, Edward. *The Visual Display of Quantitative Information*. 2nd ed. Cheshire, CT: Graphics Press, 2001.

Tymoczko, Dmitri. “The Consecutive-Semitone Constraint on Scalar Structure: A Link Between Impressionism and Jazz.” *Intégral* 11 (1997): 135–79.

———. “The Generalized Tonnetz.” *Journal of Music Theory* 56, no. 1 (2012): 1–52.

———. “Generalizing Musical Intervals.” *Journal of Music Theory* 53, no. 2 (Fall 2009): 227–54.

———. *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*. New York: Oxford University Press, 2011.

———. “The Geometry of Musical Chords.” *Science* 313, no. 5783 (July 2006): 72–74.

———. “Lewin, Intervals, and Transformations: a Comment on Hook.” *Music Theory Spectrum* 30, no. 1 (Spring 2008): 164–68.

———. “Scale Networks and Debussy.” *Journal of Music Theory* 48, no. 2 (October 2004): 219–94.

Voglewede, Matthew. “Metrically Dissonant Layers of Swing: Double Time in Two of Louis Armstrong’s Performances of ‘Lazy River.’” Paper presented at the annual meeting of the Music Theory Society of the Mid-Atlantic, Philadelphia, PA, March 2013.

Waters, Keith. “Chick Corea, Postbop Harmony, and Jazz’s Second Practice.” Paper presented at the annual meeting of the Society for Music Theory, Charlotte, NC, November 2013.

———. “‘Giant Steps’ and the ic4 Legacy.” *Intégral* 24 (2010): 135–62.

———. “Modes, Scales, Functional Harmony, and Nonfunctional Harmony in the Compositions of Herbie Hancock.” *Journal of Music Theory* 49, no. 2 (Fall 2005): 333–57.

Waters, Keith, and J. Kent Williams. “Modeling Diatonic, Acoustic, Hexatonic, and Octatonic Harmonies and Progressions in Two- and Three-Dimensional Pitch Spaces; or Jazz Harmony after 1960.” *Music Theory Online* 16, no. 3 (August 2010).

Zwerin, Charlotte. *Thelonious Monk: Straight No Chaser*. Warner Bros., 1988. VHS.

Discography

Ammons, Gene, and Sonny Stitt. *Boss Tenors*. Verve V6 8426, 1961. Reissued, Verve 837 440-2, 1991, CD.

Baker, Chet. *Cool Baker, Vol. 2*. Pacific Jazz K18P-9260, 1953. Available on *Chet Baker Quartet Featuring Russ Freeman*. Blue Note 4931642, 1998, CD.

———. *Smokin' with the Chet Baker Quintet*. Prestige PR 7449, 1965. Reissued, Prestige 9020470372, 2001, CD.

Blakey, Art, and the Jazz Messengers. *At the Cafe Bohemia, Vol. 1*. Blue Note BLP 1507, 1956. Reissued, Blue Note CDP 7 46521 2, 1987, CD.

Carter, Benny. *Elegy in Blue*. Musicmasters 65115, 1994, CD.

Coleman, George. *Amsterdam After Dark*. Timeless SJP 129, 1979. Reissued, Timeless Jazz Legacy TJL 74509, 2010, CD.

Coltrane, John. *Coltrane's Sound*. Atlantic LP 1419, 1960. Reissued, Rhino/Atlantic 7567813582, 1988, CD.

———. *Giant Steps*. Atlantic LP 1311, 1959. Reissued, Rhino/Atlantic 8122752032, 1998, CD.

Corea, Chick. *Trilogy*. Concord Jazz CJA 3568502, 2013, CD.

Davis, Miles. *Collector's Items*. Prestige PRLP 7044, 1956. Reissued, Fantasy 0712, 1990, CD.

———. *Cookin' with the Miles Davis Quintet*. Prestige 7094, 1957. Reissued, Prestige 61046, 2003, CD.

———. *Miles Davis, Vol. 2*. Blue Note BLP 5022, 1953. Reissued, Blue Note BNZ 240, 1985, CD.

——— [Miles Davis All Stars]. *Walkin'*. Prestige PRLP 7076, 1957. Reissued, Original Jazz Classics OJCCD 213, 1991, CD.

DeFrancesco, Joey. *Live: The Authorized Bootleg*. Concord Jazz 7230123, 2007, CD.

Evans, Bill. *How My Heart Sings!* Riverside RLP 473, 1962. Reissued, Original Jazz Classics OJCCD 369-2, 1995, CD.

———. *Portrait in Jazz*. Riverside RLP 12-315, 1960. Reissued, Original Jazz Classics OJCCD 088-2, 1992, CD.

———. *Sunday at the Village Vanguard*. Riverside RLP 376, 1961. Available on *The Complete Village Vanguard Recordings 1961*. Riverside 4443, 2005, CD.

Flanagan, Tommy. *Giant Steps: In Memory of John Coltrane*. Enja 4022, 1982. Reissued, Enja CD 4022-41, 1988, CD.

Gillespie, Dizzy. *Dizzy Gillespie*. RCA Victor LPV 530, 1966. Available on *Dizzy Gillespie: The Complete RCA Victor Recordings*. Bluebird 07863 66528-2, 1995, CD.

Gillespie, Dizzy, Sonny Stitt, and Sonny Rollins. *Sonny Side Up*. Verve MGV 8262, 1957. Reissued, Verve 5214262, 1992, CD.

Gordon, Dexter. *More Power!* Prestige PR 7680, 1969. Reissued, Prestige/Original Jazz Classics OJCCD 815-2, 1994, CD.

Guaraldi, Vince. *A Flower is a Lovesome Thing*. Fantasy 3-257, 1958. Reissued, Original Jazz Classics OJCCD 235-2, 1995, CD.

Hamilton, Jeff [Jeff Hamilton Trio]. *Symbiosis*. Capri 74097, 2009, CD.

Harris, Barry. *At the Jazz Workshop*. Riverside RLP 326, 1960. Reissued, Original Jazz Classics OJCCD 208-2, 1992, CD.

———. *Newer than New*. Riverside RLP 413, 1961. Reissued, Riverside VICJ 60691, 2000, CD.

Henderson, Joe. *Big Band*. Verve 5334512, 1996, CD.

———. *Inner Urge*. Blue Note BLP 4189, 1965. Reissued, Blue Note BST 84189, 1985, CD.

Jamal, Ahmad. *Jamal at the Pershing, Vol. 2*. Argo LP-667, 1961. Available on *Complete Live at the Pershing Lounge 1958*. Gambit 69264, 2007, CD.

Jones, Hank. *'Bop Redux*. Muse MR 5123, 1977. Reissued, Pony Canyon Records 30059, 2002, CD.

Kelly, Wynton. *Someday My Prince Will Come*. Vee Jay VJS 3038, 1961. Reissued, Vee Jay NVJ2-902, 1992, CD.

Kirk, [Rahsaan] Roland. *We Free Kings*. Mercury 20679, 1961. Reissued, Mercury 826455-2, 1991, CD.

Konitz, Lee, and Hal Galper. *Windows*. Inner City IC 2057, 1977. Reissued, Steeplechase SCCD 31057, 1987, CD.

Metheny, Pat. *Question and Answer*. Geffen E4242932, 1990, CD.

Monk, Thelonious. *Criss-Cross*. Columbia CL 2038, 1963. Reissued, BMG/Sony 770514, 1993, CD.

———. *Thelonious in Action*. Riverside RLP 12-262, 1958. Reissued, Original Jazz Classics OJCCD 1861032, 1992, CD.

Morgan, Lee. *Candy*. Blue Note BLP 1590, 1957. Reissued, Blue Note BNZ 67, 1987, CD.

Morgan, Lee. *Cornbread*. Blue Note BLP 4222, 1965. Reissued, Blue Note BNZ 109, 1988, CD.

Navarro, Fats. *The Fabulous Fats Navarro, Vol. 2*. Blue Note BLP 1532, 1956. Reissued, Blue Note CDP 781532 2, 1989, CD.

Parker, Charlie. *Charlie Parker, Vol. 1*. Dial LP 201, 1946. Available on *Charlie Parker: The Complete Savoy and Dial Master Takes*. Savoy SVY 17149, 2002, CD.

———. *Summit Meeting at Birdland*. Columbia JC 34831, 1977. Reissued, TriStar 80914, 1994, CD.

———. *The Magnificent Charlie Parker*. Clef MG C-646, 1955. “Blues for Alice” is available on *Confirmation: Best of the Verve Years*. Verve 314 527 815-2, 1995, CD.

Pass, Joe. *Virtuoso*. Pablo 2310 708, 1974. Reissued, Original Jazz Classics OJCCD 31990-02, 2010, CD.

Peterson, Oscar. *The Way I Really Play*. MPS 15180, 1968. Reissued, Verve B0002233-16, 2003, CD.

———. *We Get Requests*. Verve V/V6 8606, 1964. Reissued, Verve/PolyGram 5214422, 1997, CD.

Powell, Bud. *The Amazing Bud Powell, Vol. 1*. Blue Note BLP 1503, 1955. Reissued, Blue Note CDP 7815032, 1989, CD.

The Quintet [Charlie Parker, Dizzy Gillespie, Bud Powell, Charlie Mingus, Max Roach]. *Jazz at Massey Hall*. Debut DEB 124, 1953. Reissued, Original Jazz Classics OJCCD 044, 1990, CD.

Rodney, Red. *One for Bird*. Steeplechase SCCD 31238, 1989, CD.

Rollins, Sonny, and Coleman Hawkins. *Sonny Meets Hawk!* RCA/Victor LSP-2712, 1963. Reissued, RCA/Victor 09026 63479-2, 1999, CD.

Shearing, George. *I Hear a Rhapsody: Live at the Blue Note*. Telarc CD 83310, 1992, CD.

Shorter, Wayne. *Speak No Evil*. Blue Note BLP 4194, 1964. Reissued, Blue Note CDP 7465092, 1987, CD.

Taylor, Art. *Taylor’s Wailers*. Prestige 7117, 1957. Reissued, Original Jazz Classics OJCCD 094-2, 1992, CD.

Tyner, McCoy. *Remembering John*. Enja 6080 2, 1991, CD.

Williams, Tony. *Young at Heart*. Columbia 69107, 1998, CD.

Woods, Phil, and Gordon Beck. *Live at Wigmore Hall, London: The Complete Concert*. JMS 186862, 1996, CD.

Michael McClimon

michael@mcclimon.org
www.mcclimon.org

Education

2016 **Indiana University**, Bloomington, IN
PhD, Music Theory
Dissertation: "A Transformational Approach to Jazz Harmony"
Minor fields: Music History and Literature, Jazz Studies

2010 **Indiana University**, Bloomington, IN
MM, Music Theory
Outside area: Jazz Studies

2008 **Furman University**, Greenville, SC
BM, Music Theory, *Magna cum laude*

Teaching Experience

2014–present **Lecturer in Music**, Furman University

Duties: Teach both written and aural components of sophomore music theory (chromatic harmony, form, post-tonal techniques). Direct independent studies with undergraduate music theory majors (topics have included Schenkerian analysis, transformational theory, and late Romantic symphonies). Team-teach a weekly composition seminar with the rest of the theory/composition faculty.

2008–2013 **Associate Instructor**, Indiana University

Duties: Assist with courses in the undergraduate written music theory curriculum, including first-year theory (diatonic harmony), sophomore theory (repertoire spanning the 16th–19th centuries), and junior theory (repertoire from 1900 to present). From 2011–2013, had complete classroom control of an honors section of the sophomore theory sequence.

Conference Papers

Reconsidering the *Lydian Chromatic Concept*: George Russell as Historical Theorist.

⋆ Society for Music Theory (St. Louis, MO, October 2015)

Diatonic Chord Spaces in Jazz: A Transformational Approach.

▪ Music Theory Southeast (East Carolina University, March 2015)

Jazz Harmony, Transformations, and ii–V Space.

- ⋆ Society for Music Theory (Milwaukee, WI, November 2014)
- Music Theory Society of the Mid-Atlantic (Temple University, March 2013)
- *Winner, Dorothy Payne award for best student paper* (MTSMA 2013)

The Written Musical Symbol as a Medium for Interpretation: Schenkerian Analysis as Experience.

- "Technologies of Experience" Symposium (Indiana University, April 2013)

Expressing the Inexpressible: Thelonious Monk's "Crepuscule with Nellie."

- West Coast Conference for Music Theory and Analysis (University of Oregon, March 2012)
- Music Theory Southeast (Florida State University, March 2011)
- Music Theory Midwest (University of Nebraska, May 2011)

The Temporal Importance of $\hat{5}$ in Schubert's C Major Piano Sonata, D.279/ii.

- Indiana University Graduate Theory Symposium (February 2011)

Examining "Evidence": An Inquiry into Thelonious Monk's Metrical Process.

- Florida State University Music Theory Forum (January 2011)

Professional Activity

2015–present	Associate Webmaster, Society for Music Theory
2014–present	Senior Editorial Assistant, *Music Theory Online*
2014–present	Member, SMT Jazz Interest Group Awards Committee
2014–2015	Member-at-large, Society for Music Theory Networking Committee
2012–2014	Digital Projects Specialist, Center for the History of Music Theory and Literature
2012–2014	Editor, *Indiana Theory Review*
2012	Software Development Contractor, Indiana University, "Music Fundamentals Online"
2011–2014	Editorial Assistant, *Music Theory Online*
2010–2013	Various positions, Indiana University Graduate Theory Association Treasurer (2010–2012), Symposium Program Committee (2010, 2012–2013)
2009–2012	Various positions, *Indiana Theory Review* Circulation Manager (2011–2012), Editorial Staff (2009–2012)