Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective

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INTRODUCTION TO

NEO-RIEMANNIAN THEORY:

A SURVEY AND A

HISTORICAL PERSPECTIVE

Richard Cohn

The papers collected in this issue represent an emerging species of transformational theory drawn together under the *Neo-Riemannian* designation. This introductory essay sketches the origins and recent development of neo-Riemannian theory, and positions it with respect to several other genera of music theory, as well as to an evolving post-structuralist critical practice. Along the way, it seeks to situate Hugo Riemann amidst the flurry of activity occuring beneath the banner bearing his name, and to provide comfort to the perplexed observer who, innocently prying apart these covers and peering within, spies the theoretical tradition of A. B. Marx and Oettingen romping merrily with that of Babbitt, Forte, and Morris. Strange fellows indeed! . . . or perhaps, upon reflection, not so strange. . . .

I

Neo-Riemannian theory arose in response to analytical problems posed by chromatic music that is triadic but not altogether tonally unified. Such characteristics are primarily identified with music of Wagner,

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Liszt, and subsequent generations, but are also represented by some passages from Mozart, Schubert, and other pre-1850 composers. Because music of this type uses the harmonic structures and, often, the conventional cadences of diatonic tonality, it lures the attentions of analytical models designed for diatonic music. Yet it is also notoriously unresponsive to such attentions, a circumstance that was acknowledged in the first half of our century by Louis and Thuille, A. Eaglefield Hull, Ernst Kurth, Donald Francis Tovey, and Adele Katz, and has since been significantly elaborated in the writings of Mitchell, Dahlhaus, and Proctor.[1]

The assignment of a category-label to the music that concerns us here is delicate, largely for sociological reasons. 'Chromatic tonality' misleadingly points toward pitch-centricity; 'triadic chromaticism' is over-inclusive, since it also embraces chromatic harmony reconcilable to a diatonic basis. 'Triadic atonality' potentially misleads in a different direction. When first introduced by Lowinsky (1961), with reference to chromatic repertories from the turn of the seventeenth century, the conjunction oxymoronically grated on the sensibilities of readers for whom 'atonality' conjured a Second Viennese sound-world.[2] To defuse similar resistance in the current context, one feels compelled to inoculate 'atonal' with coward quotes, and to stress that the presence of consonance no more implies tonality than the presence of dissonance implies atonality. Under the circumstances, 'triadic post-tonality' may be the more appropriate term for the chromatic music of the late nineteenth century. Such is suggested by William Rothstein when he writes, of Wagner, that "Some phrases . . . are not true phrases at all from a tonal point of view because they do not contain a single coherent tonal motion. Such phrases . . . must be said to cross the hazy line separating tonality from post-tonality (or whatever one wishes to call triadic but post-tonal practice)" (Rothstein 1989, 280).

Two further points will help clarify the ways in which it is appropriate to view some nineteenth-century triadic music as to some extent tonally disunified, and, to that extent, 'post-tonal.' First, habits of thought tend to obstruct our ability to conceive of a composition as combining segments that are coherently tonal with others that are not. One such habit is the hypostasization of the tonal/atonal binary at the level of the composition (to say nothing of the composer and the entire repertory); another is the conviction that masterworks are unified by a single system. Together these obstacles force us down a default path that leads from "this phrase is difficult to understand as an instance of diatonic tonality" to "this entire composition is either lousy or atonal." Deconstruction of both obstructive habits, by now well underway, reveals alternate paths.

A related point is that periodic Classical cadences do not guarantee that the music framed by them is coherently tonal. This perspective was already articulated in Fétis's 1844 *Traité* with reference to diatonic se-

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quences,[3] and in this century it has been a tenet of German Wagnerian scholarship since Kurth. Its transmission to American scholarship, via Adorno and Dahlhaus, is manifest in the writings of Anthony Newcomb (e.g., 1981, 52–54) and Carolyn Abbate. For instance, Abbate writes with reference to Wotan's monologue from *Die Walküre*, that:

> Wagnerian tonal analysis—which tends to take note of cadence points and extrapolate their relationships—might find 'progression' or 'modulation' an adequate description, for both words are charged with comfortable values; both imply that there is a coherent arc traced by the monologue's harmonic course. Yet the cadence points are detached from the musical matter that they punctuate: they are not integral to it. It is as if these cadences are laid over unstructured harmonic improvisations; the cadences create local wrinkles, but only draw a few instants into their tonal sphere (Abbate 1991, 192).

Abbate's characterization suggests a point of common ground between the emerging neo-Riemannian and post-structuralist paradigms, but also accents a point of dissonance between them. Both paradigms recognize the potential for tonal disunity in music that uses classical harmonies, and accordingly resist shoehorning all chromatic triadic music into the framework of diatonic tonality. For the post-structuralist, the recognition of <u>tonal</u> disunity leads immediately to an ascription of disunity *tout court*, and from there to a cluster of cognate terms, *chacun tout court*, that ring the post-structuralist swimming pool like so many chaise lounges: 'unstructured,' 'incoherent,' 'indeterminate', 'coloristic', 'disjunct', 'arbitrary', or 'aimless'.

The recognition of tonal disunity could instead lead to a question: "if this music is not fully coherent according to the principles of diatonic tonality, by what other principles might it cohere?" The neo-Riemannian response recuperates a number of concepts cultivated, often in isolation of each other, by individual nineteenth-century harmonic theorists. The following exposition identifies six such concepts: triadic transformations, common-tone maximization, voice-leading parsimony, "mirror" or "dual" inversion, enharmonic equivalence, and the "Table of Tonal Relations." With few exceptions, nineteenth-century theorists incorporated each of these concepts into a framework governed by some combination of diatonic tonality, harmonic function, and dualism. Neo-Riemannian theory strips these concepts of their tonally centric and dualist residues, integrates them, and binds them within a framework already erected for the study of the atonal repertories of our own century.

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II

Neo-Riemannian theory originates in David Lewin's transformational approach to triadic relations.[4] In his 1982 essay, "A Formal Theory of Generalized Tonal Functions," Lewin proposes two classes of transformations which, although defined to act on "Riemann systems" (a concept that will not concern us here), indirectly also act on consonant triads. One class of transformations inverts a triad, mapping major and minor triads to each other. Examples are the inversion that exchanges the pitch classes that form a perfect fifth (and hence maps a triad to its parallel major or minor), and the inversion that exchanges the pitch classes that form a major third (mapping a triad to its relative major or minor). Because the inversional axis is defined in relation to the triad's component pitch classes, rather than as a fixed point in pitch-class space, this class of transformations is now referred by the term "contextual inversion."

The second class of transformations is defined with reference to Figure 1, which (following Hauptmann 1853) arranges pitch classes in a line of alternating minor and major thirds, each contiguous triplet constituting a consonant triad. Lewin proposes a set of transformations that shift triads along Figure 1 by a designated number of stations. For example, an incremental leftward shift maps a triad to its submediant; a leftward shift of two positions maps a triad to its subdominant. In general, a shift by an odd number of elements inverts a triad; consequently, there is some overlap, or redundancy, between the shift transformations and the contextual inversions. This circumstance becomes significant as the transformations subsequently evolve.

Although for heuristic reasons the characterizations of the shift transformations employed here imply a motion in diatonic space, Lewin defines his transformations generally so that they act analogously on configurations shifted along versions of Figure 1 constructed by alternating <u>dissonant</u> intervals. Thus, although triads and diatonic collections are among the objects that they produce, the transformations proposed in the 1982 article are definitionally and conceptually independent of the system of diatonic tonality normally signified by those structures.

As part of a broad exploration of musical transformations, *Generalized Musical Intervals and Transformations* (Lewin 1987, 175–80) redefines the transformations introduced in the 1982 essay so that they act directly on consonant triads (here referred to by Lewin, after Riemann, as *Klänge*). A transformation is conceived here as "something one does to a *Klang*, to obtain another *Klang*" (177). What had been an incremental leftward shift on Figure 1 is now named MED, the operation that causes the transformed triad to "become the mediant of its MED-transform" (176). What had been a double leftward-shift on Figure 1 is now

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bb Db f Ab c Eb g Bb d F a C e G b D f♯ A c♯ E g♯ B d♯

Figure 1

named DOM, and defined as transposition by five semitones (T_5). The three contextual inversions from the 1982 paper are redefined and re-named, as follows:

> We can define REL, the operation that takes any Klang into its relative major/minor. . . . We can also define PAR, the operation that takes any Klang into its parallel major/minor. . . . We can define Riemann's 'leading-tone exchange' as an operation LT (Lewin 1987, 178).

Although the transformations in GMIT remain conceptually indepen-dent of diatonic tonality, their independence is not verified there by con-struction of analogous dissonant systems as it was in Lewin 1982. Indeed, the claim of independence is somewhat occluded here by Lewin's use of informal definitions that, in the case of MED, REL, and PAR, imply the mediation of diatonic collections.[5] These interventions are heuristically advantageous, but the 1982 article demonstrates their formal superfluity.

In addition to defining triadic transformations, the brief exposition in *GMIT* marks out three areas for further exploration: compositional logic, group structure, and geometric representation. Lewin notes that two ap-plications of MED compose to produce DOM, and that MED is a group generator. Compositions of triadic transformations, consistent with the more general transformational technology developed in *GMIT*, are rep-resented as two-dimensional graphs whose elements are arranged <u>ad hoc</u> on the page.

Brian Hyer's 1989 dissertation develops these three areas, substituting a new graphing technique. Hyer extracts a subset of Lewin's triadic trans-formations: three contextual inversions (PAR, REL, and LT) and one transposition (DOM), representing each by its initial letter alone. The ap-parently diatonic MED is eliminated, its job now divided among the con-textual inversions. This marks the disappearance of the shift transforma-tions, the surviving DOM having been reinterpreted as a transposition.

To chart the composition of these transformations, Hyer recuperates a geometry favored by nineteenth-century theorists, the "Table of Tonal Relations," or *Tonnetz*.[6] The "table," more accurately a graph, coordi-nates three axes representing the three triadic intervals. On the version of the table presented here as Figure 2, perfect fifths generate the horizon-tal axis, and major and minor thirds respectively generate the two diago-nal axes. Adjacent row-pairs combine to replicate the linear pitch-class pattern of Figure 1, but in a "fan-fold" pattern. The *Tonnetz* thus sub-sumes the geometry of Figure 1.

Each triangle on Figure 2 represents a consonant triad. Arrows indi-
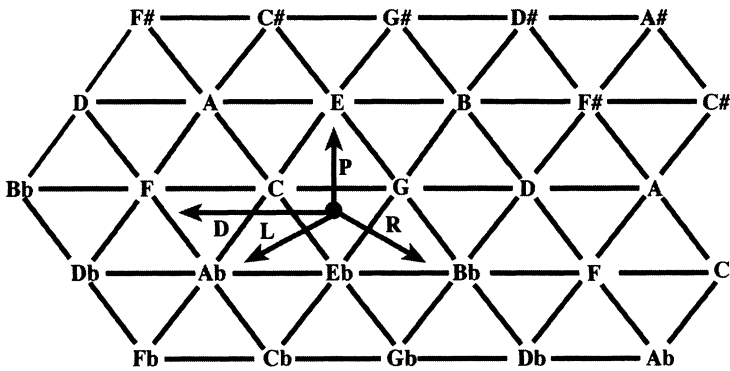
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Figure 2

cate Hyer's four transformations as they act on a C-minor triad. Each of the three contextual inversions inverts a triangle around one of its edges, mapping it into an edge-adjacent triangle. P, for Parallel, inverts around a horizontal (perfect fifth) edge, mapping C minor to C major; R, for Relative, inverts around a secondary diagonàl (major third) edge, mapping C minor to E♭ major; and L, for Leading-tone-exchange, inverts around a main diagonal (minor third) edge, mapping C minor to A♭ major. The fourth transformation, D (for dominant), transposes a triangle to the vertex-adjacent triangle to its left, mapping C minor to F minor. (Note that the D transformation is redundant, since it is produced by a composition of L followed by R.) Transformational directions on the *Tonnetz* are invariant, although the direction of the arrow is reversed when contextual inversions are applied to major triads. The *Tonnetz* thus provides a canonical geometry for modelling triadic transformations.

The objects and relations of Hyer's *Tonnetz*, unlike those of most nineteenth-century antecedents, are conceived as equally tempered, a circumstance that Hyer acknowledges by substituting enharmonically neutral integers for the enharmonically biased pitch-class names favored by Lewin (and used in Figure 2). Under equal temperament, the horizontal axis of Pythagorean fifths becomes the circle of tempered fifths, and the diagonal axes of justly tuned thirds become the circles of tempered major and minor thirds respectively. The table becomes circularized in each of its dimensions, and the entire graph becomes a hypertorus. Such a conception greatly enriches the group structure of the transformations, which Hyer explores in detail.

Hyer's appropriation of the *Tonnetz* intensifies the relationship between triadic transformational theory and nineteenth-century harmonic theory. Versions of the *Tonnetz* appeared in German harmonic treatises

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throughout the nineteenth-century, beginning with Weber's "Table of the Relationship of Keys" from 1817–1821. If the objects in Weber's table are interpreted as triads rather than as diatonic tonal spaces, his table is a rotated geometric dual of Figure 2. Like Hyer, Weber holds a cyclical conception of the table's axes, noting (and graphically showing) that the axis of minor thirds "can be bent into a circle of thirds, similar to the so-called circle of fifths."[7]

A table more closely resembling Figure 2 is presented in Arthur von Oettingen's *Harmoniesystem in dualer Entwickelung* (1866). Oettingen uses the table to chart relationships among triads, anticipating Hyer in this respect. Yet Oettingen's axes, unlike Weber's and Hyer's, are generated by purely tuned intervals, and thus extend infinitely without cyclic closure. Riemann, inspired by Oettingen, maintained the table in its infinite, non-cyclic form for more than forty years, despite his evolving ideas about tuning and temperament.[8] It is one of the odd ironies of nineteenth-century harmonic theory that those theorists who were interested in the table as a map of triadic motion were not interested in exploring its cyclic properties under equal temperament, and those theorists who were fully comfortable with equal temperament were not interested in the table's potential to map triadic motion. Through the presentist bias of current neo-Riemannian theory, it is as if nineteenth-century theorists had at their disposal both flint and fuel, yet never both together at the same time.

Hyer's work fortifies the relationship with the harmonic theories of Oettingen and Riemann in yet another respect. By eliminating MED, and thus limiting his transformations to contextual inversions and transpositions, Hyer's transformations come into close contact with the system of *Schritte* and *Wechsel* introduced in Oettingen 1866 and expanded in Riemann's *Skizze einer Neuen Methode der Harmonielehre* of 1880. Although Lewin (1987) had identified his triadic transformations with Riemann's work, he had affiliated them with the theory of harmonic functions introduced by Riemann in the 1890s, while acknowledging that Riemann did not conceive the functions in transformational terms.[9] Klumpenhouwer (1994) notes the closer affinity between the triadic transformations and the *Schritt/Wechsel* (S/W) system, which remained distinct from the functional theory introduced a decade later.[10] The affinity can be specified as follows: each species of *Wechsel* is equivalent to a single species of contextual inversion, to within enharmonic equivalence; the *Schritte*, as a genus, are equivalent to the genus of triadic transpositions; each species of *Schritt* would be equivalent to a single species of triadic transposition (again to within enharmonic equivalence) were it not for Riemann's dualism, which dictates that the *Schritte* transpose major triads by the same interval as, but in an opposite direction from, minor triads. This dualistically imposed wrinkle aside, Hyer's four transformations are a subset of the S/W system.

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The S/W system anticipates the triadic transformations of Lewin and Hyer in spirit as well as in substance, as is clarified in the recent dissertations of David Kopp (1995, 112–21) and Michael Kevin Mooney (1996, 236 ff.). For Riemann, the S/W relationship between two triads is explicitly independent of a particular tonal environment (Kopp, 124), a characteristic ideally suited to neo-Riemannian analytic purposes. Furthermore, Mooney suggests that the S/W system, for both Oettingen and Riemann, is essentially transformational in just the sense that Lewin advances: a given *Schritt* or *Wechsel* "is something one *does* to a Klang, to obtain another Klang."[11]

The point of departure for my own work in neo-Riemannian transformational theory (Cohn 1996, 1997) is the observation that each of the three contextual inversions of Lewin and Hyer feature parsimonious voice-leading. Whenever a motion between two triads involves retention of two common tones, the single "moving" voice proceeds by step: by semitone in the case of L and P, and by whole step in the case of R. This circumstance permits extended triadic progressions consisting exclusively of single-voice motion by semitone, represented as a motion along a southwest-to-northeast alley of Figure 2. Cohn 1996 designates each such alley as a hexatonic system. Figure 3 detaches these alleys and presents them as four circles, whose larger cyclical arrangement reflects the adjacencies of their respective alleys on the *Tonnetz*. The entire ensemble, termed a hyper-hexatonic system by Robert Cook, is essentially Hyer's hypertoroidal *Tonnetz*, arranged so as to accent semitonal voice-leading relations among the triads. A progression whose voice-leading is only slightly less parsimonious is represented by motion along a northwest-to-southeast alley of Figure 2, as explored in Cohn 1997, where it is called an octatonic system. Here single-voice semitonal motion (P) alternates with single-voice whole-step motion (R). The circular representations of these detached alleys are presented in Figure 5 of Douthett and Steinbach (see page 247 below).

The emphasis on common-tone preservation and semitonal voice-leading adds yet a further dimension to the relationship between triadic transformations and nineteenth-century harmonic theory. Unlike their eighteenth-century predecessors, for whom triadic proximity was a function of consonance of root relation or, alternatively, of root-relatedness on a line of fifths, many nineteenth-century theorists gauged triadic proximity by number of shared common tones, bringing to the fore exactly the three contextual inversions explored by Lewin and Hyer. The earliest source I have located that represents this new perspective, albeit only in passing, is the 1827 treatise of K. C. F. Krause ([1827] 1911, 14). A more comprehensive treatment is Adolf Bernhard Marx's *Die Lehre von der Musikalischen Komposition*, which first appeared in 1841 and remained influential throughout the remainder of the century. Kopp discusses this
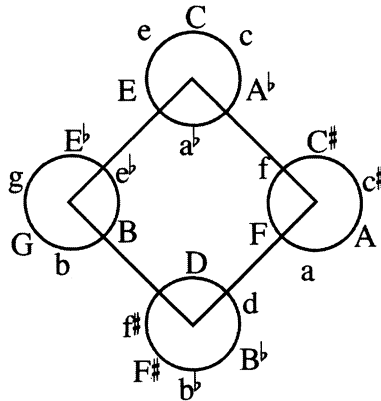
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e C c
E A♭
E♭ a♭ C♯
g e♭ f c♯
G B F A
b D a
f♯ d
F♯ B♭
b♭

Figure 3. The Hyper-hexatonic System

aspect of Marx's treatise in some depth, relating that Marx "never dis-
cusses chord progressions in terms of root relations. Chord connection
for him is completely a matter of common tones joining neighboring har-
monies" (Kopp 1995, 77). Marx also developed the concept of melodic
fluency, which essentially amounts to Schoenberg's "principle of least
motion," and maintained that harmonic relatedness and melodic fluency
competed on equal terms as determinants of harmonic progression. Com-
mon-tone preservation and semitonal voice-leading come further to the
fore in Carl Friedrich Weitzmann's 1853 monograph *Der Übermässige
Dreiklang,* which views each consonant triad as a semitonal displace-
ment of one of the four augmented triads and classifies it accordingly.[12]

It is apparent, from the foregoing discussion, that Riemann is not the
sole purveyor of those nineteenth-century ideas that are absorbed by neo-
Riemannian theory. Indeed, although Riemann is identified to some de-
gree with the six concepts itemized at the end of the first part of this
essay, none of them originated with him. This circumstance suggests that
the term 'neo-Riemannian' is most pertinently viewed as synecdochally
appropriating the name of Riemann, to represent a tradition of German
harmonic theory which his writings culminated and perpetuated in the
twentieth century.

III

Nineteenth-century music theorists had no apparent conception of
groups, sets, or graphs. (This is the case even for Helmholtz and Oettin-
gen, whose primary training was in the physical sciences.) Nonetheless,

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as the work assembled in this issue attests, these interrelated branches of mathematics provide a productive conceptual framework for characterizing many nineteenth-century ideas about harmony, as well as an efficient technology and descriptive language for making and communicating new discoveries about the properties of triads and related structures, and the relational systems in which they participate. The central category of harmonic theory, the consonant triad or *Klang*, is a species of the central higher-level category of atonal pitch-class theory, the Tn/TnI set class; Forte (1973) catalogues it as the eleventh species of cardinality three. The most prominent transformations of atonal pitch-class theory, transposition and inversion, are fundamental to nineteenth-century theory, the former deeply embedded in traditional pedagogy, the latter especially as dualism comes to the fore at mid-century. Enharmonic equivalence and equal temperament, crucial to the closure of pitch-class space, were recognized by many nineteenth-century theorists beginning with Vogler and Weber, although strenuously resisted by others. Common-tone relations among triads are associated with cardinality of pitch-class intersection, and melodic fluency with measurement of directed intervals in chromatic space, both associations proceeding by simple translation into a formal language. And graph theory provides techniques for formalizing and generalizing the relations that are portrayed in the *Tonnetz*. Although any application of modern mathematical techniques of inquiry and communication to the repertory of triadic post-tonality involves a certain degree of anachronistic retrofitting, the underlying concepts are consonant with those cultivated in the nineteenth century, and indeed were broadly explored by contemporaneous musical thinkers.

In 1990, I began to investigate the position of consonant triads within the universe of modulo-12 set classes, in the hope of understanding what properties allow its members to participate in progressions featuring parsimonious voice-leading. I discovered that only a small handful of set classes could induce extended progressions employing exclusively single-voice semitonal motion (Cohn 1996, 16). The list interacted in interesting ways with other significant features that had been identified by other explorers of the group-theoretic properties of traditional musical systems. In this way, triadic transformational theory renewed the contact (already implicit in Lewin 1982) with an ongoing tradition of speculative theory that traces its roots to Babbitt's identification of the deep-scale property of diatonic collections (Babbitt 1965).[13] Following the methodological lead of Balzano 1980, Clough and Douthett 1991, and others, I then searched for "parsimonious set classes" in chromatic universes with fewer than 12 pitch classes, in order to create a broader platform from which to test and refine my observations.

In November of 1992 some preliminary data, observations, conjectures, and a number of open questions were shared with Jack Douthett

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and John Clough. During the next six months, Douthett circulated a series of letters, cumulatively summing to approximately 115 pages, in which he sketched a variety of graph- and group-theoretic approaches to the problems posed in my correspondence. Douthett not only formalized and refined my observations, but also extended my interest in parsimonious triadic voice-leading to include relations between consonant and augmented triads, and also between various species of diatonic seventh chords. These inquiries resulted in two geometric figures that came to be known as Cube Dance and Power Towers, published here for the first time as Figures 9b and 10 of Douthett and Steinbach's paper. The significant influence of these figures, and the work associated with them, is evident in several papers included in this collection. Further circulation of Douthett's and my correspondence, at John Clough's initiative, led to the formation of the first Buffalo symposium in July 1993. David Lewin's presentation of a general algorithm for deriving parsimonious set classes became a focus of much of the 1993 meeting; it was eventually revised and expanded as "Cohn Functions" (Lewin 1996).

The eleven papers included in this issue continue along the trajectory marked out by the work that led to the 1993 Buffalo meeting. The first five of these papers generalize neo-Riemannian methodology, applying it to set classes other than the consonant triad. The first three papers share a focus on set class 4-27, the class of dominant and half-diminished seventh chords. The atonal properties of this set class had already been studied nearly thirty years ago by Boretz (1972). John Clough observed, in correspondence from 1993, that set class 4-27, like the consonant triad, minimally perturbs a symmetric division, and that this characteristic leads to parsimonious voice-leading among its members. This observation informs Adrian Childs's paper, which identifies a set of transformations that preserve two pitch classes and move the remaining two by semitone, explores the systematic foundation underlying these transformations, and applies them to passages from Chopin, Wagner, and Stravinsky. Edward Gollin represents the relationships among members of set class 4-27 from a slightly different perspective, introducing a three-dimensional *Tonnetz*, and generalizing its constructional procedure to create *Tonnetze* that portray common-tone relations within less parsimonious tetrachord-classes as well. And Stephen Soderberg probes the boundary between late-Romantic and atonal compositional practices by exploring the properties of hexachords formed by uniting pairs of 4-27 tetrachords, as part of a broader exploration of hexachordal constellations constituted by well-defined principles of construction from smaller cells.

The status of the Mystic Chord (set class 6-34) as a minimal perturbation of a whole-tone collection forms a point of departure for Clifton Callender's investigation of the parsimonious voice-leading relations that interlink the harmonic and scalar structures most characteristic of Scri-

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abin's idiom. In the final paper of this group, David Lewin explores the ordering of [013] trichords in a Bach fugue, charting their common-tone relations on a *Tonnetz* generated by dissonant intervals.

In a wide-ranging paper that bridges the two halves of the volume, Jack Douthett and Peter Steinbach explore voice-leading parsimony among the basic harmonic structures of tonal music, as they relate to various Modes of Limited Transposition of which they are subsets. Their approach synthesizes work on consonant triads and 4-27 tetrachords, but also introduces augmented triads, minor seventh chords, and fully-diminished seventh chords into the parsimonious mix. Readers wishing a brief introduction to the substance of this volume may find it useful to begin by studying Douthett and Steinbach's graphs, which are both synoptic and transparent.

The final five papers extend the focus on consonant triads that initiated the neo-Riemannian enterprise. Carol Krumhansl reports on empirical studies of listener judgments of triadic proximity. A geometric portrait of these data independently replicate the hypertoroidal *Tonnetz*, and suggest that the four neo-Riemannian transformations emphasized in Hyer's work are psychologically "real." She also finds that common-tone relations are more likely to guide the judgments of subjects without musical training, a finding that suggests some linkages with the changing constituency of concert audiences in the nineteenth century. My paper proposes bisecting the four hexatonic systems, creating eight classes that are characterized by the equivalence of their pitch-class sums, and modeling voice-leading efficiency as the differences between those sums, using music of Schubert, Liszt, and Brahms for illustration.

The final three papers focus on compositional logic and group structure within and among Riemann's *Schritt/Wechsel* transformations, neo-Riemannian triadic transformations, and the standard "twelve-tone operations." John Clough explores the abstract properties of the S/W group, and compares it with the more familiar transposition/inversion group and two other groups that combine S/W with T/I operations. Selecting one of Clough's groups for closer study, Jonathan Kochavi's study focuses on the pairing of transpositions and *Wechsel*, in the process furnishing a general mathematical backdrop against which to understand the contextual inversions emphasized by neo-Riemannian theory. David Clampitt, focusing on a hexatonic triadic passage from Wagner, compares and interrelates triadic transformations, hexatonic transpositions, and pitch-class inversion, and then introduces the concept of system modulation, adapted from Lewin 1987, to refine our understanding of triadic relations between distinct hexatonic systems.

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IV

The editors of this volume have been aware that the collaborative nature of the work presented here creates several potential pragmatic difficulties for its readers. One problem is the profusion of terms that came in and out of usage in various conversations and informally circulated documents. To take one egregious example, the individual cycles portrayed in Figure 3 have been variously referred to as "hexatonic systems," "maximally smooth cycles," "P-cycles," "P_1-cycles," "L/P cycles," and "Cohn cycles." A related problem is the interchangeability of symbolsets. Major and minor triads are distinguished using two conventional pairs of symbol sets, plus/minus and large-case/small-case. The lack of standard names and symbols for dissonant chords, too, is reflected in this volume by a certain degree of diffusion. Geometric figures, such as the *Tonnetz*, are oriented in different ways according the objectives of individual contributors. Although the volume editors have recommended some standards, ultimately individual authors have retained autonomy in these matters. The editors have sought to clarify terminological ambiguities wherever we perceived them to arise.

Finally, there is a substantial degree of cross-referencing between the individual contributions, which precludes an ideal ordering of the papers from the standpoint of the reader. We have attempted to point out linkages where we think they are significant.

NOTES

1. Louis & Thuille 1913 (1982), 408–9; Hull 1914, 36; Rothfarb 1991, 120–23 (which translates relevant aspects of Kurth 1920); Katz, 1945, 210–15; Mitchell 1962; Dahlhaus 1980 (1974), 64–71; and Proctor 1978.
2. See, for example, Fischer 1972; Dahlhaus 1990 (1968), 17–18.
3. Fétis 1844, 25–26. His view influenced Riemann; see Dahlhaus 1990 (1968), 59.
4. For a more thorough account of the development of transformational theory, see Kopp 1995, 253–69.
5. See, for example, Kopp 1995, 254, where Lewin's transformations are character-ized (wrongly, I believe) as "diatonically predicated at their core."
6. Hyer's version is actually the geometric dual of the one provided here, and its form is more consistent with the one provided in Lewin's contribution to this issue. Geometric duals are discussed in Douthett and Steinbach's contribution. For a more complete characterization of the *Tonnetz*, see Gollin's contribution in this issue.
7. Weber 1846 (1817–21), 312. For a recent *Tonnetz* interpretation of Weber's table, see Lerdahl 1994.
8. Mooney 1996 discusses the various uses to which Riemann put the table at differ-ent points of his career. See 146 ff. for a discussion of Riemann's views on tuning and their impact on his conception of the table.
9. Lewin 1987, 177. The point is echoed by Kopp 1995 and Mooney 1996. Alphonce 1988 notes that Swedish theorists in this century have interpreted Riemann's func-tion theory transformationally.
10. Although Klumpenhouwer states that the system is his own extrapolation of a sys-tem only partially filled out by Riemann, Kopp (1995, 118–21) demonstrates that Riemann 1880 presents a completed filled out S/W system, which Kopp refers to as a "root-interval system." On the conceptual independence of the S/W and func-tional systems, see Kopp , 181–84. An exposition of the S/W system is provided by Edward Gollin's contribution in this issue.
11. Mooney 1996, 71–77 and 236–39. This transformational sense is conveyed less strongly by Kopp, for whom Riemann's S/W system is a way of classifying rela-tions between triads.
12. Weitzmann's classes are the topic of Cohn 2000, and are discussed briefly on page 290 of the current issue.
13. For a summary and assessment of this tradition, see Clampitt 1997. The tradition includes Gamer 1967 on deep scales, Boretz 1971 on asymmetric maximal parti-tioning, Clough 1979 and Clough and Myerson 1985 on the special characteristics of diatonic scales, Carey and Clampitt 1989 on generated scales, Agmon 1991 on the unique voice-leading properties of triads and seventh chords in diatonic space, and Clough and Douthett 1991 on maximal evenness and hyperdiatonic scales. Similar investigations of pitch-structures in non-Western musical traditions are Rahn 1977, Krumhansl 1987, and Clough, Douthett, Ramanathan and Rowell 1993.

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