Moving beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords

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# MOVING BEYOND NEO-RIEMANNIAN TRIADS: EXPLORING A TRANSFORMATIONAL MODEL FOR SEVENTH CHORDS

Adrian P. Childs

Initial work applying the theories of neo-Riemannian triadic transformations has focused primarily on late nineteenth-century chromatic repertoire, particularly the operas of Richard Wagner.[1] While the analytical insights provided have proven rich and stimulating, a fundamental problem has also arisen: the composers whose works seem best suited for neo-Riemannian analysis rarely limited their harmonic vocabulary to simple triads. The first section of this paper examines this problem with a particular emphasis on dominant and half-diminished seventh chords, harmonies which are prominent in the chromatic repertory and which share many characteristics with the consonant triads on which the neo-Riemannian theories have been built. This examination leads, in the second section of the paper, to the creation of a new model for these seventh chords. The paper closes with a preliminary exploration of some analytical applications of the new model.

# I

Example 1 shows an excerpt from Amfortas's "Agony" aria in the first act of Wagner's *Parsifal*. Although triads abound, they typically occur as integral parts of richer harmonies, often seventh chords. To approach this passage with neo-Riemannian tools, we are forced to simplify the harmonies to focus on the triads, throwing out the sevenths of dominant seventh chords and the roots of half-diminished seventh chords.[2] To be sure, this step does not seem outlandish, and the excerpt even seems to provide us with a justification: when the passage in the first measure of the example returns in the fourth measure (initiating what is essentially a repeat of the phrase, but transposed up a half step), the initial harmony is E minor, instead of the expected C♯ half-diminished seventh chord. Wagner has thrown out the root for us. Figure 1 shows a reduction of the first two measures, with the suspensions eliminated and the integral triads labeled (note that + and – represent major and minor, respectively). We can see from this sketch that the progression features pairs of hexatonic poles in a sequential pattern.[3] This brief analysis fails to account, however, for the descending thirds of the upper voices and the stepwise descent in the "tenor" voice. That these two elements exhibit strikingly smooth voice leading, a feature generally associated with neo-Riemannian transformations, suggests that something is being lost with the simplification of seventh chords into triads.

Another example casts an even brighter light on this problem. Example 2 shows the beginning of the *cadenza* from Chopin's Prelude in C♯ Minor, op. 45, and Figure 2 shows a reduction of the first two beamed groups. In neo-Riemannian terms, each pair of chords in the sequence is generated by a Relative transformation, but this labeling is not true to the voice leading of the passage. The Relative transformation involves holding the major third constant while moving the other voice by a whole step. The E in the A-major triad should move to the F♯ in the F♯-minor triad, but instead moves to the ignored D♯ while the F♯ comes from the ignored G♮. Chopin's smooth voice-leading clearly indicates that the triadic model is inappropriate; but the passage also hints that a different model could be constructed for seventh chords, one which would involve all of the pitches to ensure consistency with the composer's text. In particular, it is worth pointing out that the relationship between each chord in this *cadenza* and its predecessor is remarkably similar: two notes remain constant, while the other two descend by half step.

Cohn (1996, n. 40) points out that set classes 3-11 (consonant triads) and 4-27 (dominant and half-diminished seventh chords) are related by more than just simple inclusion. In particular, the two set classes represent minimal perturbations from symmetric divisions of the octave.[4] With

AMFORTAS

der dort dem Er - lö - - - - ser die Wun - - - de stach, aus

der mit blut' - gen Thrä - nen der Gött - li - che weint'... ob der

Example 1. Amfortas's "Agony" aria from Act I of Wagner's *Parsifal*, mm. 1369–72

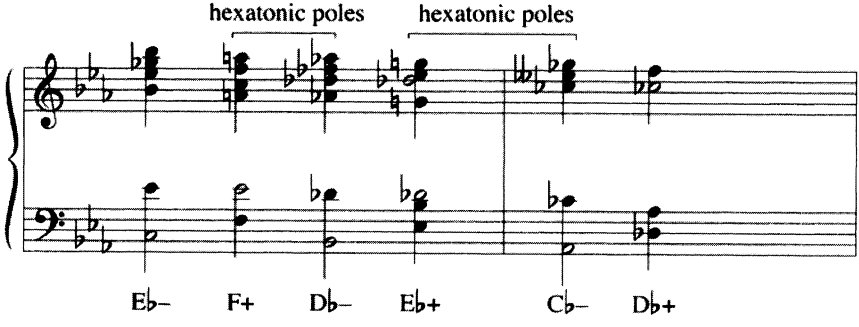
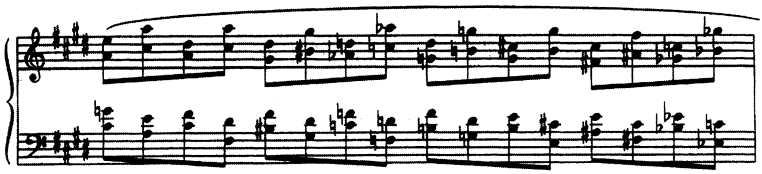



Figure 1. Reductive sketch of *Parsifal*, mm. 1369–70



Example 2. Beginning of the *cadenza* from Chopin's Prelude in C♯ Minor, op. 45

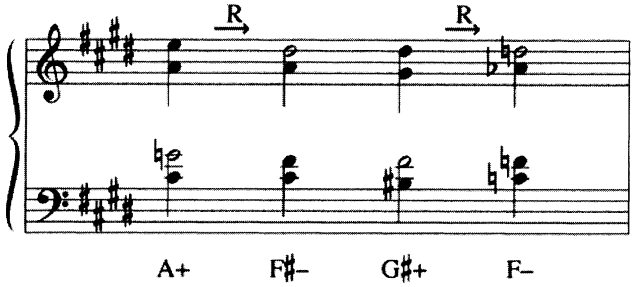



Figure 2. Reductive sketch for the Chopin *cadenza*

such minimally perturbed chords, it is possible to construct networks which involve only half-step voice leading. Such networks can be described with the $P_n$-relation (two chords are considered to be $P_n$-related if they differ by a half step in $n$ voices while the other voices remain constant).[5] Networks of $P_2$-relations among these chords are particularly easy to construct, moving between chords by returning the "perturbed" pitch to its symmetrical "home" position and then perturbing another pitch. The $P_2$-relation possibilities are even richer for set class 4-27, however, due to the set's cardinality. In addition to considering any member of the set class as a minimal perturbation of a fully-diminished seventh chord, we can compare it to a different fully-diminished collection, one from which it represents a three-voice change (by half step in each voice). Two of these voices can be "returned," creating another member of set class 4-27 in the $P_2$-relation with the first.[6]

These two $P_2$-relation properties are demonstrated in Figures 3 and 4. Figure 3a shows a symmetric fully-diminished seventh chord, and Figure 3b shows a minimal perturbation from that collection, an F dominant seventh chord (the perturbed pitch is represented with a filled-in notehead). In Figure 3c, the perturbed pitch is returned (solid arrow), and another pitch is moved (dotted arrow) to produce a different member of set-class

4-27. Figure 4a shows a symmetric fully-diminished seventh chord, and Figure 4b shows the same F dominant chord, now a three-fold perturbation from the collection in Figure 4a. In Figure 4c, two of the perturbed pitches are returned, producing a different member of set class 4-27. That set class 4-27 exhibits both of these $P_2$-relating properties suggests a transformational system for dominant and half-diminished seventh chords which would allow all four pitches to participate in parsimonious voice leading.

## II

Figure 5 demonstrates a system of seventh-chord transformations which grows directly from the analytical and theoretical considerations described above. This system consists of two distinct families of operations. The larger family is that of the S transforms, which involve holding two pitches constant while the other two move by half step in similar motion. Like the neo-Riemannian operations, each of these six transformations results in a change of mode and is involutional in nature. The individual transformations are labeled with a subscript that indicates the interval class between the two pitches being held constant and a parenthetical subscript that indicates the interval class of the two pitches that move.[7] The second family is that of the C transforms, which involve contrary motion for the non-fixed pitches. The subscripts for the three members of this family follow the same labeling convention. Since the C transforms maintain chord quality, only $C_{6(5)}$ is an involution. $C_{3(2)}$ and $C_{3(4)}$ are each other's inverses. Together, these nine transformations represent all of the possible $P_2$-relations among individual members of set class 4-27.

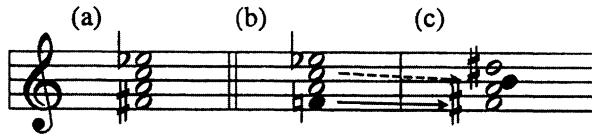



Figure 3

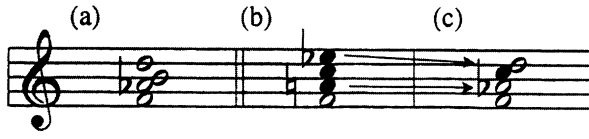



Figure 4

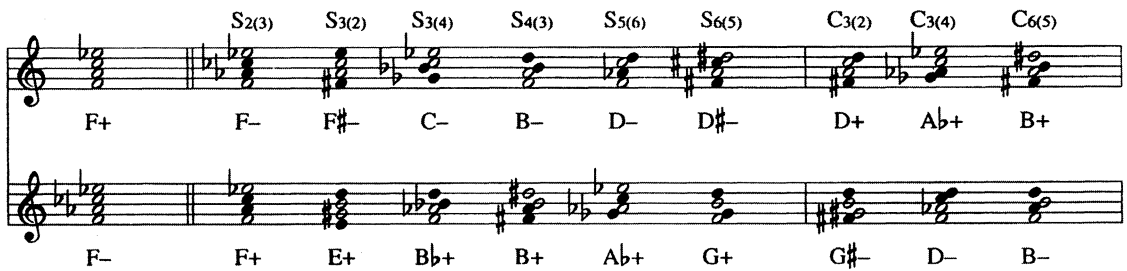



Figure 5. A system of transformations for dominant and half-diminished seventh chords (set class 4-27). + and – refer to dominant and half-diminished qualities, respectively. F+ and F– are taken as the initial chords in each example. Notes which are held constant have open noteheads, while those that move are represented by filled-in noteheads.

The parsimonious nature of these transformations also allows for the formation of various networks, one of which is demonstrated in Figure 6. This cubic network contains eight members of set class 4-27 which are subsets of the same octatonic collection. The cube involves three of the S transforms, which form the solid edges, and the three C transforms, which form the dotted-line diagonals on the faces. Each chord is adjacent to all of the other chords, with the exception of the chord that is directly opposite it on the cube, its "octatonic pole." This octatonic grouping of seventh chords is quite similar to the hexatonic grouping of triads explored by Cohn (1996), and two cyclic subgraphs of this cube—one formed by the alternation of $S_{2(3)}$ and $S_{5(6)}$ transforms, the other, by alternating $S_{4(3)}$ and $S_{5(6)}$ transforms—are analogous to the hexatonic Cohn cycle.[8] Another cubic network can be constructed by replacing each of the S transforms with its complement (that is, the transformation which moves the pitches that the original keeps fixed; the complement of $S_{a(b)}$ is $S_{b(a)}$). This cube is an analog to the Weitzmann region discussed in Cohn 2000—each constituent seventh chord is a minimal perturbation from the same fully-diminished seventh chord. It seems unlikely that any composition would be so complicated as to involve the complete network of one of these cubes, but there are many interesting subgraphs (both cyclic and not) which could be explored.

Expanding beyond these networks, we can examine the functional composition of these transformations in a more general sense. Since each C transform can be created by combining two S transforms, we can focus exclusively on the latter without loss of generality.[9] Figure 7a provides a chart that demonstrates the results of applying all double-S operations to a dominant seventh chord on C. Since the S transforms cause a change of mode, double-S transforms maintain the original mode. We can see from the summary in Figure 7b that all twelve of the same-mode chords can be reached through a double-S transform (including, of course, returning to the original chord through the duplication of any one involutional transformation). This means that it is possible to move from any member of set class 4-27 to any other member of the same modality by means of just two transformations, and to any member of the opposite modality with three (simply by appending a root-preserving $S_{2(3)}$ transform to the appropriate double transformation).[10] Each double-S transform can also be used as the source for a cycle of alternating single transforms. Since the double-S transforms exhaust the same-mode possibilities, the lengths of the implied cycles will include all of the divisors of 12 if we count each double-S transform as a single step (or all the even divisors of 24, counting each component transformation individually). This phenomenon is summarized in the second row of Figure 7b.[11] Of particular note are those cycles that contain all 24 members of set class 4-27. These include cycles created by alternation of complementary pairs (which exhaust the chords

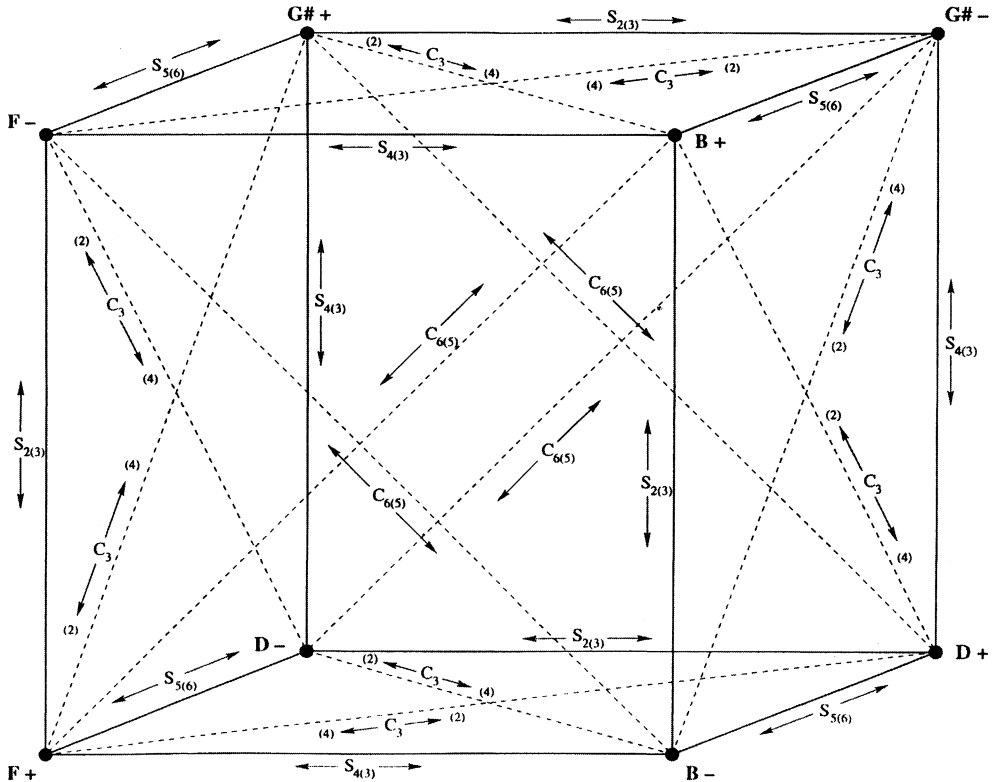



Figure 6. A cubic network which obtains among seventh chords within one octatonic collection

(a) Dominant seventh chords resulting from the application of a double-S transformation to a dominant seventh on C

| | $S_{2(3)}$ | $S_{3(2)}$ | $S_{3(4)}$ | $S_{4(3)}$ | $S_{5(6)}$ | $S_{6(5)}$ |
|---|---|---|---|---|---|---|
| $S_{2(3)}$ | C | B | F | F♯ | E♭ | D |
| $S_{3(2)}$ | C♯ | C | F♯ | G | E | E♭ |
| $S_{3(4)}$ | G | F♯ | C | C♯ | B♭ | A |
| $S_{4(3)}$ | F♯ | F | B | C | A | A♭ |
| $S_{5(6)}$ | A | A♭ | D | E♭ | C | B |
| $S_{6(5)}$ | B♭ | A | E♭ | E | C♯ | C |

(b) Frequency of appearance and length of implied cycle (in individual transformations) for double-S transformations

| | C | C♯ | D | E♭ | E | F | F♯ | G | A♭ | A | B♭ | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| frequency | 6 | 3 | 2 | 4 | 2 | 2 | 4 | 2 | 2 | 4 | 2 | 3 |
| cycle length | 2 | 24 | 12 | 8 | 6 | 24 | 4 | 24 | 6 | 8 | 12 | 24 |

Figure 7

chromatically) and those which traverse the circle of fifths (alternating $S_{2(3)}$ and $S_{3(4)}$ transforms, for example).

## III

Let us now return to the musical examples. Figure 8 shows a new reduction of the Chopin excerpt, relabeled with +/– seventh chords and S transforms. The problem with voice-leading consistency has been eliminated, and the profile of the sequence is clearly outlined by the alternating complementary $S_{4(3)}$ and $S_{3(4)}$ transforms. Figure 9 shows a relabeled reduction of the Wagner excerpt, which appears to consist of every other pair from a descending alternating sequence of the same complementary transformations. The triple-transform which passes between the pairs of chords previously identified as hexatonic poles (in the triadic analysis) provides an elegant explanation for the smooth voice-leading features which were noted previously. The single $S_{3(4)}$ transform accounts for the descending major third (interval class 4) in the upper two voices, while the pair of $S_{4(3)}$ transforms describes the descending whole-step motion

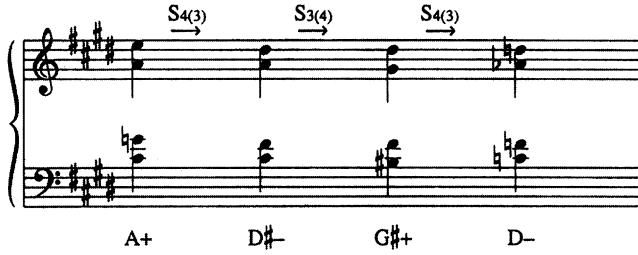



Figure 8. New reductive sketch for the Chopin *cadenza*

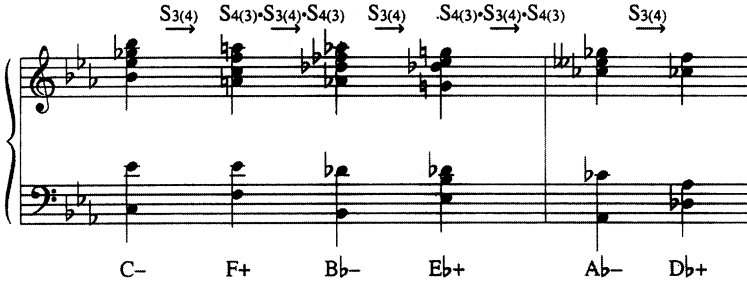



Figure 9. New reductive sketch for *Parsifal*

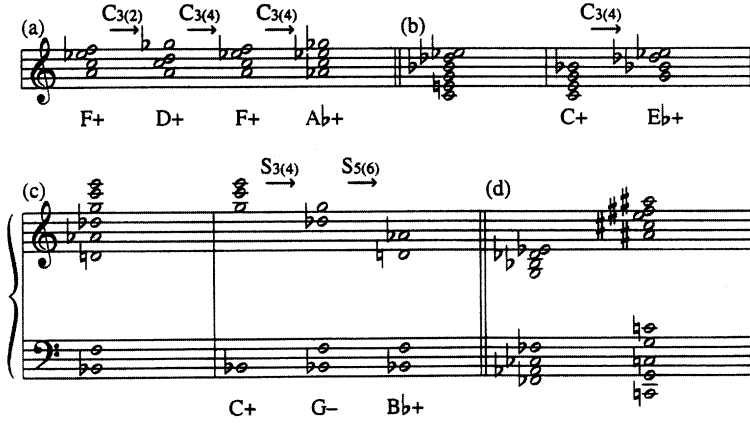



Figure 10. Excerpts from Stravinsky's *Rite of Spring*

of the tenor voice, in conjunction with the bass line (in minor tenths (interval class 3) on beats one and three of each measure).

Since this transformational model is based entirely on the set-theoretic properties of seventh chords (as opposed to their acoustic properties), it is also a useful tool for exploring atonal works in which set class 4-27 figures prominently. Stravinsky's *Rite of Spring* is one such work, and Figure 10 shows reductions of several relevant excerpts. Figure 10a shows a chord progression heard in the high violins, four measures after RN44.[12] The alternation between various dominant-seventh harmonies is achieved via parsimonious voice-leading, represented by $C_{3(2)}$ and $C_{3(4)}$ transforms. This collection of dominant seventh chords related by minor third is a prominent feature of the *Rite*, and can be represented as a triangular or tetrahedral subgraph of the cubic network in Figure 6, utilizing only the + nodes and the dotted C-transform lines. More complex harmonies can also be modeled by treating them as "vertical progressions," combining two or more S- or C-related seventh chords into a single simultaneity. Figure 10b shows a six-note chord played by the violins at RN37. The second measure of the figure shows that it can be divided into two component seventh chords related by the $C_{3(4)}$ transform. Figure 10c shows a combined harmony from the music at RN87 (the opening of the second section of the *Rite*). The second measure again shows that it can be divided into three seventh-chord components, related by the $S_{3(4)}$ and $S_{5(6)}$ transforms. Finally, Figure 10d shows how these two techniques can be combined. The first harmony is the infamous *Sacre* chord which contains an E♭ dominant seventh chord in its upper voices. When the repeated chords finally end (two measures before RN22), the subsequent harmony is a combination of F♯ and C dominant seventh chords, chords that are related to the E♭ chord by $C_{3(4)}$ and $C_{3(2)}$ transforms, respectively, and which are related to each other by a $C_{6(5)}$ transform.

We have seen how certain properties which dominant and half-diminished seventh chords share with the consonant triads have enabled the creation of a transformational model for these harmonies, and how certain properties unique to the seventh chords have imbued the model with additional possibilities. Because of its ability to track all four voices of the chords accurately, this model is more powerful in dealing with seventh chords than one based solely on neo-Riemannian triadic transformations. We have also examined the functional composition of the component transformations from the model, discovering that the individual operations can exhaust all harmonic possibilities in just three successive applications. This breadth of potential surely helps to explain the popularity of these harmonies in the later tonal repertory. Finally, we have explored a few preliminary avenues for the analytical application of the model, including expansions beyond that repertory.

# NOTES

An earlier version of this paper was read at the University of Wisconsin at Madison, Graduate Student Music Forum Symposium, April 5, 1997. I wish to thank David Clampitt, John Clough, and Richard Cohn, whose comments on previous drafts were especially helpful. Thanks also to Craig Wiegert, for the graphic rendering of Figure 6.

1. See especially Cohn 1996, Hyer 1995, and Lewin 1987 and 1992.
2. Throughout this paper, the word "root" is used in its traditional definition, and seventh chords will be labeled using their traditional roots. For an exploration of these seventh chords which incorporates notions of Riemannian dualist labels, see Gollin 1998.
3. For a definition and examination of hexatonic systems (including the notion of a hexatonic pole), see Cohn 1996.
4. Another set class with this property is Scriabin's mystic chord (set class 6-34). See Callender 1998 for further discussion of the properties of this collection.
5. The $P_n$-relation was first developed by Douthett (unpub.) and presented and explored in Lewin 1996. Some of the implications of $P_2$-networks (and their cyclical sub-networks) for consonant triads are alluded to in Cohn 1996 and 2000. A more generalized definition of this relation is fleshed out in Douthett and Steinbach 1998. As they point out, my $P_n$ becomes $P_{n,0}$ in their new model.
6. Among the three non-trivial set classes which share this minimal perturbation property, only set class 4-27 can participate in $P_2$-relations in this second manner. The analogous operation produces $P_1$-relations (specifically, the P and L neo-Riemannian transformations) in triads and $P_4$-relations in mystic chords. The generalized property produces $P_{(k-2)}$-relations in sets of cardinality $k$.
7. For most of these transformations, the first subscript alone is sufficient as a unique label to describe the operation—only those which involve interval class 3 require additional clarification, since that interval class appears twice in any member of set class 4-27. However, the use of both subscripts for all of the transformations makes their complementary grouping (into pairs of operations, one of which moves the pitches that the other maintains constant) abundantly clear. I am grateful to Charles J. Smith for suggesting this more complete system of nomenclature.
8. For a deeper discussion of the analogous properties of the hexatonic and octatonic collections with respect to set classes 3-11 and 4-27, respectively, see Douthett and Steinbach 1998.
9. Based on this fact, one could argue that the C transforms are somehow unnecessary to the system. However, they must be included as independent and equal members if all $P_2$-relations are to be considered. In a sense, the C transforms arise from an added degree of freedom (in comparison to the PLR family of triadic transformations) created by the addition of a fourth pitch and a second moving pitch.
10. While the return of the C transforms to the system expands the possible chords which can be reached with each $n$-tuple functional composition, it does not allow the exhaustion of all 24 chords (or the 12 same-mode chords) in fewer steps.
11. It is worth noting that the various pairs of transformations which have the same result arrive at their target chords with the same overall voice-leading change. Since each S operation involves moving only two voices by half step, the double-

S transforms represent the minimal change possible in moving between the initial chord and the target chord.

12. All references are to rehearsal numbers (RN) as found in the edition originally published by Izdatel'stvo "Muzyka," Moscow, available in reprint from Dover Publications.